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How to take into account the dynamics of mathematical activity?

Pedagogical and didactic constraints in teaching inquiry-based activities

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PERSONA CIÈNCIA EMPRESA
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IQS SCHOOL OF MANAGEMENT

Outline

1. 'Monumentalism' and the case of elementary functions
2. Towards the new paradigm of 'Questioning the world'
3. A ***study and research path*** (SRP) about sales forecasts
4. Analysis of the SRP:
    1. Epistemological analysis: modelling process
    2. Epistemological analysis: Herbartian schema
    3. Didactic analysis: conditions
    4. Didactic analysis: constraints
    5. Ecological (institutional) analysis
5. SRP in teachers professional development

1. “Monumentalism”

THE PARADIGM OF VISITING WORKS AND ITS SHORTCOMINGS

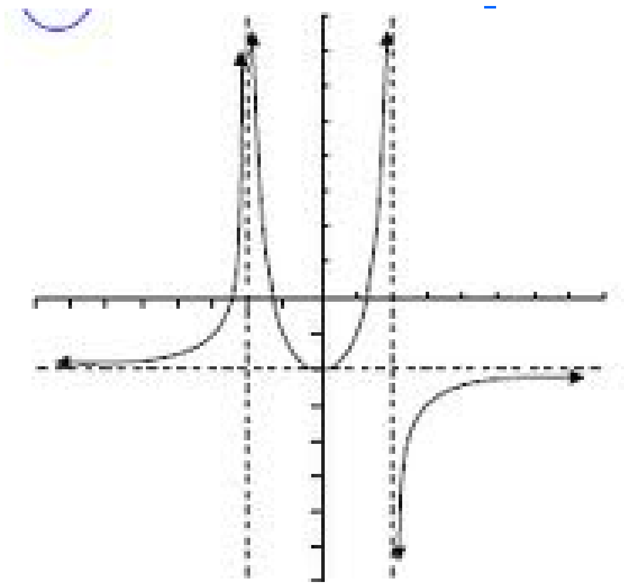
The prospective view on the didactic dimension in our societies […] can be encapsulated in a crucial historical fact: the old didactic paradigm still flourishing in so many scholastic institutions is bound to give way to a new paradigm yet in infancy. To cut a longer story short, I define a didactic paradigm as a set of rules prescribing, however implicitly, what is to be studied and what the forms of studying them are.

Y. Chevallard (2012) **Teaching mathematics in tomorrow’s society: a case for an oncoming counterparadigm**

Functions as monuments

What is done at school with functions?

- Some families are studied: linear, quadratic, rational, exponential, trigonometric, etc.
- A highly standardised technique is applied: curve sketching



- Domain
  Range
  Intercepts
  Symmetry, continuity
  Limits and asymptotes
  Derivative
  Extrema
  Graph

Once the function is sketched, the work is done. The graph is not often used to solve problems; functions are always sketched with the same precision; ...

Functions as monuments

What is done at school with functions?

Rigid notation: *f*(*x*) = ...   , *y* = ...

Functions are not identified with formulae, nor are formulae interpreted in functional terms:

$$V = \pi R^2 h \rightarrow h = \frac{V}{\pi R^2}$$

$$F = G\,\frac{m \cdot m'}{d^2}$$

No clear, explicit relationship between functions and:

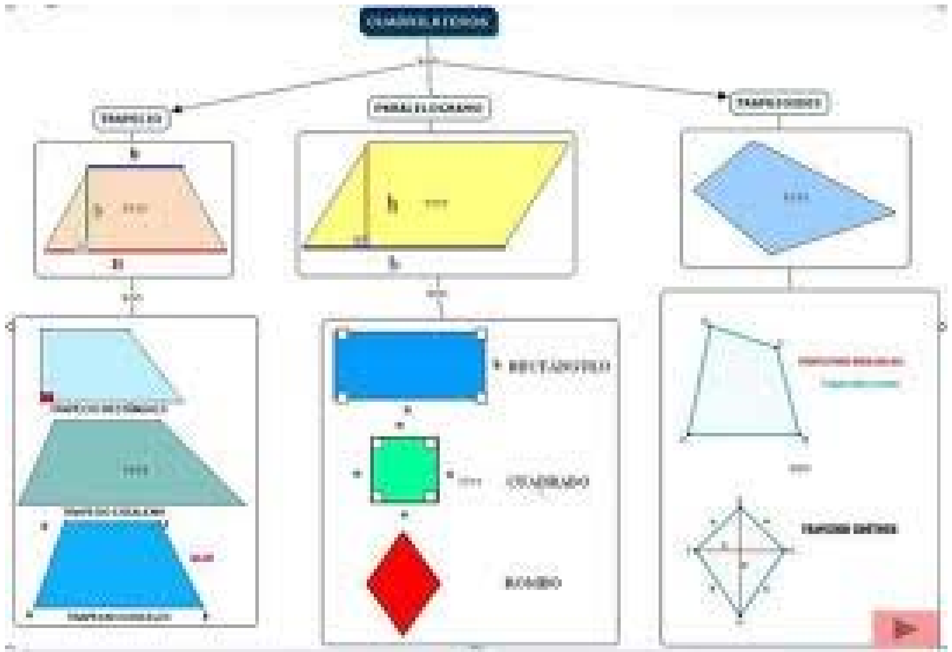
- Algebra, equations
- inequalities
- Geometry
- Physics: kinematics, mechanics, etc.

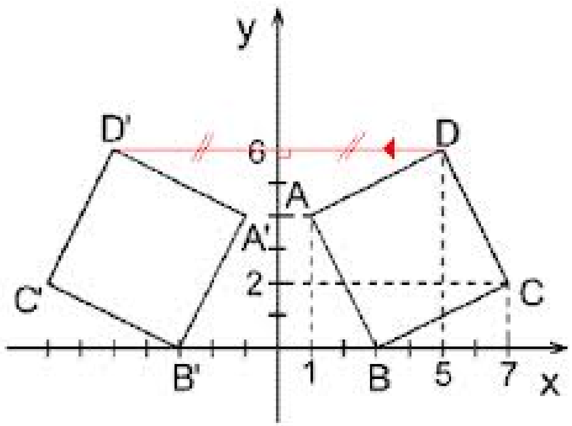
There are many other monuments...

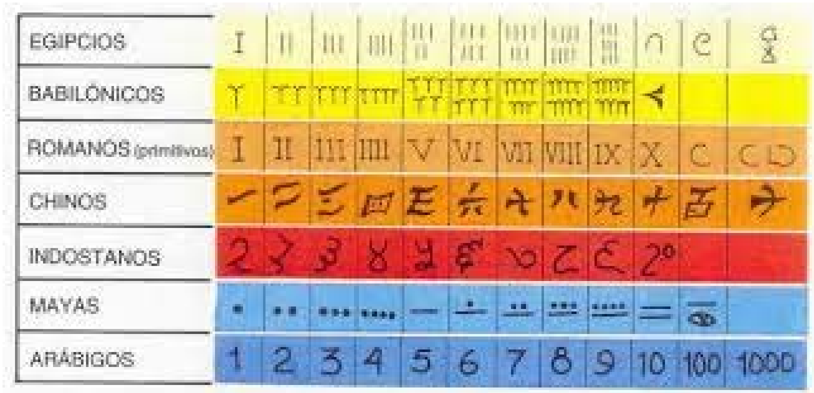
Quadrilaterals

Numeral systems

Transformations of the plane ...







Learning = “visiting works”

What is done at school with functions?

- Students need to assume that functions are very important objects (“monuments”) without really knowing what they are for. They will see that later… or not!
- They have to accept the duty of “visiting” the monuments and doing something with them, “simulating” their use
- They usually take for granted that the “real” (professional) way of using them is not the school one
- In old days, teachers “talked” about monuments (lecture); today it is students who handle and manipulate them,

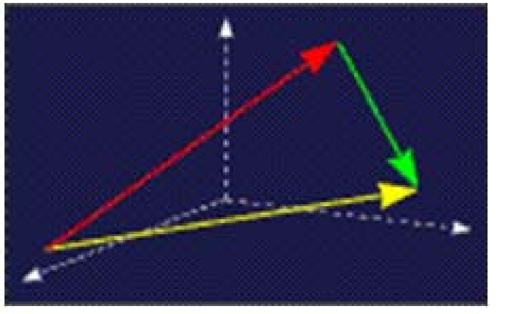
  but has the situation really changed?

PARADIGM OF VISITING WORKS OR “MONUMENTALISM”

Learning = “visiting works”
with modern tools, of course!

Gallery

maths online



The learning units of the Gallery

Functions 1

- Function and graph
- Graphs of simple power functions
- Recognize functions 1 (simple polynomial functions)
- Recognize graphs 1 (functions of at most second order)
- Polynomial of third order (cubic polynomial)
- Function plotter

Trigonometric functions

- Definition of the trig functions

Functions 2

- Recognize functions 2 (functions containing negative powers)
- Recognize graphs 2 (functions containing negative powers)
- The graphs of sin, cos and tan
- Graphs of elementary trigonometric functions
- Graphs of some exponential and logarithm functions
- Recognize functions 3 (sine and cosine functions)
- Recognize graphs 3 (sine and cosine functions)
- Function plotter

Differentiation 1

- On the definition of the derivative
- Derivative puzzle 1
- Derivative puzzle 2
- Derivative puzzle 3
- First and second derivative

Applications of differential calculus

- How to find a function's extremum

Differentiation 2

- Nowhere differentiable functions

2. ‘Questioning the world’

QUESTIONING THE WORLD: TOWARDS A NEW DIDACTIC PARADIGM

Up to a point, we might soon ditch the current didactic world in favour of a new paradigm which, when contrasted with the old one, looks like a *counterparadigm* […] What the new didactic paradigm aims to create is a new cognitive ethos in which, when any question *Q* arises, *x* will consider it, and, as often as possible, will *study* it in order to arrive at a valuable answer *A*, in many cases with a little help from some *y*.

→ Procognitive attitude and exoteric relation to knowledge

Y. Chevallard (2012) **Teaching mathematics in tomorrow’s society: a case for an oncoming counterparadigm**

2. ‘Questioning the world’

- **Transmission of contents *VS* study of questions**

  Syllabi are made of lists of themes or topics, structured in sectors and domains, within a discipline

  VS

  Syllabi contain sets of questions (no necessarily within specific disciplines) to whom answers need to be provided

What are functions? → What are functions *for*?

How to make a prediction from a time-series?

How to prepare a saving plan? How does a flu epidemy disseminate? How does a painkiller act? Etc.

Transmission of contents *VS* study of questions

José Manuel Estévez (2003)

*It has no sense **to give answers to those who have never asked the question;** therefore, **the basic task of the teacher is to recover the questions, concerns**, the process of searching for the men and women who developed the knowledge now listed in our books. [...] We need to give up with the professions of faith in the organised answers of our books. We should turn our students' looks towards the world around and **rescue the initial questions**, making them think.*

Transmission of contents *VS* study of questions

Hans van Ginkel (2004)

*Another effort that has to be made regarding curricula concerns the **problematisation of issues**. All too often university courses limit themselves to themes […] without making the effort to inquire **what issues form the core of these themes**. I believe this lack of problematisation largely accounts for the slow progress of **true understanding**. Naming the theme and going for the easy answer tend to become in this context an institutional alibi – an alibi which does not necessarily serve the advancement of knowledge and research. The problematisation of issues also helps to **bring the links between the different issues** into focus. Without a clear understanding of these interlinkages no effective understanding and management of issues will be possible.*

Herbartian schema

- **A general point of view:**
  - Johann Friedrich Herbart (1776-1841), German philosopher and pedagogue

    The university professor no longer teaches, the student is no longer taught; instead, he pursues personal research, the professor's task being to guide and advise him in this pursuit.
  - **Teaching: leading the study and research of problematic questions**
  - **Learning: the "product" of the study, obtaining/elaborating answers to the studied questions and being able to ask new questions**

Herbartian schema

Study and research paths (and activities)

MILIEU M

SRA

STUDENT(S)
TEACHER(S)
QUESTION Q

→

Answers $A_j^{\lozenge}$
Objects O_k
Questions Q_i

→

Own and 'functional' answer $A^{\heartsuit}$

⇕

'Alive' question
Not just an excuse to visit some pre-established works

UNITARIAN MODEL: can be used to describe 'small' processes (the study of a theme) as well as 'big' ones (PhD, etc.)

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3. A SRP on sales forecast in first year university

- The class becomes a **mathematical consultancy**
- Students work in **groups of consultants** (3 or 4)
- All the class approaches the same question
- The initial question can give raise to different sub-questions that can be distributed among the groups
- **Partial progress reports** are elaborated during the study and, at the end, a **common final answer** has to be written and submitted to the client by each group
- **Share of responsibilities** teacher-students: planning the work to do, searching sources of information, formulating intermediate questions, obtaining provisional answers, discussing and validating the results, etc.

3. A SRP on sales forecast

We are given some data about the evolution of shops, sales and incomes of Desigual company. We are asked to provide a short and long-term forecast for all the variables:

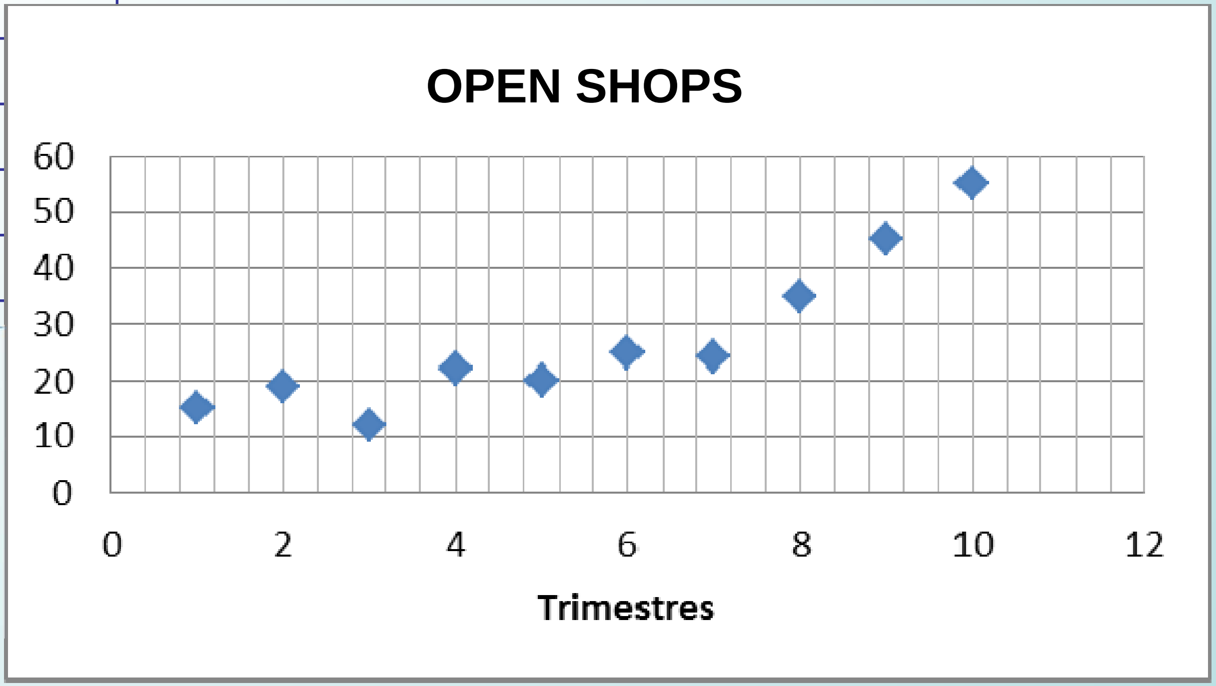
1. Number of open shops in Spain
2. Number of open shops outside Spain
3. Yearly sales in millions euros
4. Weekly incomes (€) in Passeig de Gràcia shop
5. Weekly incomes (€) in Rambles shop
6. Weekly incomes (€) in Rbla Catalunya shop
7. Weekly incomes (€) in Carrer Arcs shop
8. Weekly incomes of one-print T-shirts (Pg Gràcia)
9. Weekly incomes of one-print T-shirts (Rambles)
10. Weekly incomes of one-print T-shirts (R. Cat.)
11. Weekly incomes of one-print T-shirts (Arcs)

Case 1. Open shops in Spain

Q0: How many shops will open in the next terms?

Term	Open shops
T1 2008	15
T2 2008	19
T3 2008	12
T4 2008	22
T1 2009	20
T2 2009	25
T3 2009	24
T4 2009	35
T1 2010	45
T2 2010	55

Newly open of cumulative data?
X or ΔX ?



Q1 : Which function fits the data best?

LINEAR 1
y = ax + b

a =	4
b =	5

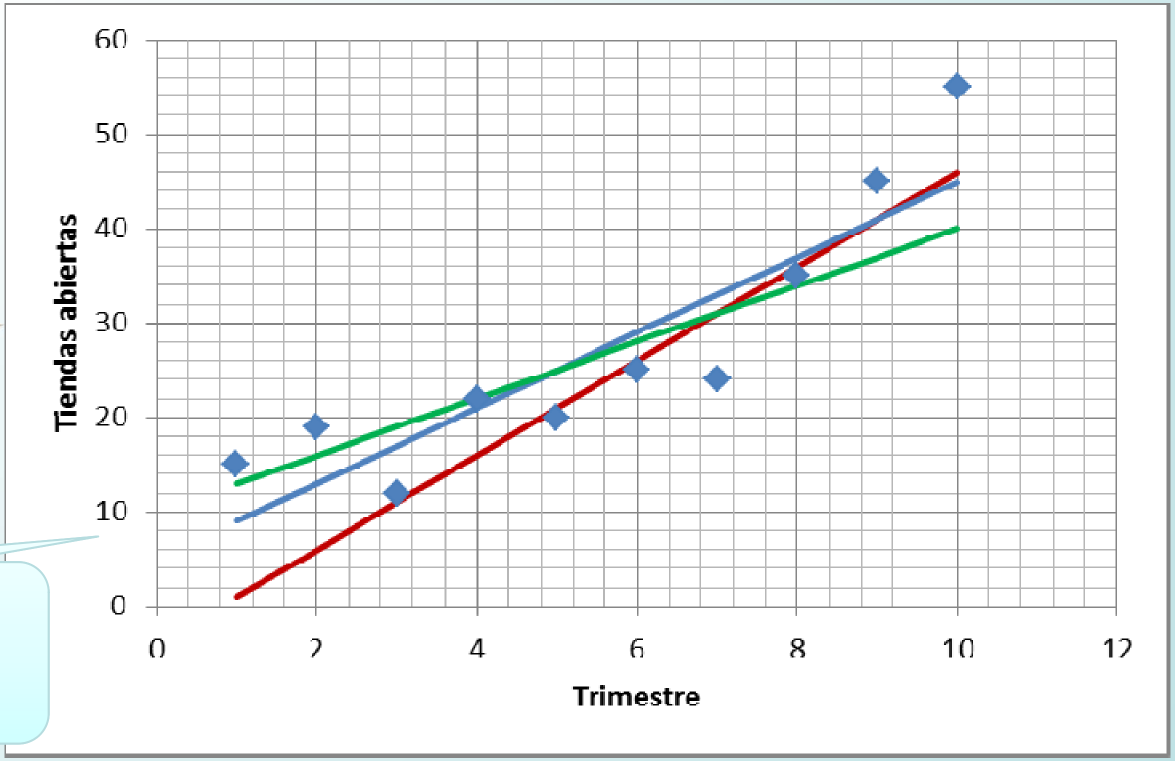
LINEAR 2
y = ax + b

a =	5
b =	-4

LINEAR 3
y = ax + b

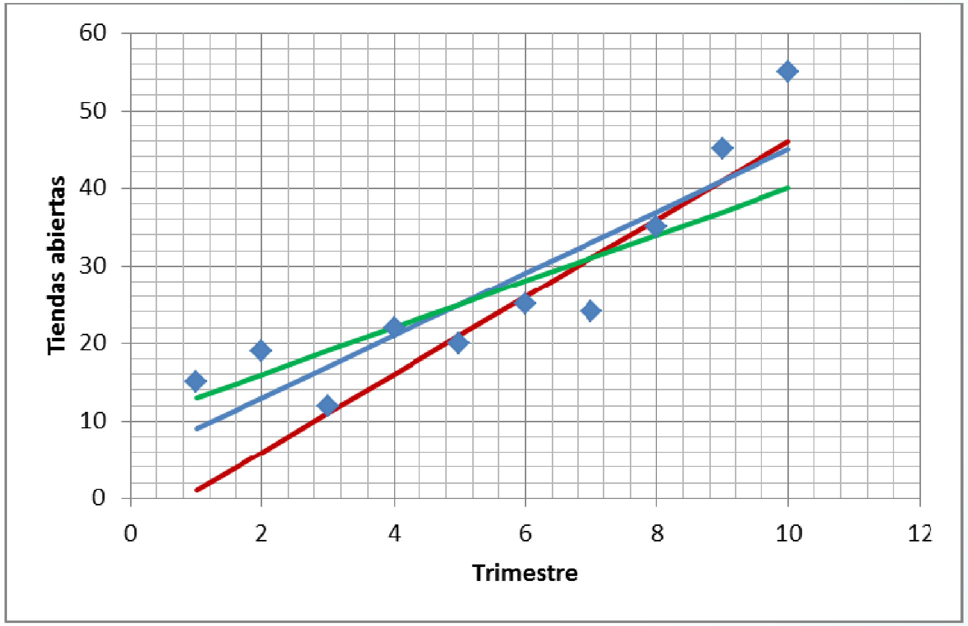
a =	3
b =	10

Term	Shops	Forecast
1	15	9
2	19	13
3	12	17
4	22	21
5	20	25
6	25	29
7	24	33
8	35	37
9	45	41
10	55	45



Q2: Which linear function fits best?

Q3: What does "fitting best" mean?



Which function is better?

> The one closest to the data (point cloud)

How to calculate the distance between a function and a set?

> Total (or average) 'distance' between each value of the function and the corresponding data

Distance? Difference? Absolute value? Squared?

Q1' : Why not trying a parabola, exponential, hyperbolic function?

PARABOLA 1

$y = ax^2 + bx + c$

a =	**0,5**
b =	**-1**
c =	**15**

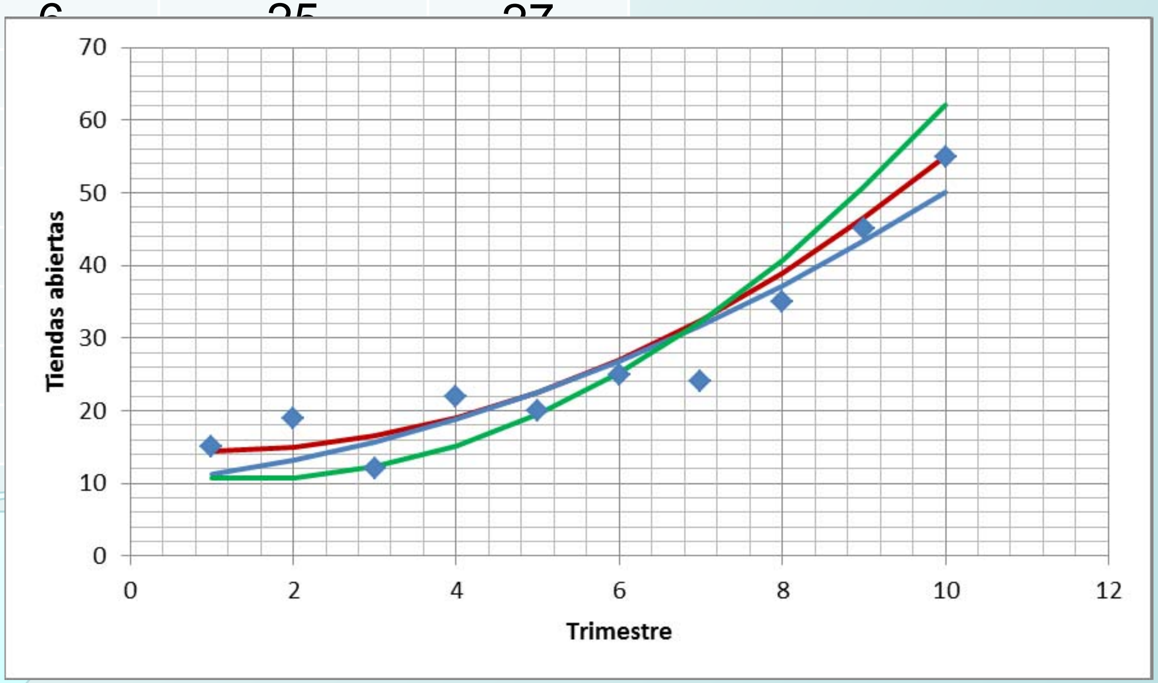
Term	Shops	Forecast
1	15	15
2	19	15
3	12	17
4	22	19
5	20	23
6	25	27
7	24	33
8	35	39
9	45	47
10	55	55
11		65
12		75
13		87

PARABOLA 2

$y = ax^2 + bx + c$

a =	**0,3**
b =	**1**
c =	**10**

Term	Shops	Forecast
1	15	11
2	19	13
3	12	16
4	22	19
5	20	23



Q5: Does a cubic/exponential/rational function fit best?

LINEAR 1

T	Shops	Forecast	Error
1	15	9	6
2	19	13	6
3	12	17	-5
4	22	21	1
5	20	25	-5
6	25	29	-4
7	24	33	-9
8	35	37	-2
9	45	41	4
10	55	45	10

2

What function fits best?

LINEAR 1 → 39
LINEAR 2 → 51
LINEAR 3 → 45
QUADRATIC 1 →14
QUADRATIC 2 →17
QUADRATIC 3 →36
EXPONENTIAL 1 →15
EXPONENTIAL 2 →24
EXPONENTIAL 3 → 18

We do not want a 'compensation' between positive and negative errors

The sum of |error| has to be minimized

New questions found

- Once we have ‘the best linear function’, ‘the best quadratic’, ‘the best exponential’, how to compare them? Comparing the sum of |errors|? Are there other criteria?
- Excel includes a ‘tendency line’ in the graphs. It is similar to our best forecast function, but not exactly the same. How is it calculated? Why?
- In the case of quadratic errors, when using the Excel function SOLVER (minimizing), the sum of errors is 0 in the case of straight lines and quadratic functions, but not with exponentials. Why?
- The forecast can be realistic for the short term. What to do for the long term? How to build S-functions?
- …

New questions to go on

- Does this way of making forecast exist outside school? Does it has a name? Are there other techniques?

- A grup of students has started considering the increase of open shops per term. How do the results change?

→ IDEA: make a double forecast, with the total number of shops (X) and with the term variation of shops (ΔX), then relate it to the derivative of the functions used as models

$$X \leftrightarrow f(x) \qquad\qquad \Delta X \leftrightarrow f'(x)$$

How does a SRP end?

- A final report with the final results is submitted:
  oral presentation or poster + written report

- Some works and disciplines have been related to the answer provided: *ordinary least squares regression*, econometrics, forecast techniques, time series,…

- **How can the final result be reinvested in future situations? How to ensure that a similar work with a similar case can be done? Where to “keep it”?**

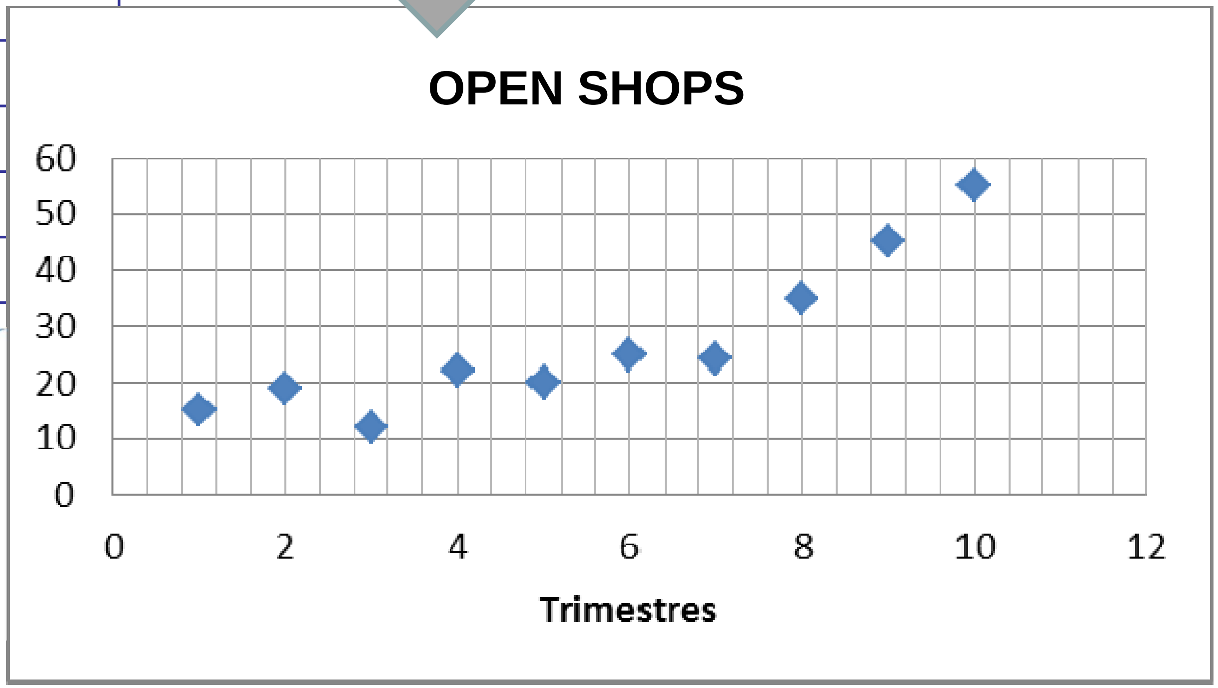
4. Analysis of the SRP

Term	Open shops
T1 2008	15
T2 2008	19
T3 2008	12
T4 2008	22
T1 2009	20
T2 2009	25
T3 2009	24
T4 2009	35
T1 2010	45
T2 2010	55

Q0: How many shops are being open in the next terms?

SYSTEM (data)

MODEL (graph)



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LINEAR 3

y = ax + b

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b =	10

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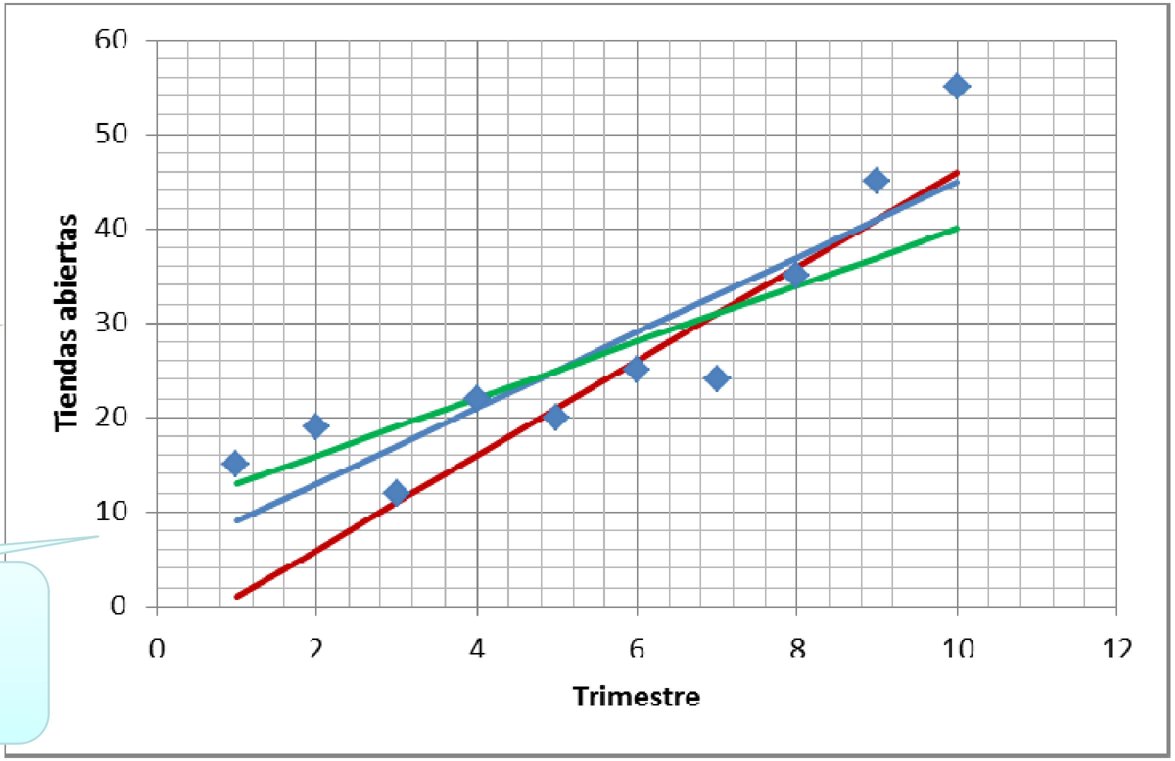
5	20	25
6	25	29
7	24	33
8	35	37
9	45	41
10	55	45

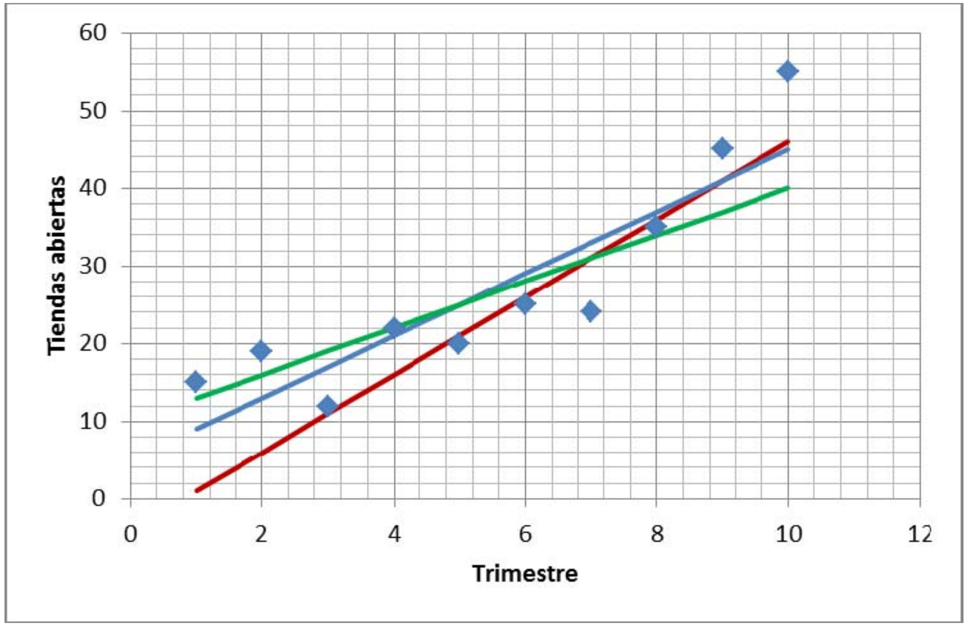
MODELS: linear functions + graphs

SYSTEM: data + point cloud



Q2: Which linear function fits best?

Q3: What does "fitting best" mean?



Which model is better?

> The function which is closest to the data (point cloud)

How to calculate the distance between a function and a set?

> Total (or average) 'distance' between each value of the function and the corresponding data

> Distance? Difference? Absolute value? Squared?

MODELS: 'distance'

The relationship between the model and the system is 'mathematised'

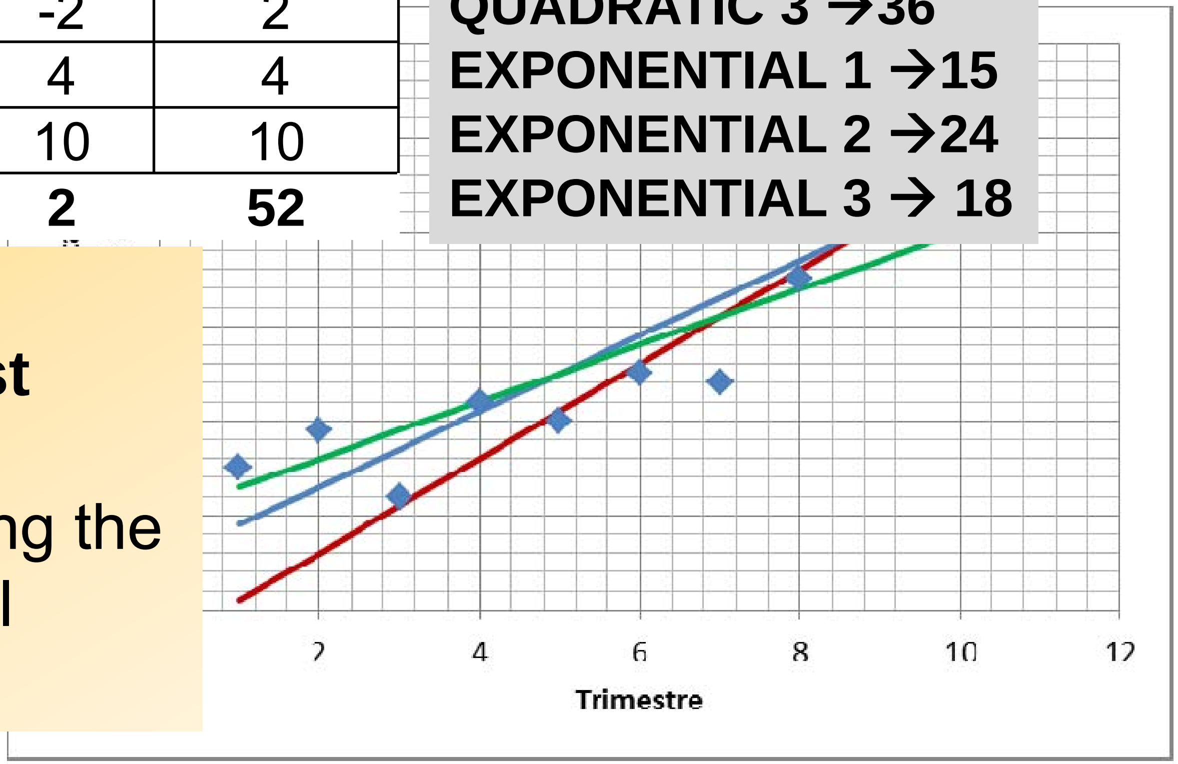
SYSTEM: data + point cloud + functions

LINEAR 1

T	Shops	Forecast	Error	|Error|
1	15	9	6	6
2	19	13	6	6
3	12	17	-5	5
4	22	21	1	1
5	20	25	-5	5
6	25	29	-4	4
7	24	33	-9	9
8	35	37	-2	2
9	45	41	4	4
10	55	45	10	10
			2	**52**

"FIT" = sum of absolute errors

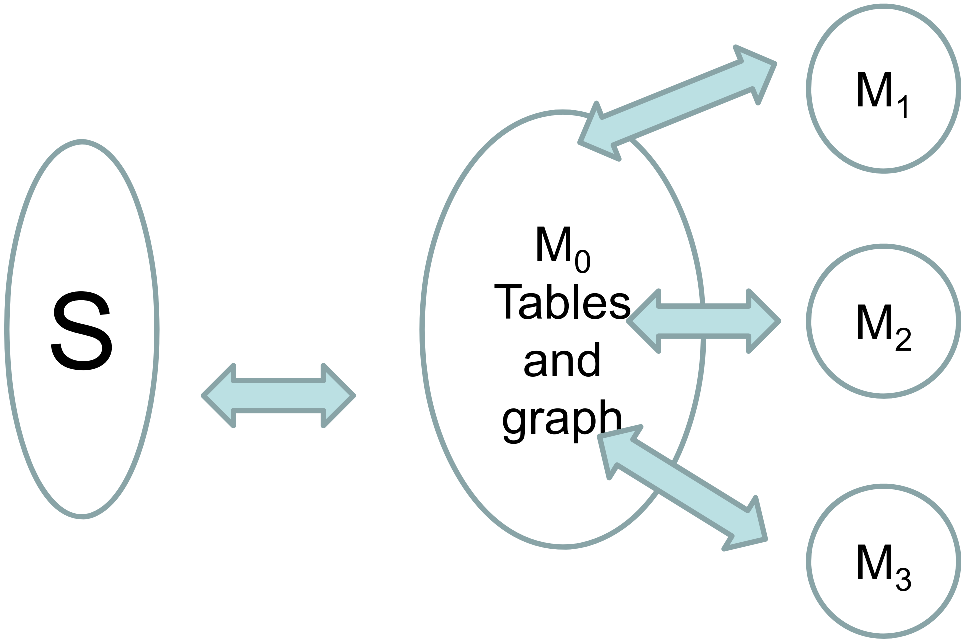
LINEAR 1 → 39
LINEAR 2 → 51
LINEAR 3 → 45
QUADRATIC 1 →14
QUADRATIC 2 →17
QUADRATIC 3 →36
EXPONENTIAL 1 →15
EXPONENTIAL 2 →24
EXPONENTIAL 3 → 18



The question of considering the best model has been mathematized: finding the minimum of a several variables function.

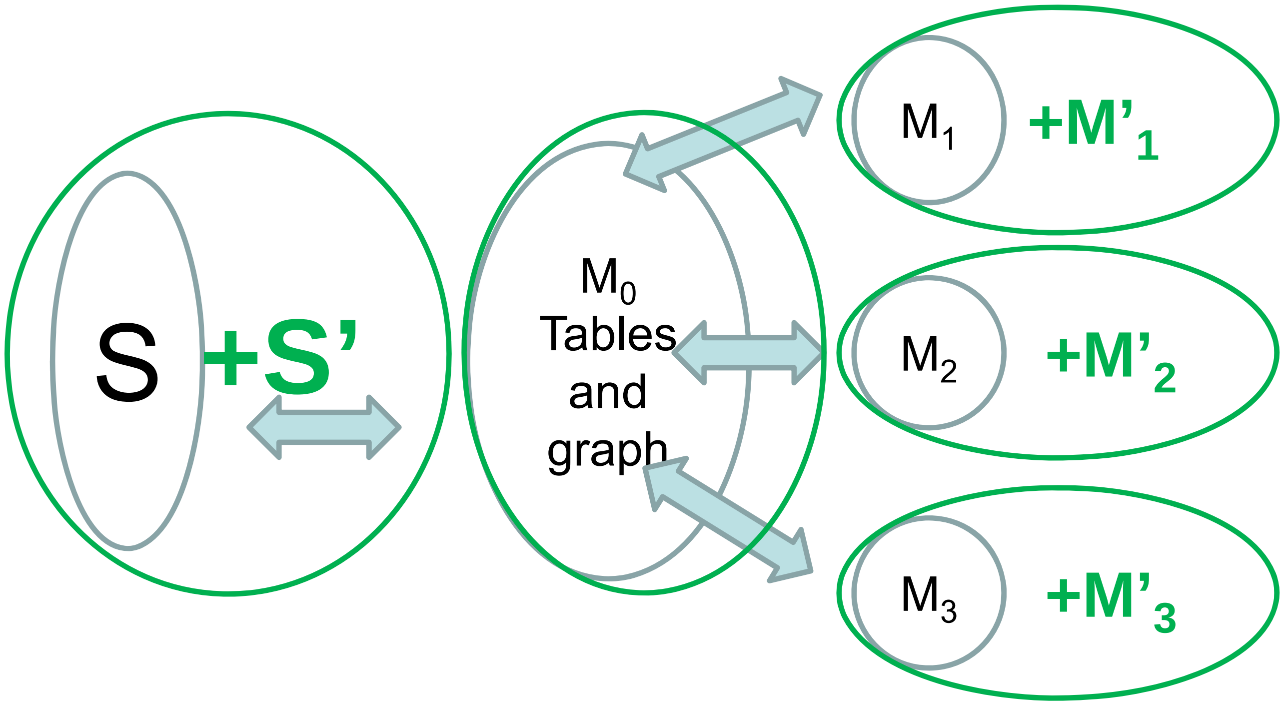
4. Analysis of the SRP

4.1. Modelling process



What model fits best?

→The average error measures the quality of the fit (model)



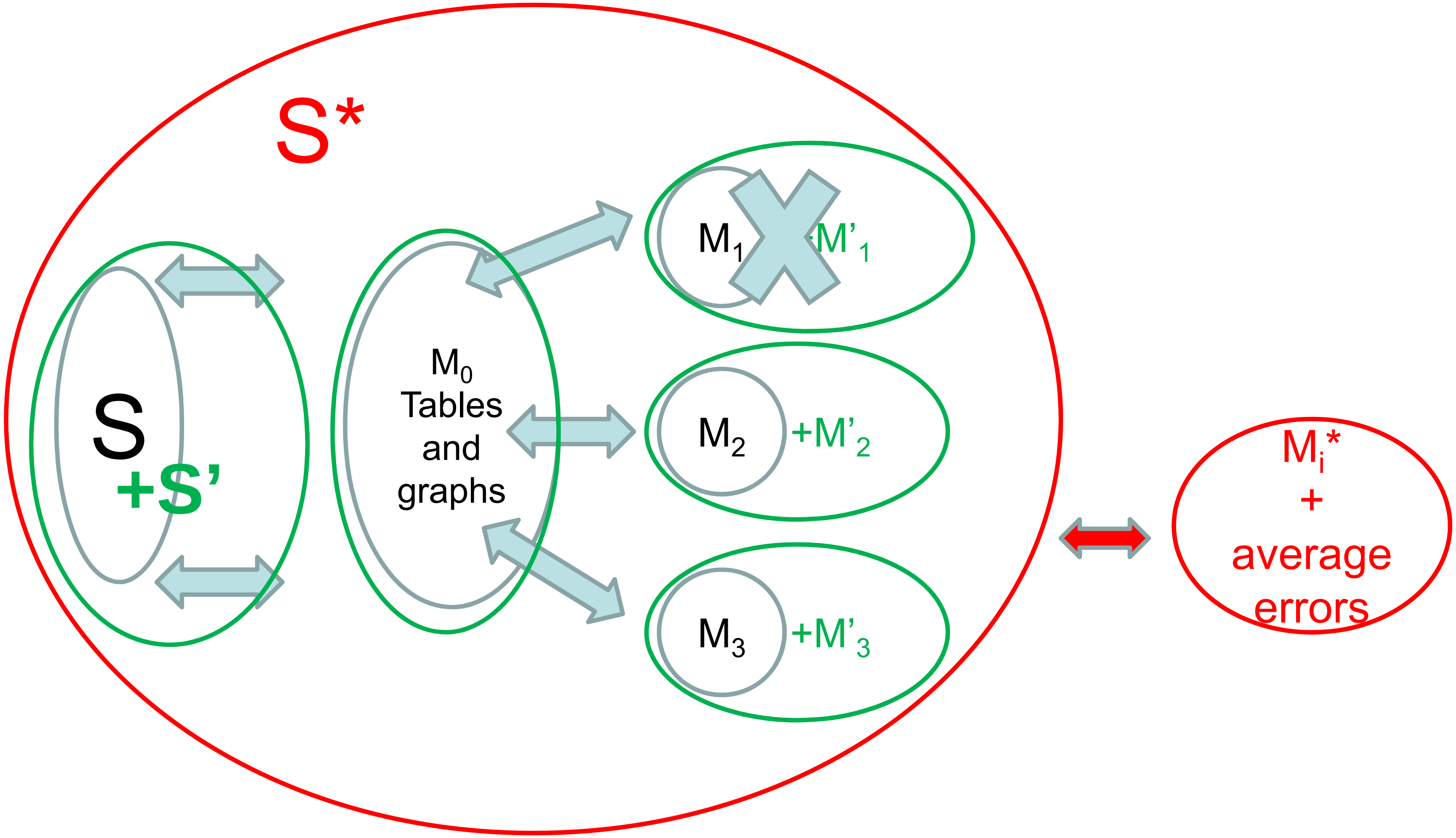
The notion of **average error** enlarges the models

The average error measures of the **'quality of the fit'**

→ **The problem of choosing the best model is** ***mathematised***

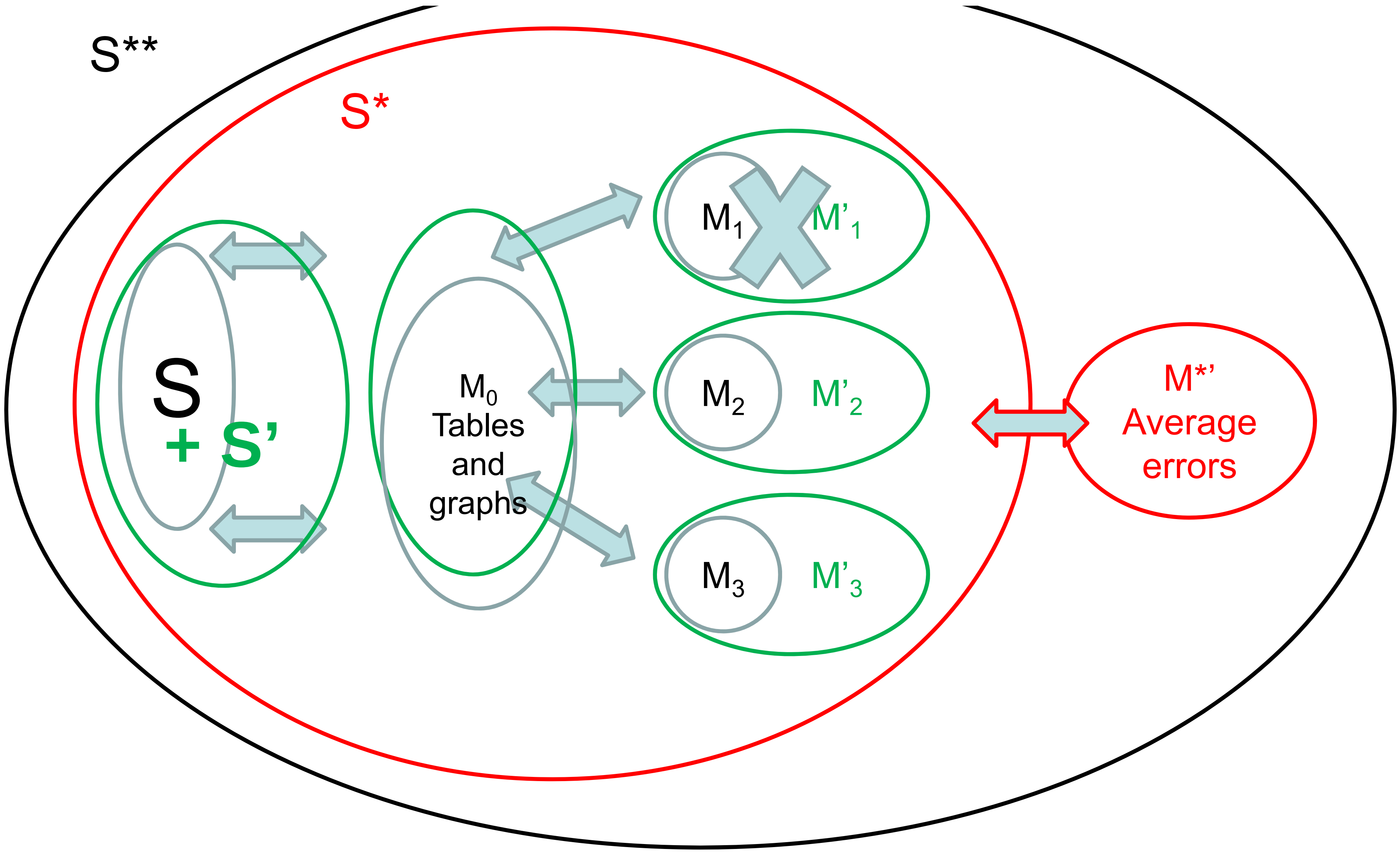
Which is the best fit to both, sales and average term sales?

→ Second enlargement of the system

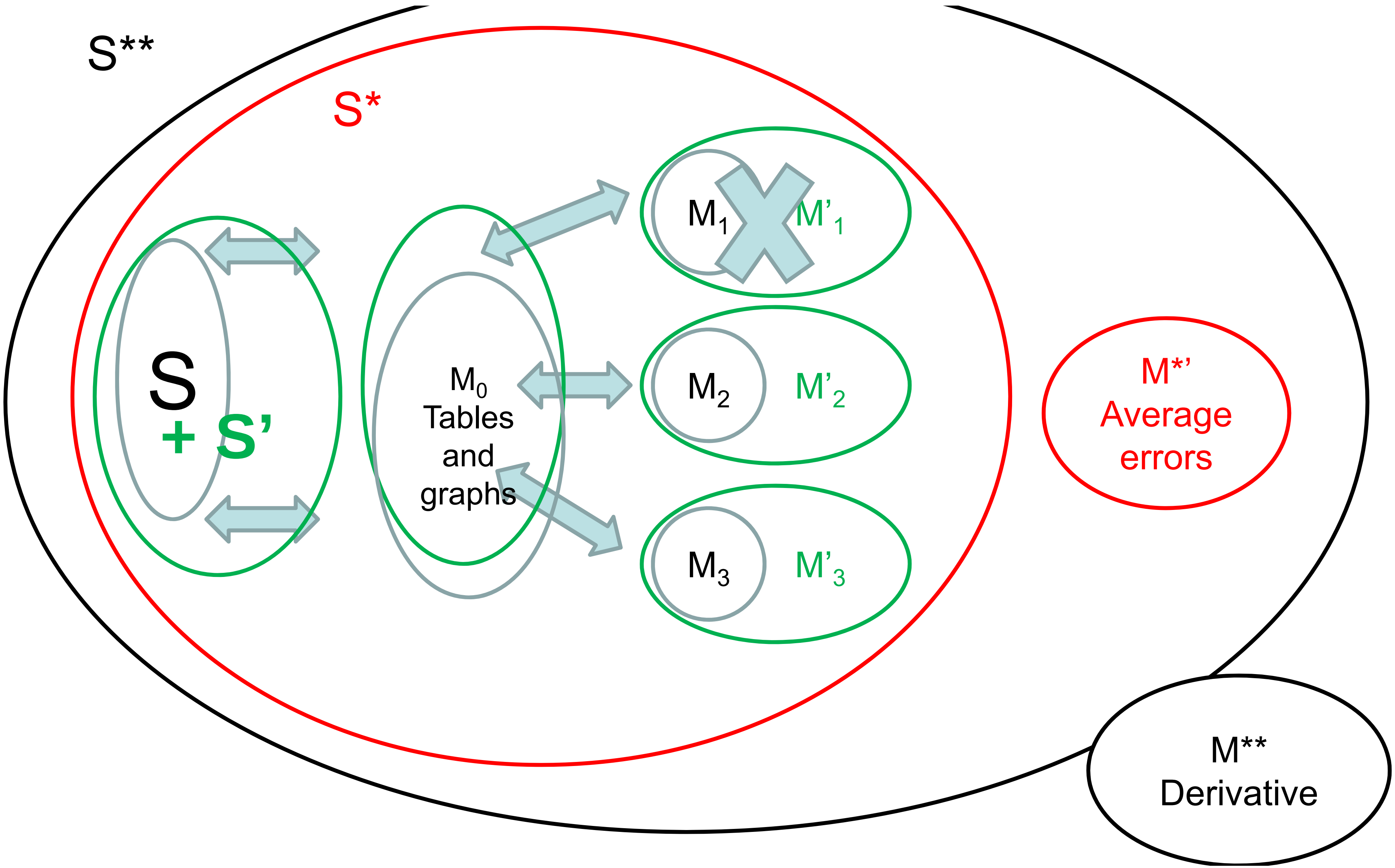


What relationship between the model of the sales and the model of the variation of term sales?

→ Third enlargement of the system



The ‘derivative’ appears as a model of the relationship between the model of the sales (*f*) and the model of the variation of term sales (Δ*f*)



The mathematizing step of the modelling process

- The process of mathematizing is rarely done “once for all”. It requires forth and back movements between the model and the system that change both (evolution).

- The successive ‘systems’ that are modelled are more and more mathematized, and the successive ‘models’ integrate the previous systems, creating new problems and, thus, generating the need to go on with the modelling process.

→ ***Dynamics of the study process in terms of the progressive modelling of enriched systems***

4. Analysis of the SRP

4.2. Herbartian schema

How to describe the 'dynamics' of the process?

MILIEU M

STUDENT(S)
TEACHER(S)
QUESTION Q

→

Answers $A_j^{\lozenge}$
Objects O_k
Questions Q_i

→

Own and 'functional' answer $A^{\heartsuit}$

'Alive' question
Not just an excuse to visit some pre-established works

UNITARIAN MODEL: can be used to describe 'small' processes (the study of a theme) as well as 'big' ones (PhD, etc.)

The herbartian schema

$$S(X\,;\,Y\,;\,Q) \rightarrow R^{\heartsuit}$$

$$[S(X\,;\,Y\,;\,Q) \rightarrow M\,] \rightarrow R^{\heartsuit}$$

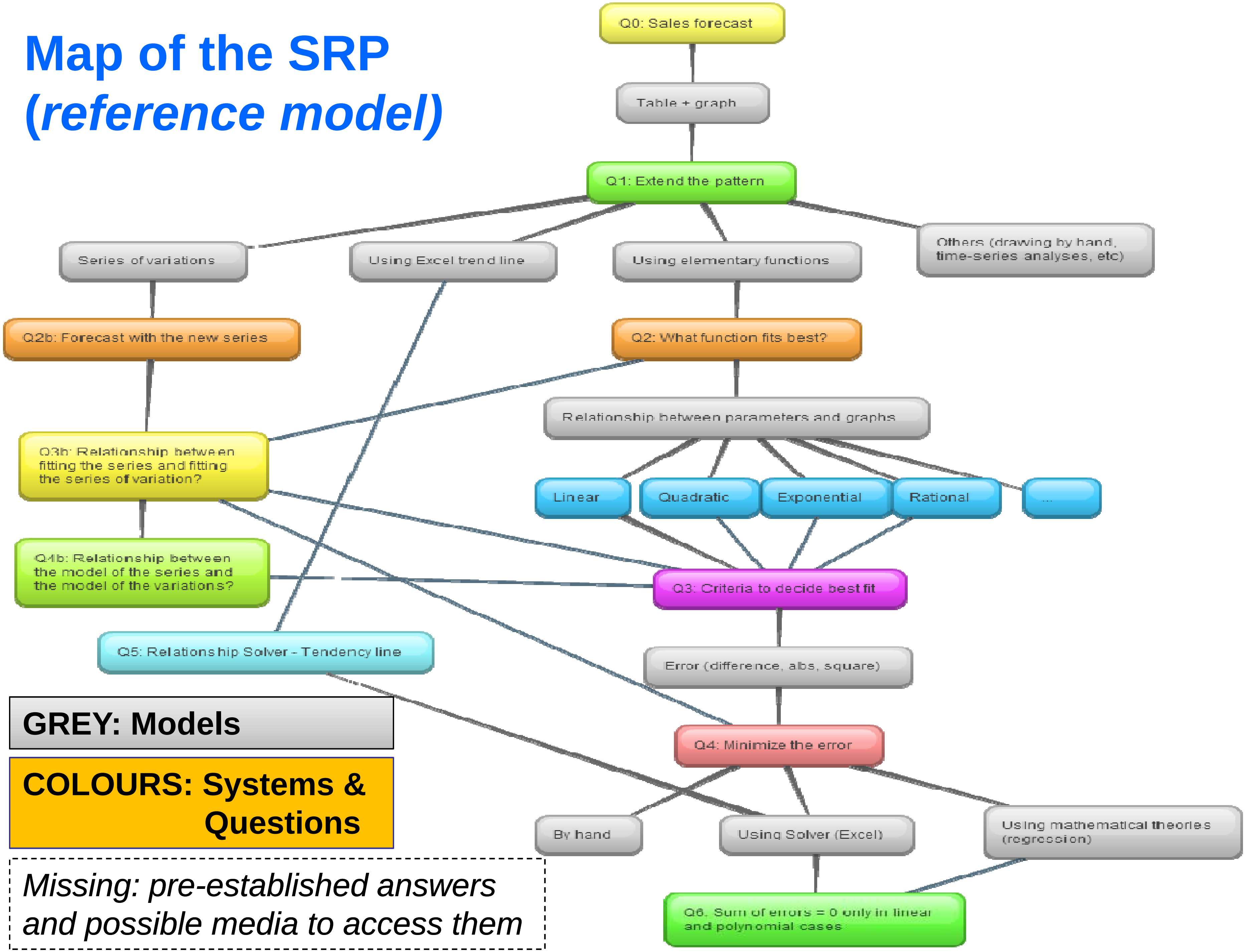
$$[S(X\,;\,Y\,;\,Q) \rightarrow \{\,R^{\lozenge}{}_1, R^{\lozenge}{}_2, \ldots, R^{\lozenge}{}_n, O_{n+1}, \ldots, O_m\,\}] \rightarrow R^{\heartsuit}$$

- The starting point is a question Q [more or less 'big']
- An own answer $A^{\heartsuit}$ has to be built [own to the group]
- To elaborate $A^{\heartsuit}$ some (material and cognitive) milieus M are necessary. They are made of other works of knowledge $A_i^{\lozenge}$ ('labeled' answers) and other (material and cognitive) objects O_k. In particular new 'partial questions' Q_j

Dynamics of the SRP

Question Q_0	How to forecast sales?
Experimental milieu	table, graphs, curves
Available answers A◊	Functions, regression, ...
Objects to contrast them	Distance, slope, ...
Derived questions	What is a good fit?, ...
New experimental milieus	Calculation of errors, Excel
New answers A◊	Excel solver, OLS
...	
Final own answer A♥	
Validation and defence of A♥	
Exploitation of A♥	

Map of the SRP
(reference model)



GREY: Models

COLOURS: Systems & Questions

Missing: pre-established answers and possible media to access them

4.3. Didactic analysis: conditions

- **The sharing of responsibilities between the teacher and the students cannot be taken for granted**
  - Who chooses and proposes the initial question **Q**?
  - Who searches the available answers $\mathbf{A}^{\Diamond}$? Where? How? What kind of ***media*** are available and how are they used?
  - How are answers $\mathbf{A}^{\Diamond}$ validated? Who determines the necessary milieus **M**? How do they evolve?
  - How do new questions **Q', Q''** appear? How the way to approach them is decided? How is it planned?
  - How is the final answer $\mathbf{A}^{\heartsuit}$ delimitated? Validated? Diffused? What status is it given? Which is its destiny? How is it related to other available answers $\mathbf{A}^{\Diamond}$?

4.4. Didactic analysis: constraints

System resistances (in spite of our effort to overcome them…)

- Students are tired to always work with the same problem, as if there was no progress, novelty → **OLD PEDAGOGY**
- Students ask the teacher to validate their answers, and it is very difficult for the teacher not to do so; they do not know how to use mathematical means to validate them
- Students are not used to planning, validating, sharing tasks, connecting their work, etc. → **NEW DIDACTIC INFRASTRUCTURES**
- Teachers tend to reproduce the traditional schema: tell the students what to do, propose short ways to the solution, provide new milieus and answers, etc.
- How to get students pose new questions during the process and how to organise their study?

4.5. ‘Ecological’ analysis

- **The strength of monumentalism**
  - ‘Didactic time’ (progress) is measured in the number of monuments that are visited; guiding through them is the teacher’s responsibility
  - Experimental milieus and media supports for the study are reduced and very tightly managed by the teacher
  - It is supposed that the teacher knows ‘more’ and ‘before’ the students; she is the last responsible of the knowledge validation

→ There are few and costly exceptions (swimming upstream)

→ Constraints do not depend on the teachers’ goodwill; institutional levels have to be considered...

The scale of didactic codetermination levels

CIVILISATION
⇳
SOCIETY
⇳
SCHOOL
⇳
PEDAGOGY
⇳
DIDACTIC SYSTEM
⇳
DISCIPLINE
⇳
DOMAIN
⇳
SECTOR
⇳
THEME
⇳
QUESTION

RELATION TO KNOWLEDGE, CURRICULA

SCHOOL ORGANISATION

MONUMENTALISM

DIDACTIC TRANSPOSITION
Knowledge organised in a 'static' way around 'concepts' and 'applications'
Lack of conceptual and linguistic tools to describe the 'dynamics' of mathematical knowledge

New didactic and mathematical infrastructures are necessary

5. SRP for teachers' professional development

Teachers' questioning (difficulties, problems) are within the ***paradigm of visiting monuments*** which is dominant in current education

Mathematical knowledge is assumed to have value by itself and the concerns are related to **organising its teaching**

When problematic questions are searched, they appear as means to ***"motivate" the study of notions*** ('applicationism')

How can teachers be "emancipated" from the dominant paradigm of "visiting works" to help them elaborate more powerful interventions?

Aim of the SRP-TPD

(1) Let teachers ***experiment a SRP*** close to what could exist in their classes (role-play or real play)

(2) Analyse the SRP using didactic tools:

- ***Mathematical analysis*** and comparison with textbooks (reference epistemological model)
- Changes in the ***didactic (and pedagogical) contracts*** that otherwise always appear as natural
- ***Ecology*** and viability of RSP: conditions and constraints

(3) *Design of new SRP and* (4) *experimentation/analysis, etc.*

Open problems

- Need of ***new mathematical and didactic devices*** to manage SRC-TPD
- Contribution of SRP as a ***teachers educational device***

HYPOTHESES

(1) SRP could be an appropriate transaction didactic device towards the new pedagogical paradigm of questioning the world

(2) TPD programmes should include the questioning of teaching and learning praxeologies, including didactic transposition processes (‘content’ praxeologies)
→ *Professional questions should be at the core of TPD*

(3) SRP-TPD could be a useful tool to
  (a) Question and study the dominant paradigm from an external point of view
  (b) Provide tools for the epistemologic and didactic analysis of teaching and learning processes
  (c) Jointly elaborate new mathematical and didactic infrastructures

SRP-TPD IN PROGRESS [secondary school teachers]

→ Alicia Ruiz Olarría (U Autonoma de Madrid)

Saving plans

→ Mabel Licera (U Río Cuarto, Argentina):

Real numbers and measure

→ Federico Olivero (U Comahue, Argentina)

Geometry

→ A. Castañeda, A. Romo, M. Sánchez (Mexico); B. Barquero, M. Bosch, J. Gascón (Barcelona); A. Ruiz-Olarría (Madrid)

Sales forecast

On-line secondary teachers education in CICATA-IPN (Mexico)

Teachers education, research in didactics and development of the profession

A teachers' education project approaching the questions raised by the practice of mathematics teaching in a real and effective way […] needs a work of tight cooperation between:

- the school system which constitutes the "field" of the teaching practice,
- research in didactics, which acts as source of questioning and production of praxeological resources to the renewal and improvement of this activity
- and the own teaching profession, which is the one who should, at the end, identify the – always evolving - needs that all its members should face.

…/…

Contribution of DM to the formation and *development* of the teaching profession

How to detect these necessities? How to conceive and develop the mathematical and didactic infrastructures needed for the development of the teaching practice?

If we locate this question in the frame of a profession, there is no mystery: do you think that generalist practitioners have to invent AIDS tri-therapies? We all know that, generally, the creation of the appropriate "medical infrastructure" cannot be done without mobilizing *huge productive efforts.* I would suggest that thinks do not need to be different with regards to educational concerns.

(Y Chevallard 2009)