

Report from the FP7 project:

Assess Inquiry in Science, Technology and Mathematics Education



ASSISTME

A definition of inquiry-based STM education and tools for measuring the degree of IBE

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Delivery date	01.10.2013
Deliverable number	D 2.5
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Dissemination level	PU

Table of Contents

SUMMARY	3
1. INTRODUCTION	4
2. REVIEW OF DEFINITIONS OF IBE IN STM	6
2.1 EU-funded projects	6
2.2 Science education	18
2.3 Technology education	27
2.4 Mathematics education.....	30
2.5 Definition of IBE	39
3. TOOLS FOR MEASURING THE DEGREE OF IBE	42
3.1 Teacher self-report questionnaires	42
3.2 Observation protocols	45
4. CONCLUSIONS	51
REFERENCES	53
FIGURES	60
TABLES	60

Summary

The ASSIST-ME project focuses on formative and summative assessment of inquiry-based education (IBE) in science, technology, and mathematics (STM). This report summarizes definitions of IBE as well as certain competences which are related to IBE. In view of the planned assessment, some of these competences have to be picked out because there are so many. Up to now, many studies only focus on 'inquiry' as a 'black box' or on single aspects of IBE (Bernholt, Rönnebeck, Ropohl, Köller, & Parchmann, 2013). Therefore, some competences should be assessed together in order to emphasize the procedural character of IBE.

When talking about 'inquiry' one has to distinguish between different perspectives. In science education, the term itself has four different meanings, summarized by Furtak, Shavelson, Shemwell, and Figueroa (2012):

- 1) scientific ways of knowing (i.e., the work that scientists do),
- 2) a way for students to learn science,
- 3) an instructional approach, and
- 4) curriculum materials.

Besides, the term 'inquiry' has a subject specific meaning. In science and technology there are several publications explicitly referring to 'inquiry' as a learning and teaching approach whereas in mathematics 'inquiry' is not a common term and approach:

Science. Within the last twenty years, scientific inquiry became a popular learning and teaching approach introduced by the National Science Education Standards (National Research Council, 1996). Most publications in this research field refer to the definition of Linn, Davis, and Bell (2004) who describe inquiry as a process of nine steps starting with the diagnose of problems and ending with the forming of coherent arguments.

Technology. The steps of inquiry in engineering design are quite similar to the steps in scientific inquiry. But the steps have different meanings because the starting point of the inquiry process is another. In engineering design the process also starts with the diagnosis of problems. However, these problems are meant as certain needs which have to be considered when constructing prototypes of certain objects.

Mathematics. Instead of inquiry, a common research field in mathematics education is problem-solving. Inquiry and problem-solving share some aspects, but there are of course differences. One major difference to scientific inquiry lies in the solution, "which is presented as a deduction from what was given in the problem to what was to be found or proved" (Schoenfeld & Kilpatrick, 2013).

1. Introduction

It had to be beyond this report to give a differentiated review of inquiry definitions in science, technology, and mathematics (STM) because there are so many. In total, the range of terms is overwhelming and not representable within one report. Therefore, on the one hand this report relates to definitions used within previous EU-funded projects and on the other hand to definitions presented in key publications, e. g. the publication by Abd El Khalick et al. (2004) about international perspectives on inquiry in science education.

Since 2008, several EU-funded projects have been initiated in the field of IBE within the Seventh Framework Program (FP7/2007-2013). Most of these projects refer to definitions or aspects of scientific inquiry and less to definitions of inquiry in technology and mathematics.

In total, these projects (S-TEAM, ESTABLISH, Fibonacci, PRIMAS, PROFILES, Pathway, INQUIRE, and SAILS) focus on the professional development of teachers in implementing IBE in their classrooms, and the promotion of the widespread use of IBE in the teaching and learning of science and mathematics. Each project has a unique emphasis, such as e. g. the development of IBE resources and materials (PRIMAS, 2009-2012) or the provision of authentic materials informed by industry (ESTABLISH, 2010-2013). The following chapter 2.1 EU-funded projects is a synthesis of the definitions of IBE used in existing EU-funded projects (see Table 1).

Table 1: Past and recent EU-funded projects working on IBE in STM

Acronym	Title	Funded period
CoReflect	Digital support for Inquiry, Collaboration, and Reflection on Socio-Scientific Debates	Mar 2008 – Feb 2010
Mind the Gap	Learning, Teaching, Research and Policy in Inquiry-Based Science Education	Apr 2008 – Mar 2010
S-TEAM	Science-Teacher Education Advanced Methods	May 2009 – Apr 2012
ESTABLISH	European Science and Technology in Action Building Links with Industry, Schools and Home	Jan 2010 – Dec 2013
FIBONACCI	Large scale dissemination of inquiry based science and mathematics education	Jan 2010 – Feb 2013
PRIMAS	Promoting Inquiry in Mathematics and Science Education	Jan 2010 – Dec 2013
PROFILES	Professional Reflection-Oriented Focus on Inquiry-based Learning and Education through Science	Dec 2010 – Nov 2014
Pathway	The Pathway to Inquiry Based Science Teaching	Jan 2011 – Dec 2013
INQUIRE	Inquiry-based teacher training for a sustainable future	Dec 2010 – Nov 2013
SAILS	strategies for assessment of inquiry learning in science	Jan 2012 – Dec 2015

(European Commission, 2011)

The ASSIST-ME project, however, goes beyond science and mathematics education and takes also technology education into account. Technology is not a common subject in many EU countries. Thus, there has been little research on IBE in technology. Most

of the reports of the EU-funded projects are based on publications from science or mathematics education. Therefore, it is helpful to define single subject-specific activities and competences which are necessary for IBE in technology but also in science and mathematics or which can be supported by IBE.

Furthermore, definitions of IBE in STM or of aspects of IBE in STM from the research literature are also presented in order to formulate a definition feasible for the ASSIST-ME project. They are described in the chapters 2.2 to 2.4. The conclusions for the ASSIST-ME project are presented at the end of the second chapter.

Within the literature, elements of IBE are described with different terms. Depending on the background of the publications, these elements are called 'abilities', 'activities', 'aspects', 'competences', 'features', 'skills', and 'standards'. The terms 'aspect' and 'feature' are often used synonymic to the term 'characteristic' in order to characterize IBE. They can also be used in order to describe what students' are engaged in when practicing inquiry. The terms 'ability', 'competence' and 'skill' usually refer to very specific student activities which are assessable by certain assessment methods, whereas the term 'activity' refers to more activities. The term 'standard' has a similar meaning because standards describe students' competences. However, in the literature the distinction is not always selective.

Another linguistic problem is the broad use of the term 'inquiry'. Furtak, Shavelson, Shemwell, and Figueroa (2012) distinguish four different meanings of inquiry:

- 1) scientific ways of knowing (i.e., the work that scientists do),
- 2) a way for students to learn science,
- 3) an instructional approach, and
- 4) curriculum materials.

These different meanings are often not distinguished. In these cases, it becomes not clear which perspective is meant. In addition, instead of 'inquiry' other terms and phrases are used, e. g. problem-based learning. According to Hmelo-Silver, Duncan, and Chinn (2007) the terms 'inquiry' and 'problem-based learning' have the same meaning.

For some investigations, it might be necessary to measure the degree of IBE in class. Possible instruments are described in chapter 3. In the literature two main tools are distinguished, teacher self-report questionnaires and observation protocols.

2. Review of definitions of IBE in STM

2.1 EU-funded projects

The following paragraphs report definitions of IBE in STM from several EU-funded projects chronically as far as they have been published.

S-TEAM

The S-TEAM project has chosen the definition of IBE from Linn, Davis, and Bell (2004, p. 4) as a common basis for discussion:

"[Inquiry is] the intentional process of diagnosing problems, critiquing experiments, and distinguishing alternatives, planning investigations, researching conjectures, searching for information, constructing models, debating with peers and forming coherent arguments."

In addition inquiry-based science teaching is characterized by activities that engage students in:

- authentic and problem based learning activities where there may not be a correct answer,
- a certain amount of experimental procedures, experiments and "hands on" activities, including searching for information,
- self-regulated learning sequences where student autonomy is emphasized,
- discursive argumentation and communication with peers ("talking science"). (Jorde, Olsen Moberg, Rønnebeck, & Stadler, 2012)

It is emphasized, however, that no common definition of IBE currently exists on a European level.

ESTABLISH

Equally, the consortium of the ESTABLISH project has adopted the definition of inquiry presented by Linn et al. (2004). Based on this definition of inquiry, nine single aspects were identified:

- diagnosing problems,
 - critiquing experiments,
 - distinguishing alternatives,
 - planning investigations,
 - researching conjectures,
 - searching for information,
 - constructing models,
 - debating with peers,
 - forming coherent arguments.
- (ESTABLISH project, 2011)

Most of these aspects are e. g. implied in the national curricula of the ESTABLISH participating countries (see Table 2).

Table 2: Elements of inquiry explicitly stated or implied in national curricula across ESTABLISH participating countries

Elements of Inquiry	CY	CZ	DE	EE	IE	IT	MT	NL	PL	SK	SE
Diagnosing problems		√	√	√		√	√	√	√	√	√
Critiquing experiments	√	√	√	√	√		√	√	√	√	√
Distinguishing alternatives		√	√	√		√		√	√	√	√
Planning investigations	√	√	√	√	√	√	√	√	√	√	√
Researching conjectures		√	√	√		√		√	√	√	√
Searching for information	√	√	√	√	√	√		√	√	√	√
Constructing models		√	√	√		√		√		√	√
Debating with peers	√	√	√	√	√			√	√	√	√
Forming coherent arguments		√	√	√	√			√	√	√	√

Identification of each country is: CY- Cyprus, CZ-Czech Republic, DE-Germany, EE-Estonia, IE- Ireland, IT- Italy, MT- Malta, NL-Netherlands, PL- Poland, SK-Slovakia and SE-Sweden.

(ESTABLISH project, 2011, p. 6)

In order to develop teaching and learning units, the partners of the ESTABLISH project specified the aspects of IBE defined by Linn et al. (2004). Table 3 shows the fundamental abilities focused by the project. McLoughlin (2011) notes that the overlap between the aspects of IBE presented in Table 3 with Wenning's (2007) hierarchy of inquiry skills (see Figure 3) is smaller than with the list of inquiry skills from the Pathway to Inquiry project published by Yiping and Blanchard (2007) (see Table 4). In order to promote IBE and its objectives, such a list of fundamental and assessable abilities is necessary. For example, 'diagnosing problems' involves not just the identification of the core of the problem or question but also the understanding and use of prior knowledge. Only both enable students to form working hypothesis.

Table 3: Fundamental abilities of IBE

Aspect of IBE	Fundamental abilities according to the ESTABLISH project
Diagnosing problems	<ul style="list-style-type: none"> • Students identify the core of the problems/ questions. • Students understand and use their prior knowledge to be able to form working hypothesis.
Critiquing experiments	<ul style="list-style-type: none"> • Students formulate arguments. • Students state outcomes in a comparative way. • Students suggest further developments. <p>In order to critique experiments intentionally and effectively, students additionally need experience, analytical skills, and reflective skills.</p>
Distinguishing alternatives	<ul style="list-style-type: none"> • Students identify key elements of the problem. • Students identify ranking level for key elements. • Students express alternatives in suitable form.
Planning investigations	<ul style="list-style-type: none"> • Students establish the hypothesis in a realistic way towards a goal. • Students consider the hypothesis and methods of answering the hypothesis. • Students set a time frame, steps involved, resources required and training in use of any equipment • Students monitor and review the approach.
Researching conjectures (hypothesis testing)	<ul style="list-style-type: none"> • Students test hypothesis which follow from their observations, facts previously gathered, or preliminary theories. • Students not just observe but consider why.
Searching for information	<ul style="list-style-type: none"> • Students define what they need to search using the right resources and how to do this and where. • Students identify possible sources of information relating to possible intervening variables.
Constructing models	<p>Students try to find something that:</p> <ul style="list-style-type: none"> • enables description, understanding, explaining, and prediction. • can be checked, proved, disproved, adapted, improved, or abandoned. <p>(The descriptions etc. can be of different types and levels (qualitative, quantitative, computer simulations, ...)).</p>
Debating with peers	<ul style="list-style-type: none"> • Students discuss and regard different interpretations of experimental results. • Students work cooperatively and collaboratively.
Forming coherent arguments	<ul style="list-style-type: none"> • Students build on evidence/ information so as to be able to present this as a logical, evidence-based communicative format, e.g. model, solution/conclusion to the process that explains and may include evidence for and against.

(adapted from McLoughlin, 2011)

Table 4: National and state science as inquiry standards

Standards	Ability to:
1. Identify questions that can be answered through scientific investigations.	<ul style="list-style-type: none"> • Refine and refocus broad/ill-defined questions • Identify and create testable questions • Identify underlying concepts related to testable questions
2. Design and conduct a scientific investigation.	<ul style="list-style-type: none"> • Make systemic observations • Accurately measure • Identify variables • Control variables • Design investigations • Interpret data • Generate explanations based on evidence • Propose alternate explanations • Critique explanations & procedures
3. Use appropriate tools and techniques to gather, analyze, and interpret data.	<ul style="list-style-type: none"> • Choose appropriate tools to answer a question or do an experiment • Use computers for collection, summary & display of evidence.
4. Develop descriptions, explanations, predictions and models using evidence.	<ul style="list-style-type: none"> • Propose an explanation based on observations • Differentiate explanation from description (providing causes for effects and establishing relationships)
5. Think critically and logically to make the relationships between evidence and explanations.	<ul style="list-style-type: none"> • Define appropriate data to use in explanation • Account for anomalous data • Review and summarize data from a simple experiment and form a logical conclusion about the cause-and-effect relationships in experiment • Begin to state some explanations in terms of the relationship between two or more variables.
6. Recognize and analyze alternative explanations and predictions.	<ul style="list-style-type: none"> • Listen to and respect the explanations proposed by other students • Remain open to and acknowledge different ideas and explanations • Be able to accept skepticism of others • Consider alternate explanations
7. Communicate scientific procedures and explanations.	<ul style="list-style-type: none"> • Communicate experimental methods • Describing observations • Summarizing results of others • Communicate explanations
8. Use mathematics in all aspect of scientific inquiry.	<ul style="list-style-type: none"> • Gather, organize and present data • Use mathematics in structuring convincing explanations

(Yiping & Blanchard, 2007)

FIBONACCI

The FIBONACCI project is one of the EU-projects in IBE focusing on both science AND mathematics education (IBSME). Within the project three key characteristics of IBSME are emphasized:

- “Through IBSME, students are developing concepts that enable them to understand the scientific aspects of the world around them through their own thinking, using critical and logical reasoning about evidence that they have gathered.
- Through IBSME, teachers lead students to develop the skills necessary for inquiry and the understanding of science concepts through their own activity and reasoning. This involves exploration and hands-on experiments.” (McLoughlin, Finlayson, & van Kampen, 2012, pp. 14–15)
- The third key characteristic is the definition of inquiry by Linn et al. (2004).

In contrast to previous projects, the FIBONACCI project explicitly describes IBE in mathematics. According to this description, IBE in mathematics is also a chronological sequence of steps:

„Like scientific inquiry, mathematical inquiry starts from a question or a problem, and answers are sought through observation and exploration; mental, material or virtual experiments are conducted; connections are made to questions offering interesting similarities with the one in hand and already answered; known mathematical techniques are brought into play and adapted when necessary. This inquiry process is led by, or leads to, hypothetical answers – often called conjectures – that are subject to validation.“ (Artigue & Baptist, 2012, p. 4)

Besides, the differences and commonalities, respectively, at the starting point of the inquiry process in science and mathematics are explained:

„In mathematics, problems are considered, and proof as to whether a claim is true or false results from a logical demonstration. In science, facts and questions are considered, and models emerge from the process of observing, experimenting, interpreting, and so on. However, the same considerations show they also have much in common. [...] In the most generally accepted meaning of the term, inquiry is an act of building and testing knowledge.“ (Artigue, Dillon, Harlen, & Léna, 2012, p. 6)

Another difference between IBE in science and mathematics is that mathematical inquiry addresses different types of questions (Artigue & Baptist, 2012). Similar to scientific inquiry they can emerge from e.g. natural phenomena (e.g. how to understand and characterise changes in the shadow of an object cast by the sun?) or technical problems (e.g. how to measure inaccessible magnitudes and objects?). However, questions can also derive from mathematical objects themselves, e. g. ‘What can it mean for two triangles, two rectangles, two polygons to have the same form?’ or ‘Given two triangles with the same area, can they be transformed one into the other by cutting and pasting?’.

In detail, inquiry-based practices in mathematics engage students in the following forms of activities:

- “articulating or elaborating questions in order to make them accessible to mathematical work;
- modeling and mathematizing;
- exploring and experimenting;
- conjecturing;
- testing, explaining, reasoning, arguing and proving;
- defining and structuring;
- connecting, representing and communicating” (Artigue et al., 2012, p. 8).

As the FIBONACCI project has adopted the definition of inquiry from Linn et al. (2004), the single aspects of inquiry are quite similar (see Figure 1). The inquiry process starts with a phenomenon or a question. After posing questions, students try to find possible explanations and hypotheses. Next, students test the hypotheses analysing if there is evidence to support a prediction based on the hypotheses. To test the prediction, new data about the phenomenon or problem are gathered and afterwards analysed. At last, the outcome is used as evidence to compare with the predicted results. “This inquiry process is led by, or leads to, hypothetical answers – often called conjectures – that are subject to validation” (Artigue & Baptist, 2012, p. 4).

However, Artigue et al. (2012) emphasize that the inquiry process is far away from linear. It’s rather a more complex scheme which makes it necessary to repeat several steps or go back to a previous step. Nevertheless, for learning and teaching inquiry it might be useful to reduce complexity by regarding inquiry as a linear process.

The inquiry process presented in Figure 1 is “the process of building understanding through collecting evidence to test possible explanations and the ideas behind them in a scientific manner” (Artigue et al., 2012, p. 7). This learning approach is defined as learning through scientific inquiry.

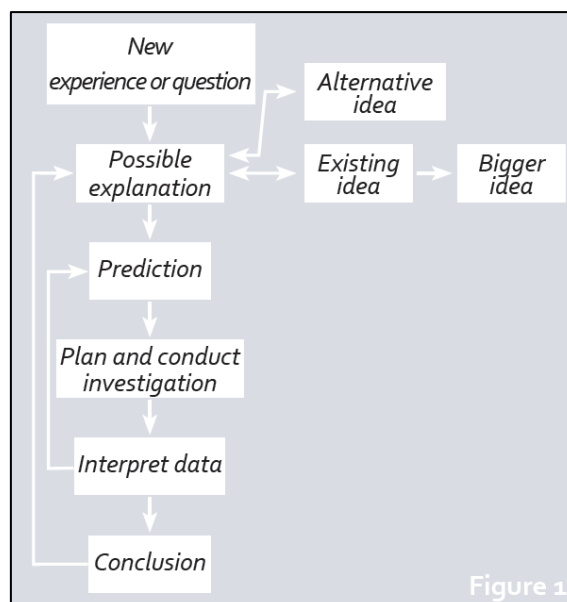


Figure 1: Process of inquiry (Artigue et al., 2012, p. 7)

Ulm (2012) also describes IBE and its characteristics within the FIBONACCI project, but in relation to science education in primary school. Even primary students can realize inquiry but on a lower level. Possible activities are e. g. the following:

- “looking at examples,
- varying given situations,
- connecting new phenomena to existing knowledge,
- formulating observations and conjectures,
- structuring situations and detecting patterns,
- describing results and giving reasons for them“ (Ulm, 2012, p. 70).

PRIMAS

Obviously, the definition of scientific inquiry by Linn et al. (2004) prevails in EU-funded projects. The PRIMAS project also refers to this definition and explicitly mentions the single aspects of inquiry given there. Above all, however, the project takes five essential features of scientific inquiry into account:

- “Learners are engaged by scientifically oriented questions.
- Learners give priority to evidence, which allows them to develop and evaluate explanations that address scientifically oriented questions.
- Learners formulate explanations from evidence to address scientifically oriented questions.
- Learners evaluate their explanations in light of alternative explanations, particularly those reflecting scientific understanding.
- Learners communicate and justify their proposed explanations.” (National Research Council, 2000, p. 25).

Since the PRIMAS project focuses on inquiry in classrooms, the project uses a broader understanding of IBE compared to other projects. It “refers to a teaching and learning culture, in which not only the described processes are involved but also certain characteristics of the teacher, the tools, the classroom atmosphere and the outcomes” (Euler, 2011, p. 6) (see Figure 2). It is thus in line with Anderson (2002) who claims that the term ‘inquiry’ does not only relate to students activities, but also to the procedural character of inquiry and especially the learning environment. In addition, the broader understanding by the PRIMAS project emphasizes that the facets ‘teacher’, ‘tools’, and ‘classroom atmosphere’ are important variables which influence students’ learning outcomes.

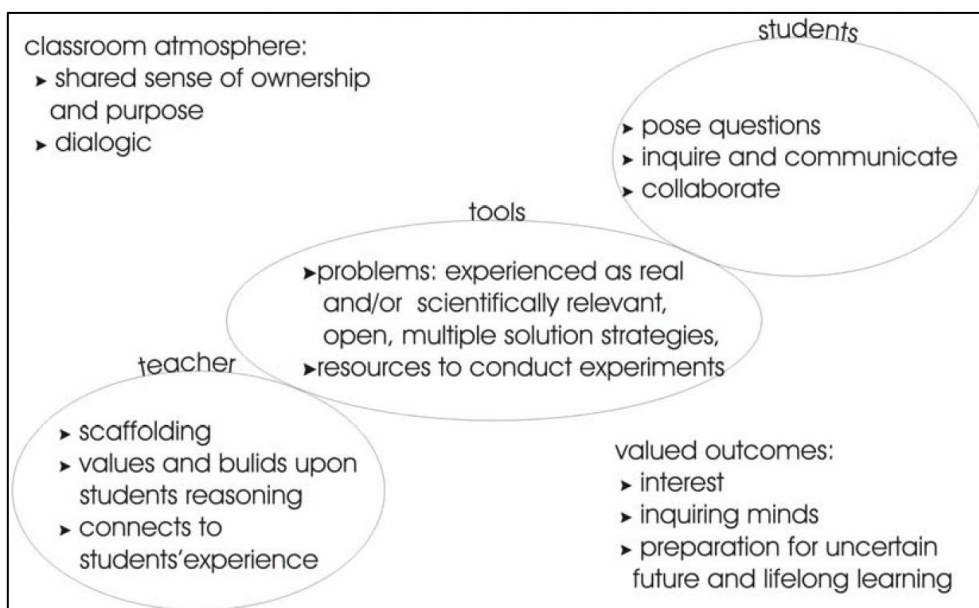


Figure 2: A multi-faceted approach to IBE (Euler, 2011, p. 6)

In view of IBE, process and self-directed learning skills are quite important objectives. Euler (2011) refers to Wu and Hsieh (2006) who define specific assessable skills students need:

- “to identify causal relationships,
- to describe the reasoning process,
- to use data as evidence,
- to evaluate” (Wu & Hsieh, 2006, p. 1290).

The four listed skills are necessary for the students to engage actively in the inquiry process. Furthermore, Euler (2011) records that these essential skills are a starting point for students to develop their own questions to examine and to make their own choices:

- “they engage in self-directed inquiry,
- they diagnose problems and develop questions,
- they formulate hypothesis,
- they identify variables,
- they collect data,
- they document their work and finally,
- they interpret and communicate the results” (Euler, 2011, pp. 7–8).

Euler (2011) concludes that the broad understanding of IBE makes it difficult to give a precise definition of IBE because definitions which focus on the process are leaving out aspects of the learning environment. Nevertheless, Engeln, Euler, and Maaß give teachers the following definition on a questionnaire:

“Inquiry-based learning (IBL) is a student-centered way of learning content, strategies, and self-directed learning skills. Students develop their questions to examine, engage in self-directed inquiry (diagnosing problems – formulating hypotheses – identifying variables – collecting data – documenting their work – interpreting and communicating results) and collaborate. The aim of IBL is to stimulate

students to adopt a critical inquiring mind and develop an aptitude for problem solving.” (Engeln, Euler, & Maaß, 2013)

Pathway

Like the PRIMAS project, the Pathway project also identified essential features of inquiry learning as a theoretical basis of the project (see Table 5).

Table 5: Framework of essential features of IBSE

Essential Features of IBSE	Variations		
	1 (Open)	2 (Guided)	3 (Structured)
QUESTION: students investigate scientifically oriented question	Student poses a scientifically oriented question	Student selects from a range of, or refines, a scientifically oriented question provided by the teacher, materials or other source	Student is given a scientifically oriented question by the teacher, materials or other source
EVIDENCE: students give priority to evidence	Student determines what constitutes evidence/data and collects it	Student selects from data/evidence provided by the teacher, materials or other source	Student is given evidence/data by the teacher, materials or other source
ANALYSE: students analyse evidence	Student decides how to analyse evidence	Student selects from ways of analyzing evidence provided by the teacher, materials or other source	Student is told how to analyse evidence provided by the teacher, materials or other source
EXPLAIN: students formulate explanation based on evidence	Student decides how to formulate evidence based on evidence	Student selects from possible ways to formulate explanation given by the teacher, materials or other source	Student is given a way to formulate explanation based on evidence
CONNECT: students connect explanations to scientific knowledge	Student independently finds and examines other resources and forms links to scientific knowledge	Student is directed to other resources and shown how to form links to scientific knowledge	Student is given other resources and shown the links with scientific knowledge
COMMUNICATE: students communicate with audience(s) and justify explanation	Student chooses how to communicate and justify explanations	Student is given broad guidelines on how to justify and communicate explanations	Student is given all the steps to justify and communicate explanations by the teacher, materials or other source
REFLECT: students reflect on the inquiry process, responses to their work, its value and impact, and their learning	Student decides independently how to structure reflection on the inquiry process and his/her learning	Student is given broad guidelines to structure reflection on the inquiry process and his/her learning by the teacher, materials or other source	Student is given a structured framework for reflection by the teacher, materials or other source
	More-----Amount of Student Self-Direction-----Less		

(Levy, Lameris, McKinney, & Ford, 2011, p. 20)

But in contrast, the project not only states that students should engage in self-directed inquiry but takes the amount of student self-direction into account. To distinguish this amount, three so called ‘types of inquiry’ are presented: ‘open’, ‘guided’, and ‘structured’. For example, the essential feature ‘students investigate scientifically oriented questions’ is subdivided into the three aspects:

- “Open: student poses a scientifically oriented question,
 - Guided: student selects from a range of, or refines, a scientifically oriented question provided by the teacher, materials or other source,
 - Structured: student is given a scientifically oriented question by the teacher, materials or other source”
- (Levy et al., 2011, p. 20).

It becomes obvious that the amount of student self-direction has an influence on the way of teaching inquiry. The teacher has to take into account which amount matches his or her students’ level of skills.

The authors of the report recommend the adoption of the framework of essential features (see Table 5) and the types of inquiry for the project (Levy et al., 2011). It is adopted from the National Science Education Standards (National Research Council, 1996).

Besides, Levy et al. (2011) emphasize that information literacy plays an essential role in view of successful inquiry although it is a less developed and thus less investigated aspect of IBE. They refer to Webber (2003) who defined information literacy as “the adoption of appropriate information behaviour to identify, through whatever channel or medium, information well fitted to information needs, leading to wise and ethical use of information in society” (Webber, 2003). Information literacy belongs to the transversal competences which are not subject-specific.

SAILS

Next to the ASSIST-ME project, the SAILS project is one of the most recent EU-funded projects. In preparation for the forthcoming work within the project, McLoughlin et al. (2012) wrote a report on mapping the development of key skills and competences onto skills developed in IBSE. The authors refer to Minner, Levy, and Century (2010) who defined the term inquiry as three distinct categories of activities:

- “what scientists do (e.g., conducting investigations using scientific methods),
 - how students learn (e.g., actively inquiring through thinking and doing into a phenomenon or problem, often mirroring the processes used by scientists),
 - and a pedagogical approach that teachers employ (e.g., designing or using curricula that allow for extended investigations)”
- (Minner et al., 2010, p. 476).

Regarding the first category, Minner et al. (2010) point out that the inquiry process itself has some essential features independently from the person who is doing or supporting inquiry. These features are the same as used by the PRIMAS project (see the paragraph about the PRIMAS project) going back to a publication by the National Research Council (2000). Furthermore, the SAILS project explicitly refers to a framework by Wenning (2007) (see Figure 3) who defines scientific inquiry skills more detailed. By this detailed description, the above mentioned aspects of IBE defined by Linn et al. (2004) become clearer. Thereby, some aspects are new, e. g. “apply numerical and statistical methods to numerical data to reach and support conclusions” (Wenning, 2007, p. 22).

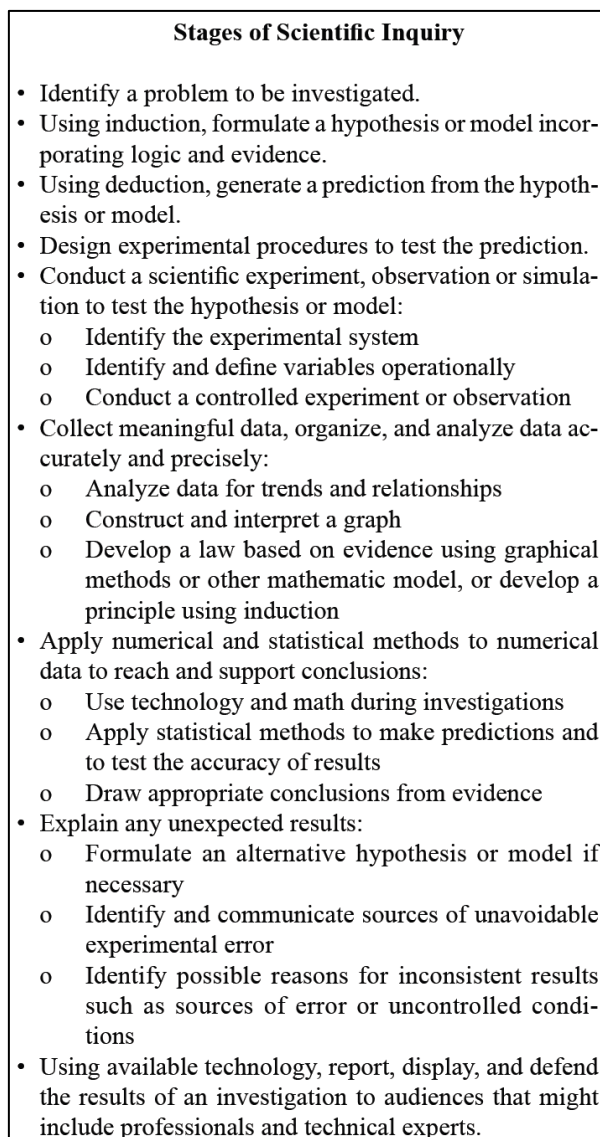


Figure 3: A limited framework defining scientific inquiry skills as a part of scientific literacy (Wenning, 2007, p. 22)

In terms of the third category, McLoughlin et al. (2012) point out that “inquiry based science education is an approach to teaching and learning science that is conducted through the process of inquiry” (McLoughlin et al., 2012, p. 11). They refer to Kahn and O’Rourke (2005) who compiled key characteristics of inquiry based teaching:

- “Students are engaged with a difficult problem or situation that is open-ended to such a degree that a variety of solutions or responses are conceivable.
- Students have control over the direction of the inquiry and the methods or approaches that are taken.
- Students draw upon their existing knowledge and they identify what their learning needs are.
- The different tasks stimulate curiosity in the students, which encourages them to continue to search for new data or evidence.

- The students are responsible for the analysis of the evidence and also for presenting evidence in an appropriate manner which defends their solution to the initial problem.” (McLoughlin et al., 2012, p. 11)

These characteristics are important because they pertain to the degree of ‘freedom’. It is the teachers’ role to assign the extent to which the students work on their own without any guidance or support. This has an influence on the difficulty of each step in the inquiry process (see paragraph about the Pathway project).

2.2 Science education

In this chapter, further definitions of scientific inquiry or its aspects are presented. Above cited publications are only mentioned again if there are additional important facts about or aspects of inquiry.

It is very important to know that “the term inquiry itself has taken on multiple meanings in the science education reform literature” (Furtak, Seidel, Iverson, & Briggs, 2012, p. 304). In total, ‘inquiry’ can be used with four different meanings: 1) scientific ways of knowing (i.e., the work that scientists do), 2) a way for students to learn science, 3) an instructional approach, and 4) curriculum materials (Furtak & Shavelson et al., 2012). When talking about inquiry as scientific ways of knowing, it is also called ‘authentic scientific inquiry’ (Hume & Coll, 2010). In this report the focus lays on the second and third meaning.

In their meta-analysis about experimental and quasi-experimental studies of inquiry-based science, Furtak & Seidel et al. (2012) mainly refer to the National Science Education Standards which can be seen as a starting point of the implementation of IBE in theory and practice. Many other publications refer explicitly to the definition and description of IBE by the National Research Council, e .g. Erdogan, Campell, and Abd-Hamid (2011) and Breslyn and McGinnis (2012).

Table 6: Abilities necessary to do scientific inquiry

K-4	5-8	9-12
Ask a question about objects, organisms, and events in the environment	Identify questions that can be answered through scientific investigations	Identify questions and concepts that guide scientific investigations
Plan and conduct a simple investigation	Design and conduct scientific investigations	Design and conduct scientific investigations
Employ simple equipment and tools to gather data and extend the senses	Use appropriate tools and techniques to gather, analyze, and interpret data	Use technology and mathematics to improve investigations and communications
Use data to construct a reasonable explanation	Develop descriptions, explanations, predictions, and models using evidence	Formulate and revise scientific explanations and models using logical evidence
	Think critically and logically to make relationships between evidence and explanations	
	Recognize and analyze alternative explanations and predictions	Recognize and analyze alternative explanations and models
Communicate investigations and explanations	Communicate scientific procedures and explanations	Communicate and defend a scientific argument
	Use mathematics in all aspects of scientific inquiry	

(National Research Council, 1996, pp. 121-123, 145+148, 175-176)

In 1996, the National Research Council published the National Science Education Standards. In these standards, inquiry was described as “central to science learning” (National Research Council, 1996, p. 2). According to the standards, two perspectives on scientific inquiry can be made: the abilities necessary to do scientific inquiry and the understanding about scientific inquiry. Table 6 shows the content standards for each school level focusing on the inquiry process. The comparison between the levels indicates the learning progressions from K to 12. Within the National Science Education Standards each standard is described in more detail. For example, when 5th-8th grade students identify questions they

“should develop the ability to refine and refocus broad and ill-defined questions. An important aspect of this ability consists of students’ ability to clarify questions and inquiries and direct them toward objects and phenomena that can be described, explained, or predicted by scientific investigations .Students should develop the ability to identify their questions with scientific ideas, concepts, and quantitative relationships that guide investigation.” (National Research Council, 1996, p. 145)

Table 7: Understandings about scientific inquiry

K-4	5-8	9-12
Scientific investigations involve asking and answering a question and comparing the answer with what scientists already know about the world.	Different kinds of questions suggest different kinds of scientific investigations. Some investigations involve observing and describing objects, organisms, or events; some involve collecting specimens; some involve experiments; some involve seeking more information; some involve discovery of new objects and phenomena; and some involve making models.	Scientists usually inquire about how physical living, or designed systems function. Conceptual principles and knowledge guide scientific inquiries. Historical and current scientific knowledge influence the design and interpretation of investigations and the evaluation of proposed explanations made by other scientists.
Scientists use different kinds of investigations depending on the questions they are trying to answer. Types of investigations include describing objects, events, and organisms; classifying them; and doing a fair test (experimenting).	Current scientific knowledge and understanding guide scientific investigations. Different scientific domains employ different methods, core theories, and standards to advance scientific knowledge and understanding.	Scientists conduct investigations for a wide variety of reasons. For example, they may wish to discover new aspects of the natural world, explain recently observed phenomena, or test the conclusions of prior investigations or the predictions of current theories.
...

(National Research Council, 1996, pp. 123, 148, 176)

Next to the mentioned abilities, students should develop an understanding about scientific inquiry. It also progresses from K to 12. Table 7 shows examples from the content standards. They illustrate that students hold the position of a scientist.

Both content standards lists indicate that inquiry is quite complex and multifaceted (see Figure 4). Therefore, it is necessary to teach students first in selected aspects of inquiry. Subsequently, they should also develop the ability to conduct complete inquiries.

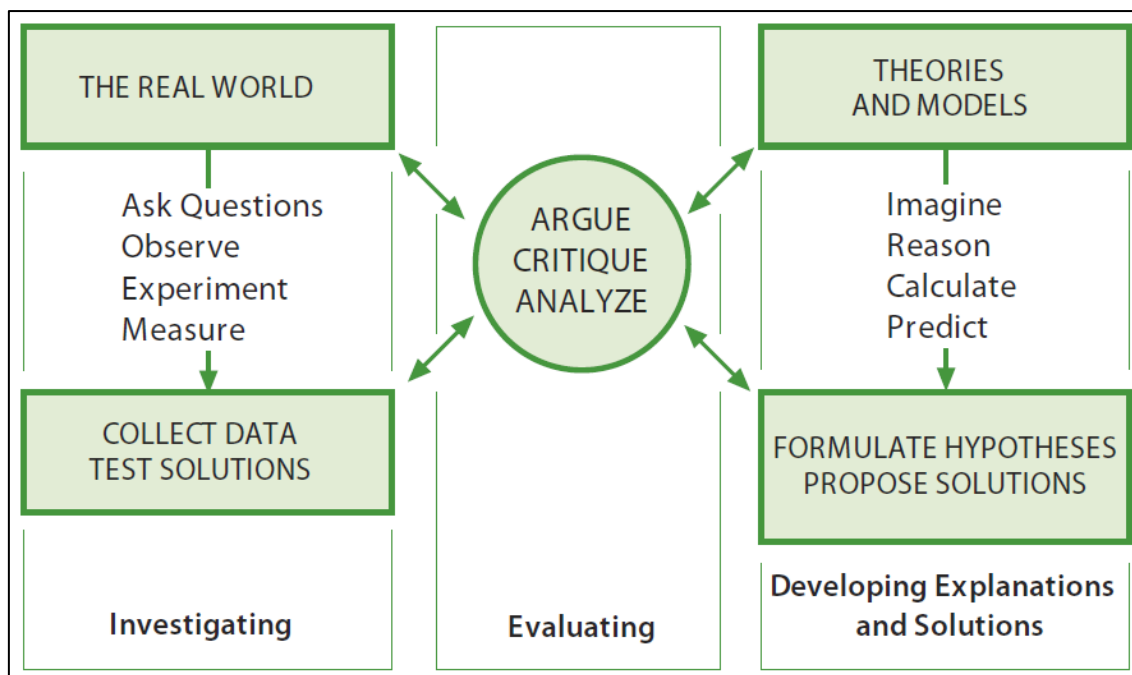


Figure 4: The three spheres of activity for scientists and engineers (National Research Council, 2012, p. 45)

To promote scientific inquiry, the National Research Council (2000) later used the description of a geologists' work as an example to make clear what is meant by inquiry:

„The geologist began his investigation with a question about an unusual and intriguing observation of nature. [...] The scientist then undertook a closer examination of the environment – asked new and more focused questions – and proposed an explanation for what he observed, applying his knowledge of plate tectonics. [...] The scientist, knowing of investigations by other scientists, used his findings to confirm the validity of his original explanation. [...] The geologist published his findings.“ (National Research Council, 2000, p. 10)

Anderson (2002) criticizes that the National Science Education Standards (National Research Council, 1996) only describe activities of scientific inquiry, but don't contain a "precise operational definition of inquiry teaching" (Anderson, 2002, p. 3). Consequently, the author concludes that "many and varied images of inquiry teaching can be expected among its readers" (Anderson, 2002, p. 3). From this it follows that the use of the term 'inquiry teaching' may not be very precise. "The dilemma this situation poses for the person attempting to synthesize what the research has to say about inquiry teaching is that making generalizations about it becomes difficult because of varied conceptions of inquiry teaching" (Anderson, 2002, p. 4).

Nevertheless, inquiry learning and teaching were picked up numerous times afterwards and aspects of both were further distinguished which makes an overview of current definitions more and more difficult.

The procedural character of scientific inquiry was first depicted by a simple model of the scientific inquiry process (see Figure 5) by White and Frederiksen (1998). In accordance with the model, students first

- “pursue a sequence of research goals in which they first formulate a question and then generate a set of competing predictions and hypotheses related to that question.
- In order to determine which of their competing hypotheses is accurate, they then plan and carry out experiments (using both computer models and real-world materials).
- Next, they analyze their data and summarize their findings in the form of scientific laws and models.
- Finally, they apply their laws and models to various situations.”
(White & Frederiksen, 1998, p. 4)

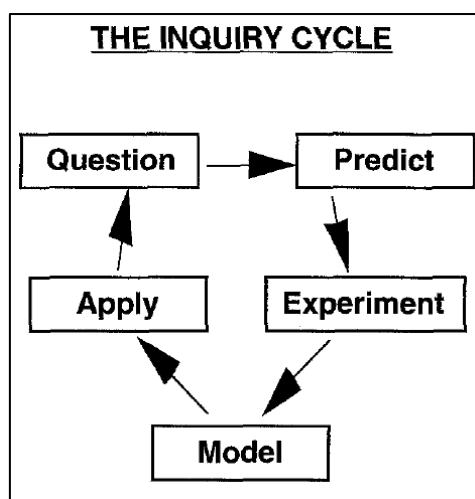


Figure 5: A model of the scientific inquiry process (White & Frederiksen, 1998, p. 5)

The advantage of this process of inquiry is that students have to “reflect on both the limitations of what they have learned (which suggests new questions) and on the deficiencies in the inquiry process itself (which suggests how it could be improved)” (White & Frederiksen, 1998, p. 4). The improvement leads the students back to the beginning of the cycle with a new or refined question and a revised approach. Afterwards, this simple model was refined, e. g. within the FIBONACCI project (Artigue et al., 2012).

In view of school relevant learning goals, White and Frederiksen (1998) point out that students should learn to do the following:

- “Formulate a well-formed, investigable research question whose pursuit will advance their understanding of a topic they are curious about [...].
- Generate alternative, competing hypotheses and predictions about what might happen with respect to that question and why it might happen [...].

- Design and carry out experiments using both the real world and computer simulations in order to determine what actually happens [...].
- Analyze their data to construct an explicit conceptual model that includes scientific laws that would predict and explain what they found [...].
- Apply their model to different situations in order to investigate its utility as well as its limitations [...]"

(White & Frederiksen, 1998, p. 10).

Additionally, Hinrichsen and Jarrett (1999) record students' needs in view of their learning process. The process can be described as constructivist:

„Students need to personally construct their own understanding by posing their own questions, designing and conducting investigations, and analyzing and communicating their findings. Students need to have opportunities to progress from concrete to abstract ideas, rethink their hypotheses, and adapt and retry their investigations and problem-solving efforts.” (Hinrichsen & Jarrett, 1999, p. 5)

The authors illustrate the advantages of scientific inquiry. In short, students construct their own understanding by taking an active role in their learning. This is one of the primary tenets of inquiry. Furthermore, Hinrichsen and Jarrett (1999) mention assessable activities characterizing inquiry. Actually, they are similar to the above listed (e. g. Linn et al., 2004). In addition, however, the authors also mention less readily assessable aspects of inquiry such as collaboration, responding to criticism, and practicing habits of mind, such as honesty and integrity in reporting findings” (Hinrichsen & Jarrett, 1999, p. 7).

In their report, Goodrun, Hackling, and Rennie (2000) point out that “inquiry [...] involves more than the processes of science [...] that stress learning skills such as observation, inference and designing a controlled experiment” (Goodrun et al., 2000, p. 145). Students rather need “to observe scientifically, not just observe, and to infer scientifically, linking observations and other evidence with scientific knowledge” (Goodrun et al., 2000, p. 145). Therefore, the authors define inquiry as follows:

“inquiry means that students combine [...] scientific processes with scientific knowledge as they reason and think critically about evidence and explanation to develop their understanding in science and ability to communicate scientific arguments” (Goodrun et al., 2000, p. 145).

Considering several publications, Abd El Khalick et al. (2004) went one step further. They concluded that a range of terms and phrases is used to characterize inquiry which include: “scientific processes; scientific method; experimental approach; problem solving; conceiving problems, formulating hypotheses, designing experiments, gathering and analyzing data, and drawing conclusions; deriving conceptual understandings; examining the limitations of scientific explanations; methodological strategies; knowledge as “temporary truths;” practical work; finding and exploring questions; independent thinking; creative inventing abilities; and hands-on activities” (Abd El Khalick et al., 2004, pp. 411–412).

For a precise definition, these overwhelming perspectives are not beneficial. Thus, Abd El Khalick et al. (2004) contrast several dichotomies to describe existing problems and the delimitation of inquiry or nature of science from more traditional approaches:

- (a) “learning science versus learning about science;
- (b) science as a search for truth versus science as a problem-solving activity;
- (c) raising and answering questions versus posing and revising explanations and/or models;
- (d) science as a cognitive activity versus science as a social activity;
- (e) demonstrating what we know (concepts) versus investigating how we know and why we believe it;
- (f) hypothetico-deductive science (causal experimental science) versus model-based science; and
- (g) science as a process of justifying and testing knowledge claims versus science as a process of discovering and generating knowledge claims”
(Abd El Khalick et al., 2004, p. 412).

Of course, these problems have an influence on teaching scientific inquiry. Therefore, in the course of problem-based or inquiry-oriented teaching approaches students should be given “the opportunity to undertake ‘research activities’ instead of just carrying out routine ‘cook-book experiments’” (European Commission, 2004, p. 125). In addition, there should be an emphasis on combining minds-on and hands-on activities. Possible methodological approaches to bring both types of activities together are of course the combination of different activities, the use of open-ended tasks, and the realization of self-directed learning.

Barron and Darling-Hammond (2008) define further characteristics of inquiry-based learning and teaching which e. g. include cooperative learning. In addition, they understand inquiry-based teaching as “a student-centered, active learning approach focusing on questioning, critical thinking, and problem solving” (Barron & Darling-Hammond, 2008, p. 11). Besides, Barron and Darling-Hammond (2008) explain the consequential students’ and teachers’ roles. Students are engaged in actively building their knowledge (see also Furtak & Seidel et al., 2012), while the teachers make students’ thinking visible, guide the small group work and ask questions to enhance students’ self-reflection. One major aspect is that the teachers support the students in working independently and thus in taking on their roles. Furthermore, teachers “also offer instruction in more traditional ways, such as lectures and explanations that are crafted and timed to support inquiry” (Barron & Darling-Hammond, 2008, p. 5). Regarding the above mentioned characteristics of the inquiry process, Barron and Darling-Hammond (2008) add that the problems to be investigated have to be meaningful and realistic and that they should have multiple solutions and multiple methods for reaching the solutions.

Table 8: Students activities in inquiry-based learning

Aspect of IBE	Students activities
1. Students begin with a question that can be answered in a scientific way.	Sometimes, questions will develop from something the students observe and suggest. Students must be able to investigate the questions in a developmentally appropriate way [...].
2. Students rely on evidence in attempting to answer the question.	This evidence can come from designing and conducting an investigation; observing a teacher demonstration; collecting specimens; or observing and describing objects, organisms, or events. The evidence can also come from books or electronic media.
3. Students form an explanation to answer the question based on the evidence collected.	Scientific explanations provide causes for effects and establish relationships based on evidence and logical argument. For students, scientific explanations go beyond current knowledge to build new ideas upon their current understanding.
4. Students evaluate their explanation.	Students consider questions such as the following: Can other reasonable explanations be based on the same evidence? Are there any flaws in the reasoning connecting the evidence to the explanation?
5. Students communicate and justify their proposed explanations.	Sharing explanations can help strengthen or bring into question students' procedures as well as their reasoning in connecting the evidence from their experiments to their explanations.

(Kessler & Galvan, 2007, p. 2)

The students' and teachers' roles in inquiry-based learning and teaching are also described by Kessler and Galvan (2007) in a much more detailed way (see Table 8). The teachers' role is the initialization of the inquiry process by providing opportunities "that invite student questions by demonstrating a phenomenon or having students engage in an open investigation of objects, substances, or processes" (Kessler & Galvan, 2007, p. 2). But of course, "teachers will likely have to modify student questions into ones that can be answered by students with the resources available, while being mindful of the curriculum" (Kessler & Galvan, 2007, p. 2). Alternatively, the teacher may also provide the question by himself.

Abd El Khalick et al. (2004) close their article about international perspectives on scientific inquiry with the suggestion of four dimensions of IBE:

- 1) types of knowledge and understandings:
conceptual, problem solving, social, and epistemic,
- 2) range of inquiry-related activities:
e. g. problem-posing; designing investigations; collecting or accessing data; generating, testing, and refining models and explanations; communicating and negotiating assertions; reflecting; and extending questions and solutions,
- 3) range of skills:
e. g. mathematical, linguistic, manipulative, cognitive and metacognitive skills
- 4) range of spheres:
e. g. personal, social, cultural, and ethical.

These four dimensions reflect the complexity inquiry learning and teaching has to face. Furtak & Seidel et al. (2012) also propose two dimensions: “the cognitive and social activities of the student and the guidance provided to students by their teacher, their peers, or curriculum” (Furtak & Seidel et al., 2012, p. 305). The first dimension covers “three categories of inquiry that include *conceptual* structures and cognitive processes that are used during scientific reasoning, *epistemic* frameworks used when scientific knowledge is developed and evaluated, and *social* interactions that shape how knowledge is communicated, represented, argued and debated” (Furtak & Seidel et al., 2012, p. 305). This dimension goes back to Duschl’s work (2008) and is quite similar to the first dimension of Abd El Khalick et al. (2004). Furtak & Seidel et al. (2012) added a fourth category called *procedural* to the three by Duschl (2008).

Table 9: Competences of inquiry defined by the TIMMS 2011 framework

4 th grade	8 th grade
Students...	Students...
<ul style="list-style-type: none"> formulate questions that can be answered based on observations or information obtained about the natural world. 	
	<ul style="list-style-type: none"> formulate a hypothesis or prediction based on observation or scientific knowledge that can be tested by investigation.
<ul style="list-style-type: none"> describe and conduct an investigation based on making systematic observations or measurements using simple tools, equipment, and procedures. 	<ul style="list-style-type: none"> demonstrate an understanding of cause and effect and the importance of specifying variables to be controlled and varied in well-designed investigations. make decisions about the measurements to be made and the equipment and procedures to use.
<ul style="list-style-type: none"> represent their findings using simple charts and diagrams. identify simple relationships. 	<ul style="list-style-type: none"> demonstrate more advanced data analysis skills in selecting and applying appropriate mathematical techniques and describing patterns in data.
<ul style="list-style-type: none"> briefly describe the results of their investigations. write an answer to a specific question based on conclusions drawn from investigations. 	<ul style="list-style-type: none"> evaluate the results of their investigation with respect to the sufficiency of their data for supporting conclusions that address the question or hypothesis under investigation. use appropriate terminology, units, precision, format, and scales.

(Mullis, Martin, Ruddock, O’Sullivan, & Preuschoff, 2009)

As inquiry is such a common learning and teaching approach, large scale assessment frameworks like TIMSS pick up scientific inquiry processes and add them to the assessment of more general knowledge. Mullis, Martin, Ruddock, O’Sullivan, and Preuschoff (2009) write: “students are expected to demonstrate the skills and abilities involved in five major aspects of the scientific inquiry process:

- Formulating questions and hypotheses

- Designing investigations
- Representing data
- Analyzing and interpreting data
- Drawing conclusions and developing explanations“ (Mullis et al., 2009, p. 88).

Concrete competences addressed by these aspects are described in view of fourth and eighth grade students (see Table 9). This differentiation is important for the ASSIST-ME project as the latter focuses on primary as well as on middle school level. In contrast to fourth grade students, eighth grade students “should demonstrate a more formalized approach to scientific investigation that involves more evaluation and decision-making” (Mullis et al., 2009, p. 89).

Next to inquiry-based teaching itself, problem-based teaching is a frequently used approach that shares characteristics with IBE. Hmelo-Silver, Duncan, and Chinn (2007, p. 100) mention typical characteristics of problem-based teaching:

- “[...] students learn content, strategies, and self-directed learning skills through collaboratively solving problems, reflecting on their experiences, and engaging in self-directed inquiry. [...]
- [Problem-based learning is] organized around relevant, authentic problems or questions. [...]
- [Problem-based learning places] heavy emphasis on collaborative learning and activity. [...]
- [In problem-based learning] students are cognitively engaged in sense-making, developing evidence-based explanations, and communicating their ideas.
- The teacher plays a key role in facilitating the learning process and may provide content knowledge on a just-in-time basis”.

Hmelo-Silver et al. (2007) emphasized that there are no clear distinguishing features between both approaches. As well as in inquiry-based teaching, students are engaged in explorations and analyses of data when doing problem-based learning. Thus, according to the authors, both terms can be used synonymic.

In their argumentation Hmelo-Silver et al. (2007) mention that content plays an important role in IBE and is closely related to the inquiry process. According to Pellegrino and Hilton (2012), the relation between the content on the one hand and the process on the other hand is one of the long-standing issues in science education. The authors define ‘content’ as facts, formulas, concepts, and theories, whereas ‘process’ means scientific methods, inquiry, and discourse (Pellegrino & Hilton, 2012). By emphasizing inquiry, the National Science Education Standards were often interpreted as only aiming at hands-on investigations. This interpretation led to a tendency to treat scientific inquiry as divorced from content. Accordingly, students were taught in scientific inquiry as a linear sequence of steps emphasizing experimental investigations. However, important aspects of inquiry such as framing questions and hypotheses or analysing and integrating data were neglected in such decontextualized investigations. Another trend which resulted from the emphasis on inquiry activities is a change in the pedagogy from passive, teacher-led instruction to active, student driven discovery (Pellegrino & Hilton, 2012).

2.3 Technology education

The International Technology Education Association (1996) sees designing and creating new products as the two main activities in technology. Both are part of a process similar to inquiry in science or mathematics. The association gives an extended definition of what is inquiry in technology containing subject specific competences:

„The technological design process involves the application of knowledge to new situations or goals, resulting in the development of new knowledge. Technological design requires an understanding of the use of resources and engages a variety of mental strategies, such as problem solving, visual imagery, and reasoning. [...] These abilities can be developed in students through experiences in designing, modeling, testing, troubleshooting, observing, analyzing, and investigating.” (International Technology Education Association, 1996, p. 18)

One major parallel to mathematics education is the aspect of problem solving. In technology education it is named engineering design that involves three main aspects: ‘defining and delimiting an engineering problem’, ‘developing possible solutions’, and optimizing the design solution (National Research Council, 2012). In contrast to science and mathematics education, however, the physical product of the design process is the intentional objective:

“After a product, system or environment is conceived, it is designed or developed. The development processes include those activities that are used to carry out the plans, create solutions, or to test ideas that are generated through a design process. The development of physical systems involves many of the common manufacturing and production processes. The development of information systems includes basic data manipulation and enhancing actions, such as encoding and decoding.” (International Technology Education Association, 1996, p. 18)

Furthermore, the International Technology Education Association (1996) emphasizes that engineering design and scientific inquiry have a number of similarities. The most obvious similarity is that both approaches are reasoning processes used to solve problems (Hume & Coll, 2010) and thus “navigational devices that serve the purpose of bridging the gap between problem and solution” (Lewis, 2006, p. 271). For both, scientists and engineers, challenging problems are moreover characterized by high levels of uncertainty that require creativity by the problem solver. Like scientific inquiry, the engineering design process is not a rigid method (Hume & Coll, 2010; NAEP 2014, 2012). When searching for possible solutions, “engineers and scientists use similar cognitive tools, such as brainstorming, reasoning by analogy, mental models, and visual representations” (International Technology Education Association, 1996, p. 39). Finally, both approaches require testing and evaluating the product – the engineering design or the scientific hypothesis.

Lewis (2006) refers to the Science and Technology/ Engineering Curriculum Framework from the Massachusetts Department of Education (2006) in order to show further parallels between inquiry in science and design in technology. The framework proposes an eight-step design process as follows:

- “identify the need or problem,

- research the need or problem,
 - develop possible solution(s),
 - select the best possible solution(s),
 - construct a prototype,
 - test and evaluate the solution(s),
 - communicate the solution(s),
 - redesign“
- (Massachusetts Department of Education, 2006, p. 84).

Lewis (2006) points out that “students follow the same inquiry trail in their classroom laboratories as practicing scientists do in their own laboratories” (Lewis, 2006, p. 258). Besides, the author marks that inquiry provides insight into the nature of science. Therefore, it is important to mention that the method follows the question, and not vice versa: “The process is not fixed but fluid“ (Lewis, 2006, p. 258).

The parallels between science and technology are also emphasized in the Technology and Engineering Literacy Framework of the National Assessment of Educational Progress (NAEP) (NAEP 2014, 2012). As well as Garmire and Pearson (2006), the authors highlight that the “engineering design process usually begins by stating a need or want as a clearly defined challenge in the form of a statement with criteria and constraints” (NAEP 2014, 2012, p. 22). This starting point is followed by several steps similar to the ones from the above mentioned process by the Massachusetts Department of Education (2006):

- investigating relevant scientific and technical information and the way that similar challenges have been solved in the past,
- generating various possible solutions,
- trying out the solution by constructing a model, prototype, or simulation,
- testing the model, prototype, or simulation to see how well it meets the criteria and falls within the constraints.

Similar to scientific inquiry, the generation of potential solutions is the most creative step in the design process. This step is usually aided by discussions to compare different solutions to the requirements of the problem or needs and then to choose the most promising solution. Another possibility is the synthesis of several ideas into an even more promising solution. Besides, an additional characteristic of engineering design is that ideas are tested before investing too much time, money, or effort. Usually, the process of engineering design ends when the constructed product meets the stated requirements. However,

“the result of an engineering design process is not always a product. In some cases the result may be a process (such as a chemical process for producing an improved paint) or a system (such as an airline control system or a railway schedule), or a computer program (such as a video game or software to forecast the weather or model financial markets).“ (NAEP 2014, 2012, p. 22)

The main difference between inquiry in science and in technology lays in the beginning of the process. Scientific inquiry is usually based on a scientific question or problem, whereas engineering design is usually based on the identification of an engineering

problem or specific need. The process of scientific inquiry ends when the question is answered by testing hypotheses and drawing conclusions. In contrast, the process of engineering design ends when a preferred solution meets the need. Therefore, “science seeks to understand, engineering seeks to meet people’s needs” (NAEP 2014, 2012, p. 22).

When describing general key principles in the area of engineering design that all students should understand at increasing levels of sophistication, the framework also mentions further activities connected to engineering design which are listed below:

- *“Engineering design is a systematic, creative, and iterative process for addressing challenges.*
- *Designing includes identifying and stating the problem, need, or desire; generating ideas; evaluating ideas; selecting a solution; making and testing models or prototypes; redesigning; and communicating results.*
- *Requirements for a design challenge include the criteria for success, or goals to be achieved, and the constraints or limits that cannot be violated in a solution. Types of criteria and constraints include materials, cost, safety, reliability, performance, maintenance, ease of use, aesthetic considerations, and policies.*
- *There are several possible ways of addressing a design challenge.*
- *Evaluation means determining how well a solution meets requirements.*
- *Optimization involves finding the best possible solution when some criterion or constraint is identified as the most important and other constraints are minimized.*
- *Engineering design usually requires one to develop and manipulate representations and models (e.g., prototypes, drawings, charts, and graphs).“ (NAEP 2014, 2012, p. 23)*

In view of assessment, Compton and Harwood (2003) mention three examples of so called ‘components of practice’ for technology education in New Zealand: ‘brief development’, ‘planning for practice’ and ‘outcome development and evaluation’. The authors highlight that the components of practice “are generic across technological contexts and thus are able to be focused to assess and plan for student progression as students experience a range of technological contexts and areas in a technology program” (Compton & Harwood, 2003, p. 9).

In connection with the description of approaches for the assessment of technological literacy, Garmire and Pearson (2006) wrote a list of linear steps in the technological design process:

- define the problem,
- identify constraints and criteria,
- conduct relevant research,
- brainstorm ideas,
- analyze alternatives (e.g., develop a trade-off matrix),
- identify a potential solution,
- research the potential solution in detail,
- design the potential solution,

- construct a prototype,
- evaluate the prototype against the criteria,
- reiterate if necessary,
- simplify if possible.“

(Garmire & Pearson, 2006, p. 43)

As Figure 6 shows this process is not linear but dynamic. The single steps are arranged in a circle to make clear that problem solving is an open process which course is influenced by results obtaining during its realization.

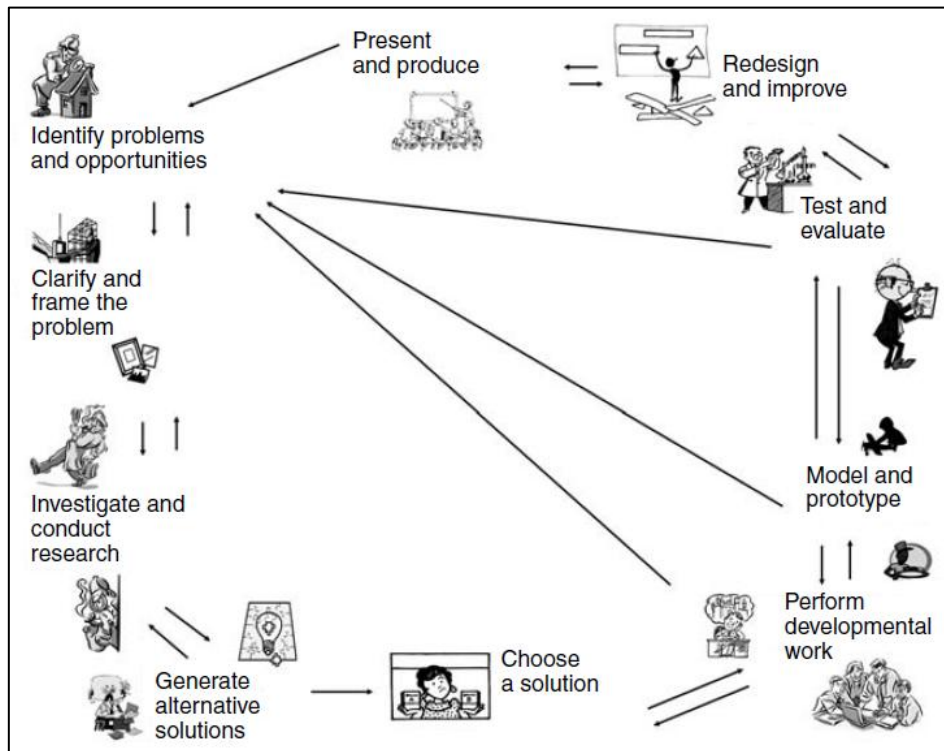


Figure 6: Design as an iterative process (Garmire & Pearson, 2006, p. 43)

Note: Typically, design begins with the identification of a problem to be solved, represented here by the detective in the upper left corner of the figure.

2.4 Mathematics education

In 1989, the National Council of Teachers of Mathematics released the Curriculum and Evaluation Standards for School Mathematics which are comparable to the National Science Education Standards (National Research Council, 1996). These standards didn't contain the terms 'inquiry' or 'inquiry-based' which might be due to the fact that 'inquiry' was and still is not a common learning and teaching approach in mathematics education or is just named differently. In their work about a national teacher training program in France, Gueudet and Trouche (2011) refer to Fuglestad (2007) who gives a definition of inquiry: "Inquiry means to ask questions, make investigations, acquire information or search for knowledge" (Gueudet & Trouche, 2011, p. 402). The authors think that in the context of mathematics teaching with a dynamic geometry environ-

ment, the definition can be interpreted as giving responsibility to students, towards both the mathematics content and the dynamic geometry environment. The authors exemplarily discuss links between the dynamic geometry environment and investigations in mathematics, e.g. the direct manipulation which is regarded as a promising agent for the transition between the processes of conjecturing and formalizing.

Teaching approaches or learning theories that include characteristics of mathematical inquiry are problem-solving (Polya, 1945; Polya, 1957), problem-centred learning (Schoenfeld, 1985), inquiry mathematics (Cobb, Wood, Yackel, & McNeal, 1992), and open approach lessons (Nohda, 2000). The FIBONACCI project (Artigue & Baptist, 2012) regards also the Dutch approach of realistic mathematics education (Freudenthal, 1973) and the French theory of didactic situations (Brousseau & Balacheff, 1997) in connection with inquiry in mathematics. Moreover, they include the Swiss concept of dialogic learning (Gallin, 2012). Another approach of inquiry in mathematics education is problem-based learning that is also mentioned in the Rocard report: "In mathematics teaching, the education community often refers to 'Problem-Based Learning (PBL)' rather than to IBE. In fact, mathematics education may easily use a problem-based approach while, in many cases, the use of experiments is more difficult. PBL describes a learning environment where problems drive the learning" (Rocard et al., 2007, p. 9).

Besides, there are several country-specific developments and considerations of problem solving. For example, Artigue and Houdement (2007), Burkhardt and Bell (2007), and Reiss and Törner (2007) give an overview of problem solving in classroom for France, the United Kingdom, and Germany respectively. Next to the above cited approaches, other country-specific approaches and traditions are described such as the more general German approach of dynamic problem solving (Dörner, Kreuzig, Reither, & Stäudel, 1983). According to the definition by Dörner et al. (1983), dynamic problem solving is a complex problem-solving process based on ill-defined problems with an uncertain result. In the United Kingdom, the approach of functional mathematics is a recent development. In the 1980's, 'mathematical investigations' and 'modelling real world problems' were practiced approaches of non-routine problem solving (Burkhardt & Bell, 2007).

Polya (1945; 1957) defined four principals of problem-solving that still have influence on recent research on this approach (e. g. Chang, Wu, Weng, & Sung, 2012). First, one has to understand the problem. Several questions can help to clarify the understanding, e. g. 'what is the unknown?', 'what are the data?', 'what is the condition?'. Next, one has to devise a plan. This means finding the connection between the data and the unknown. Once more it might be helpful to answer certain questions, e. g. 'Do you know any related problem?', 'Here is a problem related to yours and solved before. Could you use it?'. In this step it might be necessary to rethink the desired solution because one cannot solve the proposed problem. In this case, it is useful to give any support in a formative sense. Third, one has to carry out the plan. This step includes the proof of each step in order to check the correctness of the solution. At least, one has to look back in order to examine the solution obtained. Some possible questions illustrate the metacognitive character of this step, e. g. is it possible to check the re-

sult?, is it possible to check the argument?, is it possible to derive the solution differently?.

Based on Polya's principals, Schoenfeld (1985) published a further development of the learning theory of problem-solving. The main theoretical framework is the definition of four categories describing the knowledge and behaviour necessary for an adequate characterization of mathematical problem-solving performance. Table 10 shows these categories derived from Polya's principals. The first category, the resources, is defined as the mathematical knowledge. In view of problem solving, it is important to know how this knowledge is organized, stored and accessed. Schoenfeld (1985) calls this the "initial search space" (Schoenfeld, 1985, p. 17). The next category is called 'heuristics' which "become nearly synonymous with mathematical problem solving". Mainly, heuristics are "general suggestions that help an individual to understand a problem better or make progress toward its solution" (Schoenfeld, 1985, p. 23). Similar to Polya, Schoenfeld describes a category called control which includes metacognitive elements. Above all, this step focuses on the decisions "about what to do in a problem, decisions that in and of themselves may 'make or brake' an attempt to solve the problem" (Schoenfeld, 1985, p. 27). In contrast to Polya, Schoenfeld (1985) defines a new category, the belief system, and puts it next to the other three categories. According to his description, the performance of tasks is usually not only influenced by cognition including resources, heuristics, and control but also by beliefs.

Table 10: Knowledge and behaviour necessary for an adequate characterization of mathematical problem-solving performance

Category	Definition	Description
Resources	Mathematical knowledge possessed by the individual that can be brought to bear on the problem at hand	<ul style="list-style-type: none"> • Intuitions and informal knowledge regarding the domain • Facts • Algorithmic procedures • 'routine nonalgorithmic procedures • Understandings about the agreed-upon rules for working in the domain
Heuristics	Strategies and techniques for making progress on unfamiliar or nonstandard problems; rules of thumb for effective problem solving	<ul style="list-style-type: none"> • Drawing figures; introducing suitable notation • Exploiting related problems • Reformulating problems; working backwards • Testing and verification procedures
Control	Global decisions regarding the selection and implementation of resources and strategies	<ul style="list-style-type: none"> • Planning • Monitoring and assessment • Decision-making • Conscious metacognitive act
Belief system	One's 'mathematical world view', the set of (not necessarily conscious) determinants of an individual's behaviour	<ul style="list-style-type: none"> • About self • About the environment • About the topic • About mathematics

(Schoenfeld, 1985, p. 15)

Besides, Schoenfeld (1992) emphasized the relevance of problem-solving in learning to think mathematically. Above all, he pointed out that existing definitions stood in a conflict to each other. In order to distinguish of what is meant by problem solving, Schoenfeld (1992) listed three kinds of problems:

- 1) problems as routine exercises,
- 2) problems as tasks required to be done, and
- 3) problems from a mathematicians' perspective.

Problems in the sense of exercises have the following structure: First, a task is used to introduce a certain technique. Then, the technique is illustrated. At least, more tasks are provided so that the students may practice the illustrated skills. Schoenfeld (1992) assumes that the students will have a new method in their portfolio when they had worked on the exercises. This kind of problems is far away from problem-solving in the sense of inquiry. However, the second type of problems defined by Schoenfeld (1992) is closer to inquiry processes. According to Stanic and Kilpatrick (1988), this second type can be distinguished by three themes. For the first theme, problems are employed as vehicles in the service of other curricular goals, e. g. as a justification for teaching mathematics or to provide specific motivation for subject topics. In this case, problem solving is seen as facilitating the achievement of other competences. The second theme is problem solving as a specific skill. This skill is defined as being able to obtain solutions to the problems assigned. Problems from this theme can be distinguished into routine and non-routine problems. Especially, routine problems are theoretically quite close to problems as routine exercises. At least, problem solving is defined as art in order to emphasize the creativity needed for problem solving. Problems from a mathematicians' perspective are the third type of problems identified by Schoenfeld (1992). They are characterised as 'real' problems of significant difficulty and complexity motivated by practical or theoretical concerns. Because of their complexity and the required time scale (weeks, months, or even years), such problems can't be used for learning or teaching in school.

The term inquiry together with teaching or learning approaches in mathematics is only used by Cobb et al. (1992; 1991) who compare an inquiry mathematics tradition with an school mathematics tradition. Cobb et al. (1992) define inquiry in mathematics by describing students activities:

"[...] students who participate in an inquiry mathematics tradition typically experience understanding when they can create and manipulate mathematical objects in ways that they can explain and, when necessary, justify." (Cobb et al., 1992, p. 598)

In a theoretical paper, Nohda (1995) explains the teaching approach using open-ended problems. The approach is defined as an instruction in which the activities of the interaction between mathematics and students promote varied problem solving approaches using non-routine problems. The aim of teaching using open-ended problems is characterized as cultivating and fostering both creative activities by students and mathematical thinking in problem solving simultaneously. This can be reached by an instruction in which "activities of both mathematics and students are open" (Nohda, 1995, p. 57). The meaning of this expression is explained from three perspectives: (1) student

activities are open (which leads to students' generating their own questions and generalizing problems and thus eventually to problems of greater diversification), (2) mathematical activities are open (which leads to diversified and generalized procedures in problem solving) and (3) both are open (which leads to a teacher taking students' mathematical thinking in consideration and put it into mathematical activities for further problem solving). Since the qualitative and quantitative evaluation of mathematical activities that refer to the expression of mathematical ideas and problem solving processes is regarded as extremely difficult, the author suggests a model based on a matrix that represents student approaches to solving open-ended problems:

	Diversity		
Generality	A11	A12	A13
	A21	A22	A23
	A31	A32	A33

Figure 7: Matrix for the evaluation of approaches to solve open-ended problems (according to Nohda, 1995, p. 60)

As one example for the application of this matrix, an open-ended task is given in which students are asked to determine the smallest scatter among three graphic scatter patterns of marbles. In this example, 'diversity' could relate e.g. to ideas of length, area or variance whereas 'generality' could take into account whether concrete, semi-concrete or abstract examples are used.

The theory of didactic situations (TDS) is used by Hersant and Perrin-Glorian (2005) to characterize a mathematics teaching practice, currently used in secondary schools in France, which they call 'interactive synthesis discussion' (ISD). In ISD, problem solving sessions in small groups of students are followed by whole class discussions of the found solutions. The problems are chosen in a way that they partly require what the students already know, but moreover include questions or can be extended to problems whose solutions require new knowledge. In alignment with TDS, the teacher designs or adapts

“a situation including both a problem whose optimal solution involves the concept in question, and an objective milieu (in the sense of TDS). This milieu should include some material or symbolic objects that are able to provide feedback to the students' actions on them. To solve the problem, the student has to engage in actions on the milieu, to formulate hypotheses, to validate them or not, to elaborate strategies (as if trying to win a game) and to take into account the feedback from the milieu. This kind of situation, named, in TDS, the didactic situation, works, ideally, with almost no input from the teacher. Still, the teacher is responsible for obtaining that the students assume responsibility for solving the problem” (Hersant & Perrin-Glorian, 2005, pp. 115–116).

Miyakawa and Winsløw (2009) compare two didactical designs for introducing proportional reasoning to primary school pupils which are based on approaches mentioned above: “lesson study in Japan (implicitly based on the 'open approach method') and 'didactical engineering' in France (based on the theory of didactical situations)” (Miyakawa & Winsløw, 2009, p. 199). Both designs show similarities as they both are

rooted in significant theoretical ideas and as formats exist to transfer these theoretical ideas to the classroom. Both designs emphasize social interaction and independent thinking of students and require e.g. anticipating student strategies. However, they also show significant differences resulting mainly from different underlying lesson principles. Whereas the main goal of the lesson in the lesson study is that students develop and express different approaches to the problem and reflect on their own ideas by seeking to understand those of their peers, the aim in the didactical situations approach is for students to construct the one and only 'winning strategy' through interaction with the objective milieu:

“This difference of the status of strategies in principles for lessons can be explained by the different purposes of student activity in each approach. In a fundamental situation, they should lead to the personalization and institutionalization of a target mathematical knowledge (savoir), consistent with the ‘official’ mathematical knowledge. [...] In the open approach, the aim is for students to apply and test their mathematical knowledge, through two main processes: the process in which some conditions and hypotheses from a ‘real world’ problem are formulated mathematically and the process of generalization and systematization after solving a problem” (Miyakawa & Winsløw, 2009, p. 216).

In dialogic learning, instead of immediately trying to solve the problem, students should instead focus on exploring the question and related aspects in depth, thus relating it to their own world. A decisive factor for dialogic learning is that feedback is provided to the students during the exploration process (Gallin, 2012).

A recent publication by Schoenfeld and Kilpatrick (2013) summarizes similarities and differences between mathematics and science education in view of IBE based on the results from the EU-funded PRIMAS project. The authors state that “inquiry is considered the province of science education, whereas problem solving is considered the province of mathematics education” (Schoenfeld & Kilpatrick, 2013). According to Hiebert et al. (1996), problem-solving has been seen as a basis for reform in curriculum and instruction for nearly twenty years. Schoenfeld and Kilpatrick illustrate their understanding of both concepts by defining them for science and mathematics respectively:

- *“Problem solving in science is simply the finding of answers to scientific questions by gathering hypotheses, gathering evidence to test them, and drawing explanations from the evidence.*
- *Inquiry in mathematics is no more than finding connections between mathematical concepts and procedures by exploring how that mathematics might be used inside and outside school” (Schoenfeld & Kilpatrick, 2013).*

Nevertheless, nowadays the terms ‘inquiry-based science education’ and ‘problem-based learning’ are sometimes used synonymic (Rocard et al., 2007). But, “there is a tremendous gulf between the language and traditions of problem solving in mathematics and inquiry in science” (Schoenfeld & Kilpatrick, 2013). In order to clarify this gulf, Schoenfeld and Kilpatrick (2013) explain that problem-solving in mathematics from a students’ perspective means being engaged in “a task whose solution method is not known in advance”. This characteristic is also emphasized by Lane (1993) who states that the task yields multiple solutions. Furthermore, problem-solving entails the aspects

‘conjecture’ and ‘plausible reasoning’ (Schoenfeld & Kilpatrick, 2013) which are equivalent to ‘researching conjectures’ and ‘forming coherent arguments’ in science education. The difference to science education mainly lies in the “solution, which is necessarily presented as a deduction from what was given in the problem to what was to be found or proved” (Schoenfeld & Kilpatrick, 2013). Thus, proof competence might be also a crucial aspect of problem-solving in mathematics. According to Heinze, Cheng, Ufer, Lin, and Reiss (2008), ‘to prove’ means “to bridge the given condition to the wanted conclusion by intermediary hypotheses and acceptable mathematical arguments” (Heinze et al., 2008, p. 444).

Based on the above cited definitions of problem solving in mathematics education, there are many publications referring to specific aspects or characteristics of problem solving. The following paragraphs report on important specifications.

Problems that have multiple solutions are so-called open-ended problems (Kwon, Park, & Park, 2006). Even though the solutions can vary, the starting point of the problem is relatively clear. A second characteristic is that students can choose an appropriate approach on their own. In this case, they have to explain the reason for their choice. Another characteristic of open-ended problems is the employment of divergent thinking by the students when pursuing their own solutions. Kwon, Park, and Park (2006) distinguish seven types of those problems: (1) overcoming fixations, (2) multiple answers, (3) multiple strategies, (4) strategy investigation, (5) problem posing, (6) active inquiry tasks, and (7) logical thinking. Further, open-ended problems can also be defined as non-routine problems. In contrast, problems might have an open start. These open-start problems have a closed ending, in that a single answer is sought (e. g. Monaghan, Pool, Roper, & Threlfall, 2009). In this case, the problem-solver gets no information on where to start on the solution. Thus, he has to assemble from his existing knowledge a strategy that might lead to the answer.

For didactical reasons, problems can be highly structured or formulaic (Mathematical Sciences Education Board, 1993). These problems require all the same approach. In view of students’ development of competences, it is desirable that students are able to solve the more easy structured problems as well as more complex non-routine problems.

Similar to scientific inquiry, problem solving can be seen as a process. Problem posing is the first step of the process. Singer, Ellerton, Cai, and Leung (2011) summarize some empirical research results focusing on the relation between problem posing and problem solving. According to the cited studies, the complexity of posed problems has a direct influence on the whole problem solving process. Students who pose complex problems are more able to solve the problems than students who pose less complex problems.

As another aspect of problem solving, reasoning is an important student activity of IBE in mathematics (American Association for the Advancement of Science, 2009; Mathematical Sciences Education Board, 1993). On the one hand, according to the Benchmarks for Science Literacy (American Association for the Advancement of Science,

2009) reasoning in mathematics is meant as an understanding of how to use logic and evidence in making valid, persuasive arguments and in judging the arguments of others. On the other hand, Hunter and Anthony (2011) lay an emphasis on students' communication in connection with reasoning. They say that IBE "involves learning to construct representations, make arguments, reason about mathematical objects, and explain one's thinking" (Hunter & Anthony, 2011, p. 102). Finally, Hunter and Anthony (2011) suggest a framework for the engagement of students in mathematical practices within an inquiry classroom (see Figure 8). This framework covers six different dimensions, e. g. develop conceptual explanations, including using the problem context to make explanations experientially real and develop representations of the reasoning. The framework ends with the development of generalizations drawn from the previous explanations and/or justifications.

Hiebert et al. (1996) give a more general description of problem-solving characteristics, especially in view of problem-solving as a teaching approach. According to this description, "students should be allowed and encouraged to problematize what they study, to define problems that elicit their curiosities and sense-making skills" (Hiebert et al., 1996, p. 12). Consequently, problem-solving is a student centred teaching approach which should be realized as a process which is mainly influenced by students' interest:

"Allowing the subject to be problematic means allowing students to wonder why things are, to inquire, to search for solutions, and to resolve incongruities. It means that both curriculum and instruction should begin with problems, dilemmas, and questions for students." (Hiebert et al., 1996, p. 12)

Cobb, Wood, Yackel, and McNeal (1992) go one step further and state in view of students' learning that "students who participate in an inquiry mathematics tradition typically experience understanding when they can create and manipulate mathematical objects in ways that they can explain and, when necessary, justify" (Cobb et al., 1992, p. 598). The importance of students' ability to manipulate and use different systems of representations is also stressed by Elia, Gagatsis, Panaoura, Zachariades and Zouli-naki (2009). Since one representation can never fully reflect a mathematical construct, the authors regard the ability to recognize, manipulate and transfer a concept within or between multiple systems of representation as one prerequisite for the acquisition of the concept and thus for mathematical understanding.

In the context of inquiry mathematics or problem solving, several authors stress the importance of collaborative work and whole-class discussions (e.g. Cobb et al., 1991, Wood & Sellers, 1997):

"The development of an inquiry mathematics tradition requires that students have frequent opportunity to discuss, critique, explain, and when necessary, justify their interpretations and solutions. The approach of engaging students in small-group collaborative mathematical activity and then in teacher-orchestrated class discussions of their problems, interpretations and solutions was our attempt to encourage such opportunities." (Cobb et al., 1991, p. 6).

Develop conceptual explanations, including using the problem context to make explanations experientially real

- Provide a mathematical explanation. Use the context of the problem not just the numbers. Provide mathematical reasons (e.g. rather than “tidying,” state $19+7=20+6$ because $6+1=7$ and $19+1=20$).
- Develop two or more ways to explain a strategy solution.
- Analyse the explanation and construct ways to revise, extend, and elaborate on sections others might not understand.
- Predict questions that will be asked and prepare mathematical responses.

Active listening and questioning for sense making of an explanation

- Ask questions that clarify an explanation (e.g., What do you mean by? What did you do in that bit? Can you show us what you mean by? Could you draw a picture of what you are thinking?).

Collaborative support and responsibility for the reasoning of all group members

- Agree on the construction of one or more solution strategies that all members can explain.
- Work together to check, explain, and re-explain in different ways the group explanation.

Develop justification and mathematical argumentation

- Indicate agreement or disagreement (with mathematical reasons) for part of an explanation or a whole explanation.
- Justify an explanation using language (e.g., I know $3+4=7$ because $3+3=6$ and one more is 7).
- Use exploratory language (e.g., so, if, then, because, to justify and validate an explanation).
- Use questions that lead to justification (e.g., How do you know it works? Can you convince us? Why would that tell you to? Why does that work like that? So what happens if you go like that? Are you sure it's? So what happens if? What about if you say...does that still work?).

Develop representations of the reasoning

- Represent reasoning as part of exploring and making connections (e.g., How can I/we make sense of this for my/ourselves?).
- Represent reasoning to explain and justify the explanation (e.g., How can I explain, show, convince other people?).
- Use a range of representations including acting it out, drawing a picture or diagram, visualising, making a model, using symbols, verbalising or putting into words, using materials.

Develop generalizations

- Extend the explanation and/or justification to a representation of the mathematical relationship in general terms.
- Identify the rules and relationships through making and extending the connections.
- Use questions that lead to generalisations (e.g., Does it always work? Can you make connections between? Can you see any patterns? Can you make connections between? How is this the same or different to what we did before? Would that work with all numbers?).

Figure 8: A communication and participation framework to engage students in mathematical practices within an inquiry classroom (Hunter & Anthony, 2011, p. 119)

Constituting such an inquiry mathematics tradition requires developing social norms for collaborative small-group work and whole-class discussions like e.g. explaining own ideas, attempting to reach consensus, or questioning alternatives as well as a genuine commitment to communicate.

In context of classroom communication, Walshaw and Anthony (2008) highlight “the teacher’s role in establishing participation norms, in supporting and fine tuning mathematical thinking, and in shaping mathematical argumentation”. Furthermore, according to Stein, Engle, Smith, and Hughes (2008), five teacher key practices to orchestrate classroom discussions are important: “anticipating, monitoring, selecting, sequencing,

and making connections between student responses“ (Stein et al., 2008, p. 99). Another aspect mentioned in connection to problem solving in mathematics instruction is especially teachers’ ability to pose questions which is understood as one of the attributes of a teacher’s subject didactic competence and as an educational and a diagnostic tool in teacher education (Tichá & Hošpesová, 2013).

2.5 Definition of IBE

The aim of this report is to formulate an operational definition of IBE in STM feasible for further work within the project. Although there are several definitions of IBE in STM, it is not possible to just refer to one them because none of the definitions is universal. As the previous chapters show, the concept of IBE is very multifaceted and encompasses a wide range of abilities, activities, aspects, competences, features, skills, and standards students should engage in. Therefore, there should be made a selection by the ASSIST-ME project beneath the found aspects in order to focus on worthwhile and assessable issues. When making the selection one should keep in mind that the ASSIST-ME project considers three different school levels.

Besides, there is a slightly different meaning in science and technology as well as in science, technology and mathematics respectively. One important point in view of the different meanings is that IBE in science and technology is a problem-based approach but goes beyond it with the importance given to the experimental approach. In contrast, problem-solving in mathematics is also a problem-based approach. However, the focus lays only on problem-solving and not on any scientific investigations. When looking at specific aspects, it is important to keep this in mind. Another difference between science or technology and mathematics is the validation of knowledge. In science the validation is based on evidence and experience whereas in mathematics it is based on deduction or examples.

One objective of the future work should be the identification and further elaboration of certain aspects of IBE in STM. As a starting point, Table 11 gives a summary of aspects related to IBE in science and technology and problem-solving in mathematics respectively. All of the listed aspects were defined and explained within the literature cited in the chapters 2.1 to 2.4. For example, the aspects of IBE in science are mainly based on the definition by Linn et al. (2004). This definition is very common because it covers the most important steps of the inquiry process. Some aspects were found in the literature for only one subject. However, theoretically they are also possible in one of the other subjects, e. g. ‘dealing with uncertainty’. When certain aspects have been chosen for the assessment it is necessary to exactly define their meaning within the ASSIST-ME project.

Since the assessment of all aspects within the ASSIST-ME project seems unrealistic, the following recommendations are given. They address the aspects which might be focused by the ASSIST-ME assessment methods: The first recommendation is the concentration on above all subject-independent aspects such as diagnosing problems or constructing and critiquing arguments. The second recommendation is to focus on

small sequences of IBE related aspects. For example, the assessment of diagnosing problems and identifying questions should be directly combined with the assessment of formulating hypotheses in order to analyse the quality of students' formulated hypotheses. A conceivable alternative is the assessment of the whole inquiry process. In this case, the major steps should be assessed: diagnosing problems and identifying questions; formulating hypotheses; planning investigations; collecting and interpreting data; evaluating results; constructing and using models.

Table 11: Aspects of IBE and problem-solving in STM

Science	Technology	Mathematics
diagnosing problems and identifying questions	diagnosing problems and identifying needs	diagnosing problems
searching for information	searching for information	searching for information
	considering alternative solutions	considering multiple solutions
	creating mental representations	creating mental representations
formulating hypotheses	formulating hypotheses in view of the function of a device	formulating hypotheses
planning investigations	planning design	planning investigations
constructing and using models	constructing and using models	constructing and using models
researching conjectures		researching conjectures
	constructing prototypes/a prototype	
		finding structures/patterns
collecting and interpreting data		
evaluating results	evaluating results	
searching for alternatives	modifying designs	
		searching for generalizations
		dealing with uncertainty
constructing and critiquing arguments or explanations/argumentation/reasoning/using evidence	constructing and critiquing arguments or explanations/argumentation/reasoning/using evidence	constructing and critiquing arguments or explanations/argumentation/reasoning/using evidence
debating with peers/communicating	debating with peers/communicating	debating with peers/communicating

Notes.

	Aspect of IBE in STM
	Aspect of IBE in TM, SM or ST
	Domain-specific aspects

(Bernholt et al., 2013)

This last points leads to the recommendation of a definition of IBE. According to Rocard et al. (2007) and their advice for the Seventh Framework Program, the suggestion of this report is to mainly follow the definition of Linn et al. (2004) for science and tech-

nology. In mathematics, the description of the inquiry process by Artigue et al. (2012) seems a to be a good choice. By its use, it is possible to compare IBE between science, technology, and mathematics as many steps in the process are similar. In view of different grade levels, the National Science Education Standards (National Research Council, 2000) describe the progression of abilities necessary to do scientific inquiry for K-4, 5-8, and 9-12 (see Table 6). The standards also help to describe certain aspects of IBE more specific. To ensure valid and reliable assessment methods, more detailed and differentiated definitions are preferable. For example, to plan an investigation involves several competences such as drawing the design of an experiment and writing an instruction. Table 3 by McLoughlin (2011) from the FIBONACCI project is a very good example for such a specification and could be adapted by the ASSIST-ME project as well.

3. Tools for measuring the degree of IBE

For several research or evaluation studies it might be necessary to measure the degree of IBE. Tools for measuring the degree of IBE in teachers' instructional practice fall within two categories, namely teacher self-report questionnaires and observation protocols or rubrics. Among the reported tools, there are some instruments applicable for the measurement of the degree of certain aspects of IBE as described in chapter 2. Most of them are observation protocols or rubrics.

Examples from the first category are often items with a Likert-scale using item stems like e.g. 'When you teach science, how frequently/often do you ...' or 'how much time do you spend ...'. Observation protocols or rubrics often consist of reporting sheets where the observers indicate how often specific actions occur during predefined time periods within the classroom or how much time is spent on specific activities. The observation and coding mostly refer to actual instruction. There are also instruments, however, to code and evaluate e.g. teachers' lesson plans or artefacts used in instruction. In the following, several examples are presented from both categories. A summary of additional instruments (of both categories) can be found in Heinz, Lipowsky, Gröschner, and Seidel (2012). The report is based on a literature review conducted in the context of the S-TEAM project.

3.1 Teacher self-report questionnaires

Teachers self-report questionnaires that are intended to measure the implementation or degree of IBE often ask teachers to indicate how much time they usually spend on IBE activities or how often typical IBE activities (like e.g. students making predictions or share their findings with the class) occur in their regular instruction.

Within the 'Formative Assessment in Science Teaching (FAST)' project, the 'Inquiry Science Implementation Scale (ISIS)' has been developed (Brandon, Young, Pottenger, & Taum, 2009). The instrument aims at assessing teachers' implementation of student experiments. In its final form, it consists of 22 Likert-type items with a 5 point rating scale reaching from 1 = never to 5 = always. Teachers are asked to indicate how frequently they include special activities in their science teaching like e.g. having students write down the problem before doing the experiment, asking students to make predictions about an experiment, having students share their predictions, data or findings with the class or using questioning strategies to respond to students questions about experiments. Moreover, however, the instrument asks questions that are further afar from inquiry as it is understood within the ASSIST-ME project (like e.g. asking teachers how frequently they introduce new vocabulary words or check to ensure that students understand new procedures before beginning an experiment).

Further examples of questionnaire scales are found in the above mentioned report by Heinz et al. (2012). The UMAPET Teacher Survey, item 5 uses the same rating scale as ISIS and asks teachers how often students e.g. do activities that include data collection and analysis, write reflections in a notebook or journal, do laboratory experiments or complete a project that requires them to synthesize different lab activities (Heinz et

al., 2012, pp. 139–140). As part of the ‘Explain your Brain Project’ a participant survey was developed in which teachers rated on a four-point Likert-scale (from never to regularly) how often students e.g. worked in cooperative groups, participated in whole class discussions, used or made models or manipulatives, designed and carried out their own experiments or wrote descriptions of their own (Heinz et al., 2012, p. 148). Items assessing the amount of time spent on different classroom activities are also used in the ‘Survey of Instructional Practices for Science’ and the ‘Survey of Instructional Content’ (Heinz et al., 2012, pp. 162–269). In these surveys ‘time on topic’ scales are related to different scientific content like genetics, acid and bases or motion and forces. The report also lists scales from international large-scale assessments like the teacher questionnaire in TIMSS (Heinz et al., 2012, pp. 151–152) or the PISA 2006 technical report (Heinz et al., 2012, pp. 154–155).

Reference	Le, V.-N., Stecher, B. M., Lockwood, J. R., Hamilton, L. S., Robyn, A., Williams, V. L., Ryan, G., Kerr, K. A., Martinez, J. F., & Klein, S. P. (2006). <i>Improving Mathematics and Science Education: A Longitudinal Investigation of the Relationship Between Rented Instruction and Student Achievement</i> . Santa Monica, CA: RAND Corporation.
Item wording	Below is a selected list of processes that you may emphasize in teaching mathematics to fourth-grade students. How much emphasis do you place on each of the following? (Circle one response in each row)
Items	<ul style="list-style-type: none"> a. Proof and justifications/verifications (e.g. using logical argument to demonstrate correctness of mathematical relationship) b. Problem solving (e.g. finding solutions that require more than merely applying rules in a familiar situation) c. Communication (e.g. expressing mathematical ideas orally and in writing) d. Connections (e.g. linking one mathematical concept with another; applying math ideas in contexts outside of math) e. Representations (e.g. using tables, graphs, and other ways of illustrating mathematical relationships)
Item categories	Rating scale: 1 (= No emphasis), 2 (= Slight emphasis), 3 (= Moderate emphasis), 4 (= Great emphasis)

Figure 9: Example item from the Teacher Surveys: Year 2 Grade 4 Mathematics Survey – Curriculum: Item 5 (Heinz et al., 2012, p. 271)

Examples of questionnaires for mathematics and science teachers were developed within a longitudinal study aiming at investigating the relationship between rented instruction and student achievement (Heinz et al., 2012, pp. 270–297). In mathematics, teachers are asked e.g. how often their students participate in sharing ideas or solve problems in small groups, engage in hands-on mathematics activities, record, represent and analyse data, explain their thinking about mathematics problems, lead discus-

sions of a mathematics topic or use manipulatives to solve problems. Another example from this questionnaire asks teachers how much emphasis they place in their teaching of mathematics on different processes (see Figure 9).

Reference	Le, V.-N., Stecher, B. M., Lockwood, J. R., Hamilton, L. S., Robyn, A., Williams, V. L., Ryan, G., Kerr, K. A., Martinez, J. F., & Klein, S. P. (2006). <i>Improving Mathematics and Science Education: A Longitudinal Investigation of the Relationship Between Rented Instruction and Student Achievement</i> . Santa Monica, CA: RAND Corporation.
Item wording	<p>You are almost at the end of the unit on multiplying two-digit numbers. You ask students to work in pairs or groups to solve the following problem: Each school bus can hold 41 people. How many people can go on a field trip if the district has 23 buses available? Find the answer and demonstrate why it is correct.</p> <p>After the groups have worked on the problem for a while, you ask each group if they will share their work. The first group says the answer is 943 and they use the standard algorithm to show how they obtained this result [...]. A second group says the answer is 943 and they explain that they broke the numbers into tens and ones and used the distributive property [...]. After praising both groups for using effective strategies, how likely are you to do each of the following in response to these two explanations? (Circle one response in each row)</p>
Items	<ul style="list-style-type: none"> a. Ask the class if they can think of another way to solve the problem b. Suggest that the class check the results by using a calculator c. Tell them the first group's method is faster d. Tell them they are both right and move on to the next problem e. Have a classroom discussion about the differences between the two approaches
Item categories	Rating scale: 1 (= Very unlikely), 2 (= Somewhat unlikely), 3 (= Somewhat likely), 4 (= Very likely)

Figure 10: Example item from the Teachers Surveys: Year 2 Grade 4 Mathematics Survey – Teaching Scenarios: Item 7 (Heinz et al., 2012, p. 277)

Similar scales are used for science. The survey also includes scales assessing teachers' instructional practice, e.g. how often teachers use open-ended questions, require students to explain their reasoning, encourage students to communicate or explore alternative methods for solutions. Moreover, items are constructed to assess teachers' actions giving explicit teaching scenarios that hold potential for analysing not only the degree of IBE but also of formative assessment practices. One example again taken from mathematics is shown in Figure 10.

A questionnaire for students assessing was used in the ‘Constructivist Multimedia Learning Environment Survey (CMLES)’ (Heinz et al., 2012, pp. 150–151). The scale consisted of 15 items measuring students ‘learning to communicate’, ‘learning to investigate’ and ‘learning to think’. The eighth grade student questionnaire used in TIMSS also contained items asking students how often specific activities like e.g. designing, planning or conducting an experiment happen in their classrooms – in contrast e.g. to watching the teacher demonstrating an experiment (Heinz et al., 2012, pp. 388–389).

3.2 Observation protocols

Next to questionnaires, observation protocols constitute the second format of instruments that are used to assess the degree of IBE in instruction. Observation protocols often consist either of tick off sheets in which the observers are asked to mark each occurrence of a specific activity or action within a predefined time-frame or of rating sheets where teachers’ actions are rated on different point scales. Several examples for observation protocols were found within the literature review.

Erdogan, Campbell and Abd-Hamid (2011) developed the Students Actions Coding Sheet (SACS) to illuminate shifts towards student-centred science classrooms. The SACS is used in grades 6-8 science classrooms to document student actions. This is done by adding a tick mark for each action observed as many times as it occurs during the five-minute interval that is observed. The SACS focuses on assessing students actions related to three levels of cognitive domains (low, medium and high). Figure 11 shows the 24 items of the SACS. The majority of the items is related to IBE.

The Electronic Quality of Inquiry Protocol (EQUIP) has been designed to measure the quality and quantity of inquiry instruction being facilitated in K-12 mathematics and science classrooms (Marshall, Smart, & Horton, 2010). EQUIP is based on the 4Ex2 instructional model that integrates inquiry instruction, formative assessment and teacher reflection into a single cohesive model. It considers four levels of inquiry-based instruction: pre-inquiry (level 1), developing (level 2), proficient (level 3) and exemplary (level 4). The final instrument consists of 19 indicators that belong to four constructs namely instruction (e.g. instructional strategies, teacher role or knowledge acquisition), curriculum (e.g. learner centrality or integration of content and investigation), discourse (e.g. questioning level, complexity of questions or classroom interaction) and assessment (e.g. conceptual development, student reflection or type(s) of assessment). The indicators are measured by using coding rubrics. The EQUIP instrument is available via the ‘Inquiry in Motion’ homepage (Inquiry in Motion, 2009). Several other instruments influenced the development of EQUIP. To these belonged the ‘Inside the Classroom Observation and Analytic Protocol’ (Horizon Research, 2013) that provides a solid global view of classroom practice, the ‘Reformed Teaching Observation Protocol (RTOP)’ that focuses on constructivist classroom issues (Sawada et al., 2002) and the ‘Science Teacher Inquiry Rubric (STIR)’ (Beerer & Bodzin, 2003) that provides a brief protocol that is closely aligned with the National Science Education Standards definition of IBE (National Research Council, 1996).

Teacher: _____ Course: _____

Date: ____/____/____

Lesson goals: _____

Lesson objectives: _____

Directions: Please mark each occurrence observed as many times as they occur in the five-minute period. In addition, make supplementary marks for every continuous 15 seconds of observed action.

Categories	Subcategories	Number in each subcategory			
		1st five minutes	2nd five minutes	3rd five minutes	Total
Student actions indicative of lower cognitive domains <i>Remembering and initial stages of understanding</i>	(1) Positive interactions with the teacher				
	(2) Student sharing ideas with the teacher				
	(3) Student sharing observations with other students				
	(4) Student responding to teacher questions				
	(5) Student asking for attention of group (raising hand)				
	(6) Student listening to teacher				
Student actions indicative of medium cognitive domain level <i>Higher stages of understanding and applying</i>	(7) Student demonstrating excitement about activity				
	(8) Student questions				
	(9) Student elaborating on what the teacher says				
	(10) Students making their own observations beyond those planned by teachers				
	(11) Student searching for resources				
	(12) Students using resources (unsolicited)				
	(13) Student offering idea for approaching problems				
	(14) Student bringing in resources to study				
	(15) Student using alternative forms of communication				
	(16) Student making observations				
	(17) Student communicating data				
	(18) Student designing experiment				
	(19) Student following teacher's directions				
Student actions indicative of the highest cognitive domain level <i>Analyzing, evaluating and creating</i>	(20) Student negotiating/sharing and refining ideas				
	(21) Student defending responses with evidence				
	(22) Student providing explanation for phenomenon				
	(23) Student collaborating				
	(24) Student recognizing errors in process				

Figure 11: The Student Actions Coding Sheet (Erdogan et al., 2011, pp. 1333–1334)

An instrument to measure preservice teachers' ability to develop appropriate 5E (Engage, Explore, Explain, Elaborate and Evaluate) learning cycle lesson plans was developed and analysed by Goldston, Day and Sundberg (2010). The 5E inquiry lesson

plan (ILP) rubric is comprised of 12 items with a scoring range of zero (unacceptable) to four (excellent) points per item:

“The ILP items associated with the 5E learning cycle begin with the engage, a single item that includes the criteria of ascertaining students' prior knowledge, generating student questions and motivating learning. The next three items focus on the explore stage, which includes criteria associated with active, student-centered learning tasks, eliciting student questions, and formative assessment of learning. The criteria of the three items of the explain phase focus on teachers eliciting student discussion of the explore activities leading to clarification of the targeted concept(s). Providing students with a variety of ways to illustrate the concepts/skills and a formative assessment for learning is also included in the explain phase. The elaboration phase, comprised of two items, focuses on activities for children to apply and extend their newly acquired concepts in a different context with real-life connections to the concepts or skills. Evaluation, as the last phase, was comprised of three items. The evaluation criteria focused on its alignment with the lesson's objectives, its appropriateness for the concepts/skills, and rubric development if appropriate.” (Goldston et al., 2010, pp. 639-640).

A science lesson plan analysis instrument (called SLPAL) was also developed by Jacobs, Martin and Otieno (2008). It aimed at the formative and summative evaluation of a teacher education program and consisted of four subscales of which mostly the last - portrayal and uses of the practices of science – has relevance for the assessment of IBE instruction. For each item teachers can be rated on a three point scale as exemplary, making progress or needs improvement. Figure 12 shows the rating rubric for one item of the subscale ‘portrayal and uses of the practices of science’.

Item	Exemplary	Making Progress	Needs Improvement
Student practitioners of scientific inquiry (weight = 3)	Students are consistently engaged firsthand in learning content through inquiry or doing, rather than being told “answers”; inquiry process skills are taught in context	Some effort at engaging students in doing science is evident, with an emphasis on telling students science <i>or</i> Inquiry is taught out of context as a separate content area, rather than as a set of process skills to be applied	Students learn science exclusively by being told accepted scientific knowledge without discussion of how the knowledge was developed

Figure 12: The SLPAL item ‘student practitioners of scientific inquiry’ belonging to the subscale ‘portrayal and use of the practices of science’ (Jacobs et al., 2008, p. 1104)

Martinez, Borko, and Stecher (2012) investigated the properties of a teacher-generated instrument (the Scoop Notebook) to measure instruction. The Scoop notebook combines features of portfolios and self-report. Classroom artefacts and teacher reflections were collected from samples of middle school science classrooms on a daily basis and rated along 10 dimensions of science instruction derived from the National Science Education Standards. Among these 10 dimensions, several are directly related to IBE like e.g. hands-on, inquiry, scientific discourse community or explanation/justification. Detailed rubrics were developed to characterize each dimension of practice on a five-point Likert-scale ranging from low (1) to high (5).

In a study that focused on teachers' use of an 8-week chemistry curriculum that explicitly supports students in one particular inquiry practice, the construction of scientific arguments to explain phenomena, video tapes of lessons were coded with respect to the teachers instructional practices (McNeill, 2009). However, the coding scheme is very teacher-centred (asking questions like 'Does the teacher explicitly discuss what a scientific explanation is?', 'Does the teacher identify the different components of explanation as he or she models the explanation?' or 'Does the teacher discuss the key science principles for the explanation?').

As with the teacher questionnaires, additional instruments can be found in Heinz et al. (2012). Within the 'Explain your brain' project, classroom observation protocols were used to assess types of instruction, student engagement and cognitive activity (Heinz et al., 2012, pp. 141–143). Information was collected throughout the lesson in five-minute intervals. Moreover, observers were asked to rate the occurrence of a list of key indicators on a five point scale Likert ranging from 1 = 'not at all' to 5 = 'to a great extent'. Examples of key indicators are e.g. 'This lesson encouraged students to seek and value alternative models of investigation or problem solving' or 'Students were encouraged to generate conjectures, alternative solution strategies and to interpret evidence' (Heinz et al., 2012, p. 144). They should also rate the degree to which students are engaged in higher order thinking activities (combining, synthesizing, generalizing, explaining, hypothesizing, concluding) as compared to lower order thinking activities as reciting factual information or employing rules and algorithms through repetitive routines (Heinz et al., 2012, pp. 145–146). Another item looks at conversation and communication within the classroom (Heinz et al., 2012, pp. 146–147).

Within a longitudinal investigation of the relationship between rented instruction and student achievement, items assessing the extent to which teachers employ certain activities like focusing on reasoning and problem-solving, encouraging students to come up with more than one way of solving a problem, modelling scientific curiosity, scepticism, openness and an emphasis on analysis, reasoning and reflection or encouraging discussions were administered in mathematics and science classrooms. Observers could rate the extent as high (8,7,6), medium (5,4,3) or low (2,1,0) (Heinz et al., 2012, pp. 313–315).

A comparison of international patterns of scientific inquiry in science teaching and learning was conducted by Kobarg et al. (2011) using data from PISA 2006. The investigation was on five items from the student questionnaire which belong to the four

scales i) interactive science teaching ('Students are given opportunities to explain their ideas', ii) hands-on activities ('Students spend time in the laboratory doing practical experiments' and 'Students are asked to draw conclusions from an experiment they have conducted'), iii) student investigations ('Students are allowed to design their own experiments' and iv) real-life applications ('The teacher uses science to help students understand the world outside school'). The items were answered using a four point rating scale reaching from 'never or hardly ever' to 'in all lessons'.

In teaching mathematics to this class, how often do you usually ask students to do the following?

Check **one** circle for each line.

Every or almost every lesson
About half the lessons
Some lessons
Never

a) Listen to me explain how to solve problems ----- - - -

b) Memorize rules, procedures, and facts ----- - - -

c) Work problems (individually or with peers) with my guidance ----- - - -

d) Work problems together in the whole class with direct guidance from me ----- - - -

e) Work problems (individually or with peers) while I am occupied by other tasks ----- - - -

f) Apply facts, concepts, and procedures to solve routine problems ----- - - -

g) Explain their answers ----- - - -

h) Relate what they are learning in mathematics to their daily lives ----- - - -

i) Decide on their own procedures for solving complex problems ----- - - -

j) Work on problems for which there is no immediately obvious method of solution ----- - - -

k) Take a written test or quiz ----- - - -

In teaching science to the students in this class, how often do you usually ask them to do the following?

Check **one** circle for each line.

Every or almost every lesson
About half the lessons
Some lessons
Never

a) Observe natural phenomena and describe what they see --- - - -

b) Watch me demonstrate an experiment or investigation ----- - - -

c) Design or plan experiments or investigations ----- - - -

d) Conduct experiments or investigations ----- - - -

e) Read their textbooks or other resource materials ----- - - -

f) Have students memorize facts and principles ----- - - -

g) Use scientific formulas and laws to solve routine problems ----- - - -

h) Give explanations about something they are studying ----- - - -

i) Relate what they are learning in science to their daily lives ----- - - -

j) Do field work outside of class - - - -

k) Take a written test or quiz ----- - - -

Figure 13: Scales from the Mathematics and Science Teacher Questionnaires in TIMSS 2011 grade 8 (IEA International Association for the Evaluation of Educational Achievement, 2013)

In summary, several instruments to measure the degree of IBE in teachers' instructional practice exist in the literature. Examples of such instruments, either teacher or student questionnaires or observation protocols, respectively, have been presented both from the fields of mathematics and science education. Several studies that investigated both subject areas showed that it is possible to use similar scales in science and mathematics. One example is given in Figure 13. The presented scales are from the questionnaires for mathematics (left) and science teachers (right), respectively, in TIMSS 2011. They show a similar structure but are adapted to the specific contents and pro-

cesses of the respective subjects. This suggests that although the majority of instruments found within this research comes from the field of science education, it should be possible to adapt these scales to mathematics or technology using expert knowledge from these fields.

4. Conclusions

There is a huge range of publications defining IBE or certain aspects of IBE, especially in science. Most of them are going back or refer to the National Science Education Standards (National Research Council, 1996). Even the descriptions of engineering design have been deduced from science education. Another very common definition was posed by Linn et al. (2004). It is also recommended by Rocard et al. (2007) for use within in the Seventh Framework Program. However, in technology and mathematics IBE is defined slightly different.

On the one hand, there are several similarities between science inquiry and engineering design which are described in the literature. On the other hand, there are some differences which are coming from the subject itself. Although IBE is a common approach in science and nowadays also in engineering design, it is not developed for mathematics education. Therefore, this report is limited in its conclusions conferring to mathematics. This subject uses problem-solving as an approach for learning and teaching.

Science. Within the last twenty years, scientific inquiry became a popular learning and teaching approach introduced by the National Science Education Standards (National Research Council, 1996). Most publications in this research field refer to the definition of Linn, Davis, and Bell (2004) who describe inquiry as a process of nine steps starting with the diagnose of problems and ending with the forming coherent arguments.

Technology. The steps of inquiry in engineering design are quite similar to the steps in scientific inquiry. But the steps have different meanings because the starting point of the inquiry process is another. In engineering design the process also starts with the diagnosis of problems. However, these problems are meant as certain needs which have to be considered when constructing prototypes of certain objects (International Technology Education Association, 1996).

Mathematics. Instead of inquiry, a common research field in mathematics education is problem-solving. Inquiry and problem-solving share some aspects, but there are of course differences. One major difference to scientific inquiry lies in the solution, “which is presented as a deduction from what was given in the problem to what was to be found or proved” (Schoenfeld & Kilpatrick, 2013).

Within the ASSIST-ME project it will be necessary to work with subject-specific definitions of IBE in order to address these differences. But, none of the found definitions is universal. Most of the definitions describe IBE by mentioning aspects of IBE. Therefore, a list of aspects of IBE might be a good solution. Such a list is presented in chapter 2.5 (see Table 11). This list makes it possible to identify subject-specific foci but also subject-independent aspects and to develop specific assessment methods for certain aspects of IBE.

Especially in science, there are several approved tools to measure the degree of IBE. However, most of them focus on general classroom activities, especially on characteristics of the teaching approach (e. g. students designing, planning or conducting an

experiment in contrast to watching the teacher demonstrating an experiment). Nevertheless, there are some tools which are usable for the measurement of the degree of IBE.

This report is a summary of important definitions of IBE in science and technology as well as problem-solving in mathematics. It reflects the current state of the work and provides a solid basis for the prospective challenges. However, within the ASSIST-ME project other publications might be found that are also important for the further work. In this case, the publications could be added to this report.

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Figures

Figure 1: Process of inquiry (Artigue et al., 2012, p. 7)	11
Figure 2: A multi-faceted approach to IBE (Euler, 2011, p. 6).....	13
Figure 3: A limited framework defining scientific inquiry skills as a part of scientific literacy (Wenning, 2007, p. 22)	16
Figure 4: The three spheres of activity for scientists and engineers (National Research Council, 2012, p. 45)	20
Figure 5: A model of the scientific inquiry process (White & Frederiksen, 1998, p. 5) .	21
Figure 6: Design as an iterative process (Garmire & Pearson, 2006, p. 43)	30
Figure 7: Matrix for the evaluation of approaches to solve open-ended problems (according to Nohda, 1995, p. 60).....	34
Figure 8: A communication and participation framework to engage students in mathematical practices within an inquiry classroom (Hunter & Anthony, 2011, p. 119)	38
Figure 9: Example item from the Teacher Surveys: Year 2 Grade 4 Mathematics Survey – Curriculum: Item 5 (Heinz et al., 2012, p. 271).....	43
Figure 10: Example item from the Teachers Surveys: Year 2 Grade 4 Mathematics Survey – Teaching Scenarios: Item 7 (Heinz et al., 2012, p. 277).....	44
Figure 11: The Student Actions Coding Sheet (Erdogan et al., 2011, pp. 1333–1334)	46
Figure 12: The SLPAL item ‘student practitioners of scientific inquiry’ belonging to the subscale ‘portrayal and use of the practices of science’ (Jacobs et al., 2008, p. 1104)	47
Figure 13: Scales from the Mathematics and Science Teacher Questionnaires in TIMSS 2011 grade 8 (IEA International Association for the Evaluation of Educational Achievement, 2013).....	49

Tables

Table 1: Past and recent EU-funded projects working on IBE in STM	4
Table 2: Elements of inquiry explicitly stated or implied in national curricula across ESTABLISH participating countries	7
Table 3: Fundamental abilities of IBE	8
Table 4: National and state science as inquiry standards	9
Table 5: Framework of essential features of IBSE	14
Table 6: Abilities necessary to do scientific inquiry	18
Table 7: Understandings about scientific inquiry	19
Table 8: Students activities in inquiry-based learning	24
Table 9: Competences of inquiry defined by the TIMMS 2011 framework	25
Table 10: Knowledge and behaviour necessary for an adequate characterization of mathematical problem-solving performance	32
Table 11: Aspects of IBE and problem-solving in STM	40