ASSESSMENT IN INQUIRY BASED EDUCATION IN PRIMARY MATHEMATICS

[1]✉Alena Hospesova, [2]Iva Zlabkova

[1]Department of Pedagogy and Psychology, Faculty of Education, Jihočeská univerzita v Českých Budějovicích, Jeronýmova 10, České Budějovice, 371 15, Czech Republic, +420 606 606 905, hospes@pf.jcu.cz

[2]Department of Pedagogy and Psychology, Faculty of Education, Jihočeská univerzita v Českých Budějovicích, Czech Republic

Abstract

The contribution reports on the findings of research in the area of peer-assessment and self-assessment in inquiry based mathematics education on primary school level. On the basis of analyses of 20 inquiry based lessons that included pupils' self-assessment and peer-assessment we identify the phenomena that hinder this approach: problem of supporting self-assessment and formative peer-assessment (form of working sheets, support from the teacher), formulation and operationalization of learning objectives, understanding vaguely or inaccurately formulated pupils' solutions, identification of correct solutions of the problem, institutionalization of knowledge.

Keywords

Inquiry based mathematics education, formative assessment, peer-assessment, self-assessment

Introduction

Inquiry based education in mathematics has become a promoted way of developing understanding in natural sciences and mathematics and of raising interest in scientific disciplines. The nature of this teaching method requires specific approaches to assessment of findings pupils come to in their "inquiries".

What do we understand by inquiry based education?

In simple terms, inquiry based education means that the teacher creates such conditions at school that allow the pupil to discover on their own a part of the knowledge to be learned. It has recently been regarded as a promising way that promotes interest in the study of natural sciences and mathematics. The goal of this approach is to prepare a situation in which pupils will use in the classroom methods that are similar to methods used to gain knowledge in scientific research (Artigue and Blomhoj, 2013). Jorde et al (quoted from Ropohl et al, 2013) stress that pupils must get involved in authentic problem solving, experimenting and rich communication with their peers and their teacher.

Inquiry in primary school mathematics is launched by an appropriate problem (properties of these problems are listed e.g. in Samková et al, 2015). Pupils grasp the problem, look for ways of its solution (experiment with numbers or geometrical objects, create suitable models, search for regularities and relations), interpret results, formulate conclusions and generalizations, communicate their findings to classmates and the teacher. The teacher initiates the educational situation, helps pupils during the inquiry if requested (scaffolding), directs the final discussion (institutionalizes the discovered knowledge). Without any doubt this situation is more difficult for the teacher than standard transmissive approaches, as he/she must often react to unclear, sometimes only partially correct or incorrect pupils' contributions. Despite these difficulties the teaching public tend to speak highly of the potential of inquiry based education. However, at the same time they point at the difficulties related to implementation of inquiry based education in a classroom. One of these objections is the problematic nature of assessment of pupils' progress, especially if learning in inquiry based education is to be assessed in the traditional summative way, focusing more on knowledge than competences.

Since 2013 we have been actively involved in the solution of the project of 7th EU framework programme Assist-me (Assess Inquiry in Science, Technology and Mathematics Education), whose goal is to create and test a set of concrete procedures of formative assessment, e.g. continuous teacher's interactions with pupils (remarks, recommendations, "*ad hoc*" corrections), self-assessment and peer-assessment, marking and grading of pupils' solutions. The Czech team is responsible for application of peer-assessment in mathematics and science education. The here presented study focuses on teaching mathematics on primary school level.

What do we understand by formative assessment?

Assessment is a mental process that gives feedback information on the quality of the assessed object (i.e. the concerned object or activity) and can be communicated (Slavík, 1999). Assessment as part of the learning process includes collection and interpretation of data relevant for a pupil's learning. The purpose is (a) to help learning of the pupils in the process of learning (the formative use of assessment, in other words *assessment for learning*), (b) to find out what the pupils have learned (the summative use of assessment, simply said *assessment of learning*). (Black and Wiliam, 1998; Black et al, 2004; Earl and Katz, 2006).

Formative assessment is especially important in the situation of inquiry based education. Peer-assessment, which is the focus of this study, is understood here in accordance with Slavík (2003) as a way to autonomous assessment, i.e. deeper reflection on one's own learning and its results "… that learners use on their own, master it, that they understand to the needed extent, that they can explain or defend" (Slavík, 2003: 14). Development and deepening of autonomous assessment partially depends on self-assessment and partially on assessment of somebody else's performance (most likely of a classmate, i.e. peer-assessment) through which pupils learn to reflect on their work (Slavík, 1999). Pachler et al (2010) stressed that learner self-regulation is a core factor in formative assessment and that it is linked to motivation and emotional factors which affect learners' engagement.

The goal of the research study is to identify those phenomena that could be observed in a planned implementation of inquiry based education in mathematics at primary school level. Integral part of this teaching experiment was pupils' formative peer-assessment.

Materials and Methods

This study is based on a qualitative analysis of 20 teaching experiments conducted in mathematics lessons on primary school level. The experiments were conducted in 3 stages: 6 experiments were conducted in the 5th grade (pupils aged mostly 10) in autumn 2014, 6 experiments in the 2nd grade and 2 in the 3rd grade in spring 2015, 3 experiments in 5th grade and 3 in 4th grade in autumn 2015. Each stage was conducted in collaboration of two teachers from parallel classes and the authors of this study (the last stage in cooperation of 2 teachers from neighbouring classrooms and authors of this study). The teacher, authors of the paper and one person technically supporting the recording of video were present in the classrooms during the experiment. The number of pupils varied from16 to 24. They solved the problems and assessed their peers in groups of 4.

The teaching experiments were conducted with the goal of creating space for pupils' independent inquiry and subsequent peer-assessment of the findings from this inquiry. The planning of the experimental lessons always had the form of a discussion among the teachers and the authors of this paper. In this discussion the learning objective of the particular teaching unit was formulated, the possible ways of achieving this objective proposed and the possibilities of implementation of formative assessment sought. Each of the teachers then created a detailed lesson plan for the teaching experiment in her own class. In most cases this meant creation of a worksheet into which the pupils recorded their solutions as well as assessments of their peers and reactions to this assessment.

The teaching experiments were 45 to 90 minutes long. All of them were video recorded and transcribed. Pupils' group discussions were recorded on voice recorders. All pupils' written production was documented. Also other data were collected to get as accurate view of the teaching experiment as possible: teachers' self-reflections, questionnaires structuring assessment of teachers' self-reflection, self-assessment tools for pupils.

The data were analysed qualitatively as we were entering *terra incognita.* Video recordings were watched more times, transcripts of them and pupils' written production studied to pinpoint the phenomena connected to formative assessment.

Results and Discussion

In this paper we will not analyse how pupils' cognition and knowledge has changed thanks to inquiry based education. We will comment on significant phenomena identified in implementation of assessment in the teaching experiments.

Supporting self-assessment and formative peer-assessment

The teaching experiment involved pupils with different experience with self-assessment and peer-assessment. That is why worksheets were prepared for assessment in the experiment. These worksheets were used both for recording the solution of the problem and assessment (a sample worksheet created for *Lentils problem* is in Appendix 1).

At first we asked questions to encourage assessment in general: Do you like your classmates' solutions? If not, what advice would you give them? However, answers to these questions gave no supporting feedback to the solvers. One of the possible ways to dealing with this problem seemed to split the solution into stages and ask for separate assessment of each stage. E.g. the pupils' task was to find how long "path" they will build if they put spaghetti from one package into a line. In this case we could have asked a sequence of questions: Did the solvers state correctly that there were about 500 spaghettis in one package? Did they measure that the length of one spaghetti is 23 cm? Did they calculate correctly the length of 500 spaghettis in one line? However, these questions make the evaluators acknowledge only one correct way of solving the problem. But the solvers could have proceeded in a different way. For example, they could have divided the spaghetti into ten roughly equal piles, out the spaghetti from one pile into one line, measure the length and multiply the result by ten.

This is why we were looking for a compromise. In case of the *Spaghetti problem* we asked the following questions: Did your classmates state the number of spaghetti correctly? Was their procedure of stating the length of the line correct? Do you understand their solution? What advice would you give them?

Formulation of learning objectives

It is essential and not only in an inquiry based lesson that the didactical goal to be achieved in the lesson should be clearly defined. This premise is often discussed in the materials for teachers. When teaching mathematics at primary school level in the Czech Republic it is often only the topic of the lesson that is defined and sometimes also the teacher's activities: "I will explain…", "We will revise…". The teacher is more likely to plan what problems will be included in the lesson, how they will be arranged and what form of class management will be used. We think this is partly the consequence of availability of workbooks and worksheets that enable this style of work. The teacher does not plan the lesson with respect to what the pupils will learn but with the objective of solving problems provided in textbooks, workbooks and worksheets correctly.

When planning the teaching experiments, we gradually found out that the quality of the feedback that pupils may gain from their solutions depends on accuracy of the definition and operationalization of the learning objective of the "inquiry". At the beginning the objectives defined in cooperation with the teachers were quite general (e.g. "get experience", "apply a known procedure in a new environment"). The experiments showed that it is essential for assessment to know what the pupil was expected to learn, to state the objective in terms of the expected pupil's performance. This can be illustrated on a problem in which the pupils were asked to formulate comprehensible instructions on how to determine the number of tiles needed for "tiling" the triangle in Fig. 1. Evaluators were assessing the correctness of the instructions by using these instructions for determining the number of tiles needed for tiling other triangles. This assessment could have been initiated by a concrete question: Did the solvers determine correctly that 15 tiles are needed? But asking this question would not correspond to the defined learning objectives: Pupils brush up pre-concepts that form the basis of measuring the area of a triangle, namely their experience with filling in a triangle with e.g. a square (i.e. by a selected unit).

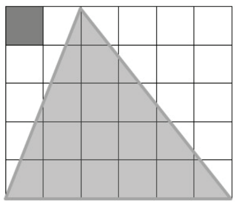

Figure 1 "Triangular" tiling

Correctness of solution of the problem and peer-assessment

In some problems the pupils had been solving during inquiry it was difficult to decide whether the problem had been solved correctly, in other words to assess the individual steps of the solving procedure described by their classmates, and it was equally as difficult to communicate this assessment in a comprehensible way. The pupils used two ways to overcome this difficulty:

- they compared their classmates' solution with their own solving procedure, which they naturally saw as correct (S1 in Table 1)
- they commented on the single steps of the solving procedure rather vaguely (S2 in Table 1).

In Table 1 we present solution and peer-assessment of the problem: How many lentils are there in a 500 g package? (worksheet in Appendix 1)

Solvers	Evaluators
First we found that there are 80 lentils in 5 g. 500: 5 = 100, 100. 80 = 8 000 So there should be 8 000 lentils in one package.	☺We like that they have the same principles as we do. ☹ But the weighing should be accurate.
First we found out what the mass of the package was. Then we counted the number of lentils in one gram. At the end we calculated a problem and got the result 93 258.	☹ We can't assess this. They didn't state what problem this was. So we don't know how they solved it.

Table 1: Examples of pupils' solution of *Lentils problem* **and peer-assessment**

- Solution S1 is correct. Evaluators express this by emotional "We like it" and support this by comparing it to their own solution. The comment on accuracy of weighing comes out of the evaluators' own experience.
- Solvers of S2 described some steps that were not needed for the solution (the mass of lentils in the package was given in the assignment); other steps were not described clearly enough to make a decision on their correctness. The evaluators commented on the second part of the solution quite clearly. However, they did not comment on the fact that it was not needed to find out "what the mass of the package was". If the problem was assessed by the teacher she would have undoubtedly drawn pupils' attention to this point, i.e. to assessment of whether all the steps were really needed for the solution. Other problem is that number of lentils is approximately 10 times higher than the right solution. The peers did not comment this fact.

Essential in inquiries are those erroneous contributions that move the solution forward. However, these are not often assessed by an evaluator who is familiar with similar methods of work as the solver. The situation becomes even more complicated if the solution is not described clearly and comprehensibly by the solver.

Peer-assessment and institutionalization of knowledge

Formulation of the objectives of an inquiry is connected to the issue of institutionalization of the gained knowledge. We found out that at the end of inquiry based lessons the pupils expected an unequivocal decision on what had been done correctly. They expected the result of their solving – the discovered knowledge – to be shared by the group, critically discussed and then accepted. The final summary was in the hands of the teacher. However, if the teacher had not stated the learning objective clearly enough, their summary was very vague ("You worked very nicely", "I am pleased with your work."). Our findings are in agreement with the Theory of Didactical Situations. The need for the inclusion of institutionalization phase was theoretically grasped by Brousseau and Balacheff (1997) and introduced in the model of so called a-didactical situation (a situation in which the teacher let the pupils to discover (part of) mathematical knowledge). The presence (and necessity) of institutionalization phase in situations independent problem solving have been confirmed in the Czech educational context (Novotná and Hošpesová, 2013).

Conclusions

Our analyses show there are phenomena that make peer-assessment in inquiry based education in primary school mathematics difficult:

- Inquiry may result in finding more ways of solving the problem; some of these ways might not be understood by peers and then may be assessed as incorrect.
- Children's ability to describe solving procedures is in the process of construction on primary school level. Sometimes even the teacher finds it difficult to understand what exactly the children were doing while solving the problem and what they achieved.
- Mathematical knowledge is of cumulative nature. If the evaluator has no idea about the structure of mathematical knowledge, they are likely to fail to formulate appropriated learning objective and discern a valuable or substantial mathematical idea.
- It is very difficult for the teacher to define the objectives of the inquiry and what and how to assess. In this the teacher needs support as well as time and space for discussion with other teachers.

Our experiments showed that pupils adapted to inquiry based education and peer-assessment very quickly. Pupils' willingness to inquire and to assess each other and their success in these activities developed as they were gaining experience. Becoming independent both in case of individuals and groups while solving problems and development towards autonomous assessment is a gradual process which must be given enough space at schools. In our project these opportunities were given in the way expressed by the following schema (Fig. 2).

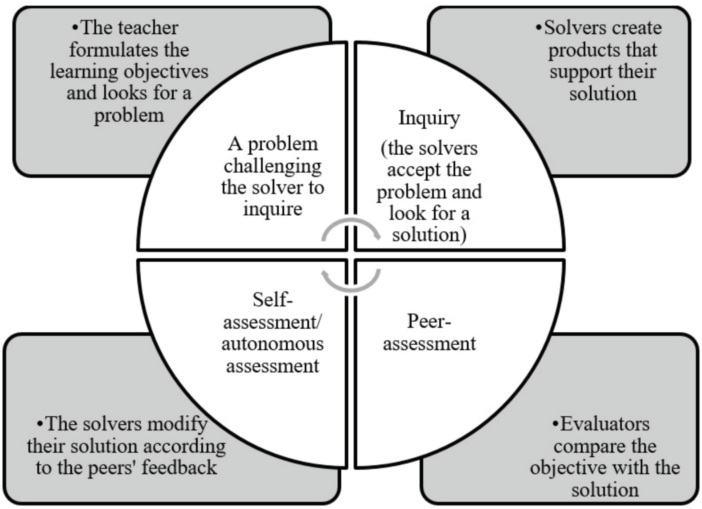

Figure 2: The process of formation of autonomous assessment (source: authors´ own schema)

Acknowledgements

This contribution was supported by the European Union's Seventh Framework Programme for Research and Development under Grant ASSIST-Me (Assess Inquiry in Science, Technology and Mathematics Education, number 321428).

We would like to thank teachers from primary schools Plešivec (Český Krumlov), Uhelný trh (Praha) and Jesenice for the willingness to look with us for the ways to inquiry based education in primary mathematics and its assessment.

References

Artigue, M. and Blomhoj, M. (2013) 'Conceptualizing inquiry-based education in mathematics', *ZDM Mathematics Education,* vol. 45, no. 6, pp. 797-810. http://dx.doi.org/10.1007/s11858-013-0506-6

Black, P. and Wiliam, D. (1998) 'Assessment and Classroom Learning', *Assessment in Education: Principles, Policy & Practice*, vol. 5, no. 1, pp. 7-74.

Black, P., Harrison, C., Lee, C., Marshall, B. and Wiliam, D. (2004) 'Working inside the Black Box: Assessment for Learning in the Classroom', *Phi Delta Kappan,* vol. 86, no. 1, pp. 8-21. http://dx.doi.org/10.1177/003172170408600105

Brousseau, G. and Balacheff, N. (1997) *Theory of didactical situations in mathematics: didactique des mathématiques, 1970-1990,* Boston: Kluwer Academic Publishers.

Earl, L.M. and Katz, S.M. (2006) *Rethinking classroom assessment with purpose in mind assessment for learning, assessment as learning, assessment of learning,* Winnipeg: Manitoba Education, Citizenship & Youth.

Novotná, J. and Hošpesová, A. (2013) 'Students and their teacher in a didactical situation. A case study', *Student Voice in Mathematics Classrooms around the World,* Rotterdam: Sense Publishers, pp. 133-142.

Pachler, N., Daly, C., Mor, Y., and Mellar, H. (2010) 'Formative e-assessment: practitioners cases', *Computers & Education*, vol. 54, no. 3, pp. 715-721. http://dx.doi.org/10.1016/j.compedu.2009.09.032

Ropohl, M., Rönnebeck, S., Bernholt, S. and Köller, O. (2013) *A definition of inquiry-based STM education and tools for measuring the degree of IBE (No. D2.5),* [Online], Available: http://assistme.ku.dk/project/workpackages/wp2/131015_del_2_5_IPN.pdf/ [14 Mar 2016]

Samková, L., Hošpesová, A., Roubíček, F. and Tichá, M. (2015) 'Badatelsky orientované vyučování matematiky', *Scientia in Educatione*, vol. 6, no. 1, pp. 1-33.

Slavík, J. (1999) *Hodnocení v současné škole: východiska a nové metody pro praxi,* Prague: Portál.

Slavík, J. (2003) 'Autonomní a heteronomní pojetí školního hodnocení – aktuální problém pedagogické teorie a praxe', *Pedagogika*, vol. 40, no. 1, pp. 5-25.

Appendix 1: Assessment Tool (Worksheet 1 - How many lentils are there in a 500 g package? (Coloured parts are intended for peer assessment. In the original version there was more space for pupils´ writings.)

Names of the pupils solving the problem:			
How did we proceed?			
Result:			
Names of pupils who assess the solution:			
Is the result correct? ☺ ☹	☺ What did you like in the solution?		
☹ Recommend your classmates how to get the correct number.			
Revision:			
Did the advice from your classmates help you? How? ☺ ☹			