Report from the FP7 project:

Assess Inquiry in Science, Technology and Mathematics Education

ASSISTME

Assessment method description for 'modeling in Mathematics' competence

Interactions on-the-fly on students' discourse

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1. Summary

This descripiton will provide ideas and inspiration on how to formatively assess 'modeling competence' using the on-the-fly assessment method. There will be a description of what students and what the teacher are expected to do (their task) and how students' learning working process could be formatively assessed.

Interaction on-the-fly certainly could be used in many fields of competence. Here the focus lies on the modeling competence in a paradigmatic example in Mathematics.

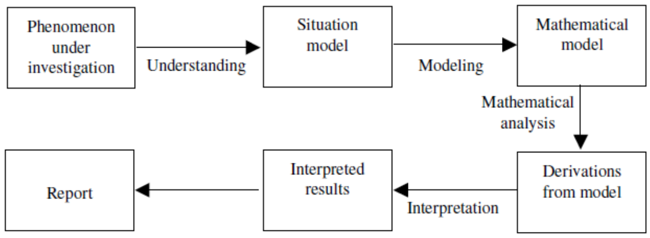
Subject	• Modeling competence generally integrateable in all Science subjects, in Mathematics and Technology education. • Paradigmatic example in a Mathematics unit on “paper, scissors, stone” game, approximately two lessons of 40 minutes (over one week).
School level	• Modeling competence applicable in lower and upper secondary and primary education level • Paradigmatic example in primary education
Assessed competences in the paradigmatic example	Key for the sub competences in modeling: • Understanding the phenomenon under investigation (situation model). • Constructing a mathematical model of the relevant elements, relations and conditions available in the situation model. • Working through the mathematical model using disciplinary methods in order to derive some mathematical results. • Interpreting the outcome of the computational work to arrive at a solution to the real – word problem situation that gave rise to the mathematical model. • Evaluating the model by checking if the interpreted mathematical outcome is appropriate and reasonable for the original problem situation.  The mathematical model proposed by Verschaffel et al. (2000)
Data collection about student learning	• Debate, discourse, role-play
Feedback method	• Interaction on-the-fly
Combination with summative assessment	• Description, guidelines and paradigmatic example for formative assessment, assessment criteria also usable for summative assessment.

Table 1. Main characteristics of assessment method "Interaction on-the-fly on students' debate and role-play"

2. Modeling competence

Modeling in Mathematics Education is a process of representing real world problems in mathematical problems using the mathematical terms in an attempt to find real world solutions to those problems (Ang, 2001). “A mathematical model can be considered as a simplification or abstraction of a (complex) real world problem or situation into a mathematical form, thereby converting the real world problem into a mathematical problem.” (Ang, 2001, p. 64).

A mathematical model is the one proposed by Verschaffel, Greer and De Corte (2000) (see figure 2). The main steps of this model are the subsequent (Panaoura, Gagatsis, & Demetriou, 2009, p. 67):

- Understanding the phenomenon under investigation, leading to a model of the relevant elements, relations and conditions that are embedded in the situation (situation model).
- Constructing a mathematical model of the relevant elements, relations and conditions available in the situation model.
- Working through the mathematical model using disciplinary methods in order to derive some mathematical results.
- Interpreting the outcome of the computational work to arrive at a solution to the real – word problem situation that gave rise to the mathematical model.
- Evaluating the model by checking if the interpreted mathematical outcome is appropriate and reasonable for the original problem situation.
- Communicating the solution of the original real – word problem.

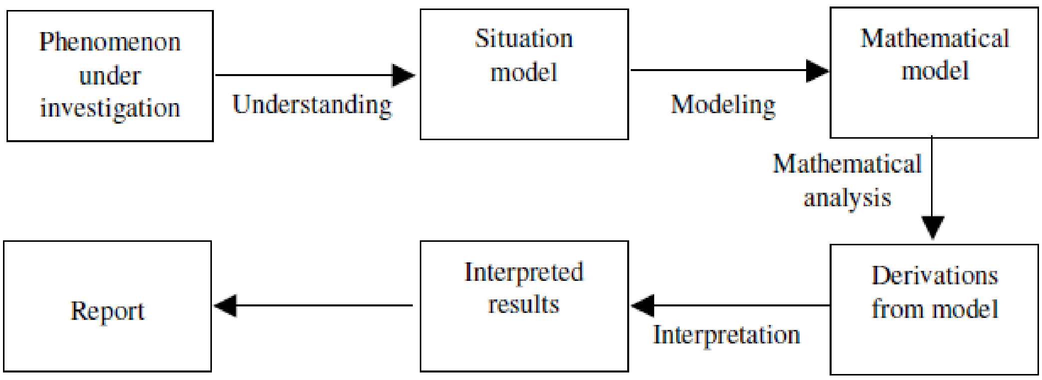


Figure 2: The mathematical model proposed by Verschaffel et al. (2000).

The several steps of the model proposed by Verschaffel et al. (2000) correspond to the modeling practices of the aforementioned Modelling Competence Framework (Papaevripidou, Nicolaou, & Constantinou, 2014). In particular, the first four steps of the Verschaffel et al. (2000) model are in line with the sub-competences of *constructing, using, evaluating and revising a model* and the last two steps are in line with the sub-competences of *comparing* and *validating a model* as presented in figure 1 in the Modelling Competence Framework.

A strategy commonly used to enhance problem solving ability in mathematics, is the usage of an analogy in an effort to create mental model of similar problems the transfer of knowledge of the constructed models into new situations/ problems. According to Panaoura, Gagatsis and Demetriou (2009) "*in order to successfully use analogy to create a mental model, students must be able to extract the relevant facts from the problem, compare it to prior knowledge base in the problem domain, and recognize relevant similarities between the current problem and previous ones that they had encountered*" (Panaoura, Gagatsis, & Demetriou, 2009, p. 67).

3. Description of the assessment method with guidelines how to use it

The feedback method "interactions on-the-fly" describes informal formative feedback. This chapter will provide a description of the principle along with short summaries of different varieties.

Principle of interactions on-the-fly

"On-the-fly formative assessment arises when a "teachable moment" unexpectedly occurs, for example, when a teacher circulating and listening to the conversation among students in small groups overhears a students say that, as a consequence of her or his experiment, 'density is a property of the plastic block and I doesn't matter what the mass or volume is because the density stays the same for that kind of plastic.' The teacher recognizes the student's grasp of density and challenges the student with other materials to see if she or he and her or his group-mates can generalize the density idea." (Shavelson et al., 2008, p.300).

Complementary to 'on-the-fly formative assessment' is 'planned-for-interaction formative assessment'. Planned-for-interaction formative assessment includes marking (see chapter 6.2); peer- and self-assessment (see chapter 6.3); open classroom discussion and structured classroom dialogue (see chapter 6.4).

Varieties (non-exhaustive list)

Assessment conversation (Duschl, 2003; Duschl & Gitomer, 1997; Ruiz-Primo & Furtak, 2006)

Ruiz-Primo and Furtak (2004), Ruiz-Primo and Furtak (2006a), and Ruiz-Primo and Furtak (2006b), describe typical assessment conversations as a four-step cycle, where the teacher elicits a question, the student responds, the teacher recognizes the student's response, and then uses the information collected to student learning (see figure 3). 'Eliciting' means evoking, educing, bringing out, or developing. To describe a teacher's actions as eliciting during informal formative assessment is thus an accurate description, as teachers are calling for a reaction, clarification, elaboration, or explanation from students. Typical examples of such eliciting questions include "Why do you think so?" or "What does that mean?" (Ruiz-Primo & Furtak, 2006b). During informal formative assessment, teachers must react on the fly by recognizing whether a student's response is a scientifically accepted idea and then use the information from the response in a way that the general flow of the classroom narrative is not interrupted (e.g., calling students in the class to start a discussion, shaping students' ideas).

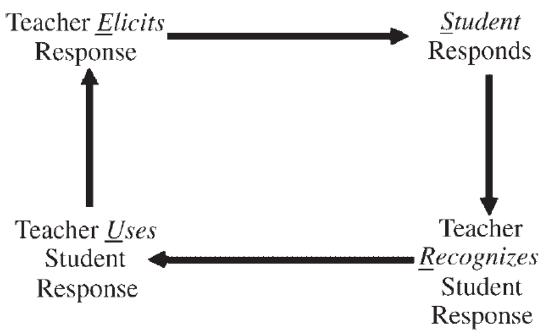


Figure 4: The ESRU model of informal formative assessment (taken from Ruiz-Primo and Furtak, 2006).

(Taken from ASSIST-ME report D4.7, p. 18-20)

4. Paradigmatic example: Mathematics, primary education

In this chapter, the use of a method for formatively assessing students' competence in creating a mathematical model for solving a mathematical problem given will be illustrated by an example. The example is for a Mathematics unit for the elementary school level and it taken from a booklet[1] for teachers developed during the LEMA project. LEMA[2] (Learning and Education in and through Modelling and Applications) was an EU Comenius funded project in which mathematics educators from six countries worked to produce materials to support teachers' professional development. In the context of this project, a booklet with thirty modeling tasks was developed, from where the following activity was inspired.

The task is to play the game "paper, scissors, stone" which is a popular two-person hand game. The game is often used as a selection method in a similar way to coin flipping, drawing straws, or throwing dice to randomly select a person for some purpose.

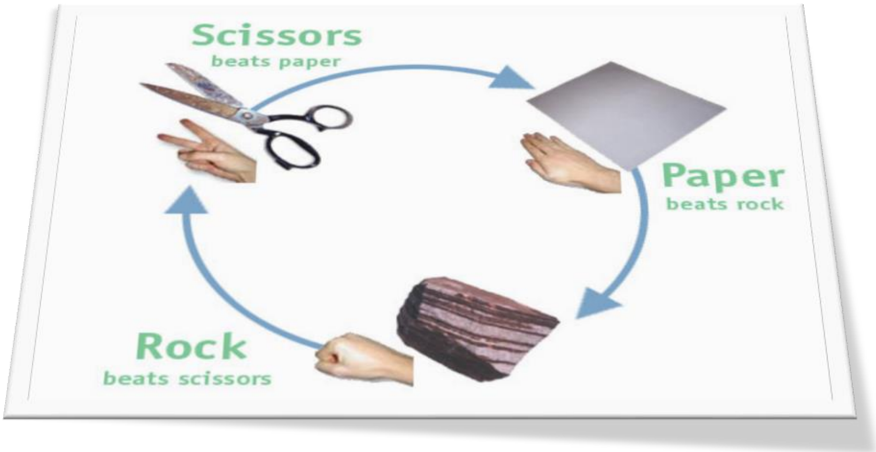


Figure 5. The "paper, scissors, stone" game (LEMA Project).

The rules of the game are:

- Paper beats Stone because paper wraps up stone.
- Stone beats Scissors because stone blunts scissors.
- Scissors beats Paper because scissors cuts paper.

As an additional resource, a simulation of this game can be found in the following link: http://www.weebls-stuff.com/games/Scissors+Paper+Stone/

The students work in groups and try to answer the following questions given by the teacher: "*Is this a fair game? Is there a way to win more often?* Those questions could represent the first step of the four-step cycle of the assessment conversation, described in the previous sub-chapter. The teacher could act as a facilitator for each group, hearing students' discussions within the groups and eliciting crucial questions in specific points in an effort to call for clarifications, elaboration and explanation from students and therefore collecting information about students' learning.

[1] http://www.primas-project.eu/servlet/supportBinaryFiles?referenceId=3&supportId=1245
[2] http://www.lema-project.org/web.lemaproject/web/eu/tout.php

In this example students need to understand the situation/ problem given and identify the elements and conditions embedded. Then, they should construct a mathematical model. In this example the students could work with probabilities (possible combinations between the three elements: paper, scissors, stone). A next step is to think of various ways of achieving a win more frequently. This could be answered by using the mathematical model of probabilities and derive some results. Those results could be confirmed with empirical data (playing the game). See figure 6 for some of the possible answers. Students could determine different ways of achieving this target. There is no correct answer; the aim of this task is mainly to engage students in problem solving and modeling situations.

Using these results, students should discuss within their groups and terminate a way to win more often. The teacher could prompt students with questions such as "*Why do you think so?*" or "*What does that mean?*" or "*Is it a fair game or not? Why?"* in order to extract information about students' understanding, reasoning and modeling skills. Depending on students' responses, the teacher could extent this task by asking "*What happens if you have a 4th element (for instance, a match that strikes stone and burns paper but is cut by scissors)?"*

In the context of the informal formative assessment, teachers should react on the fly by recognizing whether a student's response is a well-documented idea and then use the information from the response in a way that the general flow of the group narrative is not interrupted.

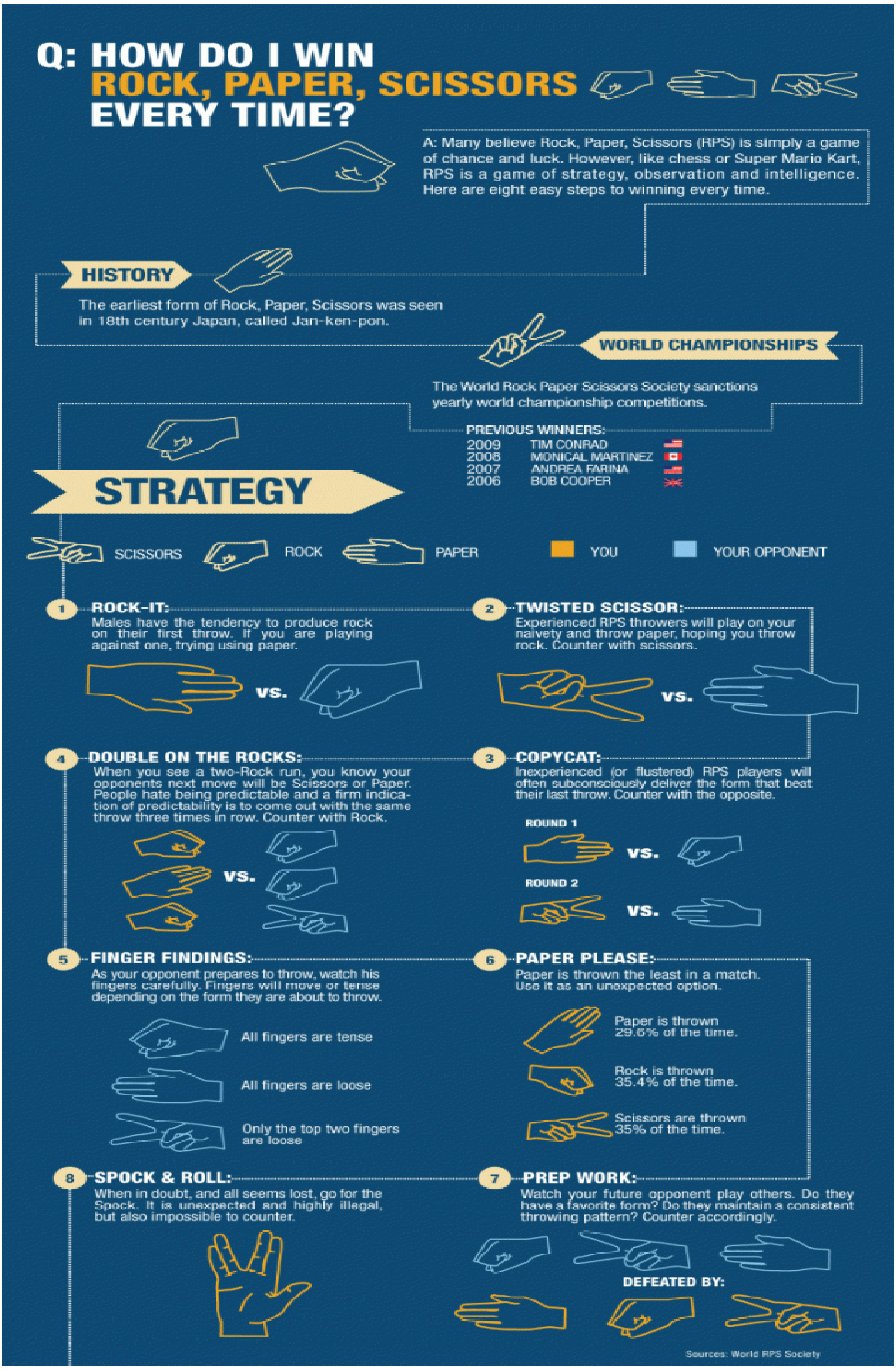


Figure 6. Ways of winning the game “paper, scissors, stone”
http://flowingdata.com/2010/07/30/how-to-win-rock-paper-scissors-every-time/

5. Assessment criteria

The following table displays typical teacher's strategies for the different dimensions of the ESRU cycle (compare to the first chapter).

Eliciting	Recognizing	Using
Epistemic frameworks		
Teacher asks students to:	Teacher:	Teacher:
• Compare/contrast observations, data, or procedures	• Clarifies/Elaborates based on students' responses	• Promotes students' thinking by asking them to elaborate their responses (why, how)
• Use and apply known procedures	• Takes votes to acknowledge different students' ideas	• Compares/contrasts students' responses to acknowledges and discuss alternative explanations conceptions
• Make predictions/provide hypotheses	• Repeats/paraphrases students words	• Promotes debating and discussion among students' ideas/conceptions
• Interpret information, data, patterns	• Revoices students' words (incorporates students' contributions into the class conversation, summarizes what student said, acknowledge student contribution)	• Helps students to achieve consensus
• Provide evidence and examples	• Captures/displays students' responses/explanations	• Helps relate evidence to explanations
• Relate evidence and explanations		• Provides descriptive or helpful feedback
• Formulate scientific explanations		• Promotes making sense
• Evaluate quality of evidence		• Promotes exploration of students' own ideas
• Suggest hypothetical procedures or experimental plans		• Refers explicitly to the nature of science
• Compare/contrast others' ideas		• Makes connections to previous learning
• Check students' comprehension		
Conceptual structures		
Teacher asks students to:		
• Provide potential or actual definitions		
• Apply, relate, compare, contrast concepts		
• Compare/contrasts others' definitions or ideas		
• Check their comprehension		

Table 2: Typical teacher's strategies for the different dimensions of the ESRU cycle. Taken from Ruiz-Primo and Furtak, 2006.

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