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Assess Inquiry in Science, Technology and Mathematics Education

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Assessment method description for 'problem solving in mathematics' competence

Written comments and self-assessment to student protocol

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1. Introduction and summary

This description will introduce a method for formatively assessing pupils' competence of problem solving in general. There will be a description of what pupils are expected to do (their task) and how teacher's written comments and pupil's self-assessment could be provided.

Pupils individually work on a task to understand the decimal system. They form, read, write, display, arrange, and compare numbers over 1000 on the given place value rubic (picture below) and recognize, handle, and describe place value problems. They follow a description of the task that has 5 subtasks und provide a protocol of their performance. In an assessment criteria grid, pupils make a self-assessment and the teacher gives comments.

the given place value rubic

There is a paradigmatic example "Form an arrange numbers on the rubric" for grade 4 and one for grade 6. Furthermore, for both grades there is a practice orientated, simplified alternative problem solving task "Form and arrange amounts of money".

Subject	• Feedback method generally adaptable to all subjects • Paradigmatic example in mathematics; arithmetic and quantities. Topic: numer domain and decimal system, money
School level	• Assessment method generally adaptable to all levels • Paradigmatic examples for primary school (grade 4 and 6)
Assessed competences	Describing and understanding mathematical or „real world" problems • Planning and carrying out a problem solving strategy • Making and analyzing connections • Evaluating the strategy, the conjectures and the meaningfulness of the results • Generalising and systematizing the results and the specific problem solving strategy
Data collection about student learning	• Student protocol
Feedback method	• Written comments • Self-assessment
Combination with summative assessment	• paradigmatic example and feedback method for formative assessment, but generally usable for both formative and summative assessment

Table 1. Main characteristics of assessment method "Written comments and self-assessment to student protocol".

2. Description of the feedback method with guidelines how to use it

To assess pupils' competence of problem solving, an assessment criteria grid is filled out by the pupil itself (self-assessment) and the teacher (or maybe also another pupil). The criteria grid is structured in an approach to the basic idea (Z) and 4 or 5 subtasks (A to D/E). The creation and implementation of the criteria grid is explained in this chapter.

Suggestion for a competence model as a basis for the creation of a criteria grid

Following the proposed competence model with problem solving competences the modelling of selective criteria is conceivable. This way aspects of problem solving competences could be differentiated regarding levels of performance as well as be assigned to accents of mathematical operational competences. Yet this won't be selectively possible and shouldn't, as mentioned, be carried out in a segmentive way but rather pose a guideline and show tendencies:

	Task for the observation of problem solving competences	Accents of mathematical operational competences (())	
Z	**Approach to the idea of the basic task** Identify and grasp the task and comprehend the basis of the idea of the task	Know, perceive and describe* Operate and calculate*	Elemental
A	**Subtask A** (basic requirement) Develop and implement problem solving strategies	Research and explore* Operate and calculate*	Simple
B	**Subtask B** (basic requirement) Arrange, verify, complement, and represent problem solutions	Argue and justify* Display and communicate*	
C	**Subtask C** (extended requirement) Extend, transfer, and apply the task and the problem solution	Mathematize and model* Research and explore* Operate and calculate*	challenging
D	**Subtask D** (extended requirement) Reflect on, describe and evaluate the task and the problem solution	Interpret and reflect* Display and communicate* Argue and justify	

*=aspect of activity mathematics basic competence EDK (educational standards Switzerland)

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Criteria grid:

	Observation criteria for the fulfillment of the subtask of a comprehensive task	Self-assessment	Assessment by others	
Z	I am able to… To have access to the idea of the basic task (Fundament to deal with the subtasks A to E			basic
A	I am able to… **Subtask A** (basic requirement)			simple
B	I am able to… **Subtask B** (basic requirement)			simple
C	I am able to… **Subtask C** (advanced requirement)			challenging
D	I am able to… **Subtask D** (advanced requirement)			challenging

Evidence grid for the teacher

According to this competence model a grid can be developed for the teachers which describes, with the meaning of an alike teacher comment, even more precisely the requirements for the fulfillment of the observation criteria, records the reference to mathematical operational competences and support approaches. The support approaches should contribute to the assurance of the mathematical operational competences and are therefore rather oriented towards the implemented subject related competence model as well as local curricula teaching materials. Therefore they can only be used conditionally respectively hardly be specifically formulated.

	As to the criteria	competences	Support approach	
Z	Introduce the task (didactical advices)	Know, recognize, describe*	Leave out ... Use instead of the ... Repeat basic terms to the ...	Simple
A	Reduce the minimum number of solutions...	Research and explore*	Support via ...	

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		Operate and calculate*	Repeat basis competences to ...	
B	Offer arrangement-/ display help for ... (Support for the problem solution and the logging)	Argue and justify Display and communicate*	Edit deepening tasks to ... Learn to use arrangement-/ display help ...	
C	Set the focus of the observation on the skill of the ... Offer extension variation ...	Mathematize and model* operate and calculate*	Implement additional material as ... Train the imagination of the ...	Challenging
D	Accept as well formulations as ... Offer help for the display for the ...	Interpret and reflect* Display and communicate*	Exercise the description with ...	

To support the pupils in specific steps of the each subtask, teachers find a list with approaches in the appendix.

3. Paradigmatic example: Mathematics, primary level

In this paradigmatic example, the pupils work individually on a task about number domain and decimal system, additionally money. They form and arrange numbers or amounts of money on a rubric and follow different subtasks. See the worksheets in the appendix. In this chapter is an overview of the tasks. (All tasks from Hengartner, Hirt, Wälti. 2010)

1a. "Form and arrange numbers on the rubric" (grade 4)

Topic: Number domain (N^0) and decimal system

Learning target: To learn and deduce the number domain over 1000

1b. "Form and arrange numbers on the rubric" (grade 6)

Note: Same setting of task as in task 1a (higher level by expanding the number dimension from only whole numbers to Q with decimal fraction)

Topic: Number dimension (Q^0) and decimal system

Learning target: To learn the domain of rational numbers

2a. "Form and arrange amounts of money2 (grade 3/ 4)

Note: This is a practice oriented, simplified alternative of the foregoing problem solving task with plates on the rubric.

The thousands digit can also be displayed with banknotes in Swiss francs (or dollars). Therefore is this task rather simple with euros for the grade 4.

Topic: Number dimension (N^0), decimal system, money (euro)

Learning target: To learn and tap the number domain over 1000; handle decimal money pieces (banknotes or coins) and understand the decimal system

2b. "Form and arrange amounts of money" (grade 5/ 6)

This task can be implemented with any currency concerning the significant values (hundred, tenner, tenth, and hundredth).

Topic: Number dimension (Q^0), decimal system, money (euro)

Learning target: To learn and tap the number domain of rational numbers; handle decimal money pieces (banknotes or coins) and understand the decimal system

4. Didactical hints for the implementation in class

In this chapter, there is a guide for the teacher on how to introduce, accompany and assess the learning environment.

1. Production in the learning environment (initiate assessment event)

- **Introduce mathematical learning environment:**
  - ➢ join into the richer task situation
  - ➢ name the topic, inspire previous knowledge, tie in (*)
  - ➢ explain important terms (*)
  - ➢ show means, material, tools (*)
- **Explain objectives**
  - ➢ point up the task - visualize
  - ➢ play through examples (*)
  - ➢ clarify expectations and minimal objectives
  - ➢ open up free space, advanced requirements
  - ➢ name used social settings

(*) Possibility to activate previous to the actual phase of the self-activity with partner discussions, class discussions, pupils' contributions in front of the class, demonstrations and naming of tools by pupils, playing together with the class through a simple example etc.

2. Phase of self-activity (moderate assessment event)

- **Open up the learning environment**
  - ➢ observe, check, ensure understanding of the tasks
  - ➢ conduct individual final clarifications (avoid final clarification for the whole class and undertake only in an "emergency" if there's a lot unclear for a lot of children > final clarification interrupt started initiated thinking processes and disrupt the learning)
- **Accompany the learning environment**
  - ➢ advise and support individually (ask, listen, give advice, clarify requirements, confirm, challenge, apprehend difficulties, offer "help for self-help", strengthen social-dialogic learning, appreciate self-activity of the children, possibly note larger degree of private lessons and support for particular children on the pupil documents or the observation sheets respectively criteria in question)
  - ➢ mind a good learning climate, avoid exam situation and deficit orientation
  - ➢ Learning environments often times reach over several lessons. This allows for children to try out particular tasks a bunch of times, to delve into them, make improvements and corrections during the learning process, and to gradually accomplish the defined achievement criteria. For this reason learning environments offer support possibilities during the learning process. It should be evaluated if a pupil reaches the defined criteria in the end of the learning process (and not already in the beginning!).
  - ➢ observe, diagnose, estimate skills in different general competence areas (subject-, self-, and social competence)

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3. Exchange - Assessment (evaluate the event of assessment)

- **Exchange concerning learning installation (right after the realization)**
  - ➢ Exchange between pupils; show each other document possible s and record insights, if need be arrange a small exhibition (Let them examine works of other children, enable exchange conversations between the children: What did you discover, detect, realize...)
  - ➢ If need be enable self-assessment concerning the implementation and the learning results of the learning environment (short oral or written learning report: How did it go? What did I learn? What am I able to do now? etc.)
  - ➢ Give feedback concerning the learning environment: commitment of the pupils; highlight discoveries; name difficulties and problems (possible to let them be named by the pupils)
  - ➢ Possibly give hints regarding the continuations(), deepening, evaluations, exhibitions, application of the works out of the learning environment
- **Evaluation of the learning environment**
  - ➢ Overview the documents of the pupils and go over them in a first sighting
  - ➢ Refer to the results of the exchange and a possible self-assessment (Mind the quality of the task and the implementation and relate it to the criteria of evaluation)
  - ➢ If need be adjust the criteria of evaluation/ assessment
  - ➢ Evaluate and give feedback (criteria)
  - ➢ If need be give individual support (on criteria or on student document)
  - ➢ Gather the data of the whole class (for example with the overall assessment tool)
  - ➢ Generate possible support approach for the class
  - ➢ *Briefly comment the return of the student documents (highlight possible findings and support offer) for the whole class or deepening/ continuation of the learning content from the learning environment; if need be do individual supportive talks), possibly meetings with academic remedial teacher, therapists with student documents as illustration of the needs of support.*
  - ➢ *Support is possible for the individual learning environments if they are implemented in the following school year on a higher level (especially in the area of "number dimension extension and operations"). If so the children with a known difficulty should be treated and supported specifically.*

5. References

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Appendix

Problem solving task 1a: "Form and arrange numbers on the rubric" (grade 4)

Task with basic idea and 5 subtasks

Z) Place different numbers with one (1) plate.

Which numbers can you display? Read the numbers and write them down.

A) Place different numbers with always *two* plates. Write them down sorted by value. (You always have to place two plates.)

B) Place a number with *three* plates. Move a plate in the rubric a position to the right or left. Which numbers accrue now? Can you explain why the place number changes in this manner? (What happens when you move several plates?)

C) Form different numbers between 1'000 and 10'000 with always *four* plates. How many numbers can you form? (You always have to use all 4 plates! For this task you can also merely imagine the rubric or do a drawing.)

D) Describe how you approached task C.

What do you think? Can form more or less numbers between 1'000 and 10'000 with *five* plates? Describe and justify your opinion.

E) Chose a number of plates between 1 and 9. Form with the same number of plates different numbers. (You can as well take the formed numbers from task A, B or C).) Divide these numbers by 9 and always write as well the residual of the division down. What do you notice?

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		●		●	
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Problem solving task 1a: "Form and arrange numbers on the rubric" (grade 4)

Criteria grid for the self-assessment and assessment by other

	Observation criteria for the fulfillment of the subtask of a comprehensive task	Self-assessment	Assessment by others	
Z	I can place, read, and write down different numbers with 1 plate			basic
A	I can form and arrange at least 20 different numbers with 2 plate.			simple
B	I can explain alterations caused by shifting 1 plate to another position in the rubric.			
C	I can form all possible numbers in a number domain (with 3 plates).			challenging
D	I can coherently explain my approach for the solution of the task C.			
E	I can divide numbers, which are formed with plates, by 9 and describe what I notice.			

Z: Access (foundation to deal with the task A to E)
A, B: Task for all children (basic requirement)
C, D, E: Task for individual children (extended requirement)

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Problem solving task 1a: " Form and arrange numbers on the rubric" (grade 4)

Hints for the teachers:

	As to the criteria	**competences**	**Support approach**	
Z	Briefly introduce the task and clarify terms (demonstrate, possibly also with or by pupils)	Know, recognize, describe* *(problem capturing)*	Narrow down the number dimension (until 10'000 or 1'000, Leave ((HT)) and ((ZT)) "empty" on the rubric); Apply practical and simpler version with money.	Simple
A	The minimal number can be adapted if needed (there is a total of 28 different numbers possible). Integrity of all the combinatorial possibilities is not (yet) a criteria (see subtasks C and D). What matters is the forming and arranging of the numbers.	Research and explore* *(Develop and apply problem solving strategies)*	Simplification: Arrange given numbers up to 1'000 or 10'000; check and reduce significant value (see Z); Moderate support through exchange between the pupils concerning possible problem solving strategies (mathematics-meeting). *Hints: Add and arrange selectively and strategically from small up to large or the other way around?* *PLEASE NOTE: Don't give these hints for a selectively and strategically approach too fast but encourage good pupils to find already here all solutions (possibly name the total number of 28 as motivation).*	
B	Also accept drawings and calculations as reasons. If need be support with the formulation	Operate and calculate* Argue and justify*	Let the operation be executed or be taken place only in the imagination. Give and put a small number. Simplification: Give small numbers; enrichment: Give as large numbers as possible!	
C	Direct the observation focus on the strategical approach (concerted from the lowest to the largest possible number or vice versa or complement formed numbers; 19 solutions)	Mathematize and model* *("Generalization" of problem solving strategies)*	Let the operation be taken place or draw solutions or let it be developed in abstract form with numbers lists. Simplification: Reduce the number range or the number of plates; enrichment: Larger number range or number of plates.	Challenging
D	Also accept descriptions with number examples.	Interpret and reflect the results* *(problem reflection)*	Wording support with catchwords and repetition respectively visualization of terminologies concerning significant values.	
E	Challenging extra task concerning the topic 10er system and 9er rest. Direct the focus of observation on the ability of concerted	Operate and calculate* Research and	Give hints that the subtask can only be solved by dividing with the residual.	

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strategical research and accept simple descriptions (Residual is always the same, residual is always the same as the numbers of plates).	explore *("Generalization" of problem solving strategies)*		13

*=aspect of activity mathematics basic competence EDK (educational standards Switzerland)

Problem solving task 2a: "Form and arrange amounts of money (grade 3/ 4)

Task with basic idea and 5 subtasks

Z) Which coins and banknotes do you know? Count them.

Take for the following tasks only a bunch of 1 euro coins, 10 euro bank notes and 100 euro bank notes. Place different amounts of money with these pieces (1 euro coins, 10 euro bank notes and 100 euro bank notes) and name them.

A) Place different amounts of money with always *two* pieces*?. Write them down sorted by value. How many different amounts of money can you place?

B) Now place an amount of money with always *three* pieces?. How many different amounts of money are there now? (You always have to take three pieces!)

C) How many different amounts of money are there if you take from now on always *four* pieces? What do you think? Imagine this task and write down all amounts of money sorted by value!

D) Describe how you approached task C.

What do you think? How many amounts of money are there in each case with *five, six, seven, eight or nine* pieces? Can you describe the solution? What do you notice?

E) Divide the amounts of money from task A, B, C, and D always respectively on 9 persons. (Divide the amounts of money as well by 9 and always write down the residual of this division). What do you notice?

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* for example two 1 euro coins; one 1 euro coin and one 100 euro bank note; two 10 euro bank notes.

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Problem solving task 2a: "Form and arrange amounts of money (grade 3/ 4)

Criteria grid for the self-assessment and assessment by other

	Observation criteria for the fulfillment of the subtask of a comprehensive task	Self-assessment	Assessment by others	
Z	I know the euro money and can place and name different amounts of money with 1 euro coins, 10 euro bank notes, and 100 euro bank notes.			basic
A	I can form and arrange different amounts of money with 2 pieces. I know the euro coins and can place and name different amounts of money with 1 euro coins, 10 euro bank notes, and 100 euro bank notes.			simple
B	I can form and arrange every possible amount of money with 3 pieces.			
C	I can write down every possible amount of money with 4 pieces.			challenging
D	I can coherently explain my approach for the solution of the task C.			
E	I can divide formed amounts of money by 9 and describe what I notice.			

Z: Access (foundation to deal with the task A to E)
A, B: Task for all children (basic requirement)
C, D, E: Task for individual children (extended requirement)

Problem solving task 2a: "Form and arrange amounts of money (grade 3/ 4)

Hints for the teachers

	As to the criteria	competences	Support approach	
Z	Show euro money and clarify all terms for all coins and bank notes. Point to the limited input of decimal money value.	Know, recognize, describe* *(problem capturing)*	Repeat terms for money with game money and pictures.	Simple
A	Out of the 6 possible solutions most of the time all of them should be found. Yet integrity of the combinatory possibilities is not the central criteria (see subtask C and D). Important is here the competence of forming and arranging the money values.	Research and explore* *(Develop and apply problem solving strategies)*	Leave money values and/or draw. Develop an order system. Moderate support through exchange between the pupils concerning possible problem solving strategies (mathematics conference). *Hints: Add and arrange selectively and strategically from small up to large or the other way around?* *PLEASE NOTE: Don't give these hints for a selectively and strategically approach too fast but encourage good pupils to find already here all solutions (possibly name the total number of 28 as motivation).*	
B	Out of the 10 possible solutions all should now be found.	Operate and calculate* Argue and justify*	Let the operation be executed or be taken place only in the imagination. Enrichment: Give also different euro bank notes and examine changes.	
C	Direct the observation focus on the strategical approach (concerted from the lowest to the largest possible money value or vice versa or complement formed money values; 15 solutions)	Mathematize and model* *("Generalization" of problem solving strategies)*	Let the operation be taken place or draw solutions; invite to develop lists in most abstract form as possible. Enrichment: Change to the alternative with the rubric or add different euro bank notes.	Challenging
D	Also accept simple descriptions with examples, drawings or money value series.	Interpret and reflect the results* *(problem reflection)*	Wording support with catchwords and repetition respectively visualization of terminologies concerning money (coins and bank notes).	
E	Challenging extra task concerning the topic 10er system and 9er rest. Direct the focus of observation on the ability of concerted strategical research and accept	Operate and calculate* Research and	Give hints that the subtask can only be solved by dividing with the residual.	

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| most simple descriptions (residual is always the same, residual is always the same as the numbers of pieces). | explore

("Generalization" of problem solving strategies) | | 17 | |

*=aspect of activity mathematics basic competence EDK (educational standards Switzerland)

Problem solving task 1b: "Form and arrange numbers on the rubric" (grade 6)

Task with basic idea and 5 subtasks

Z) Place different numbers with one (1) plate.

Which numbers can you display? Read the numbers and write them down.

A) Place different numbers with always t*wo* plates. Write them down sorted by value. (You always have to place two plates.)

B) Place a number with *three* plates. Move a plate in the rubric a position to the right or left. Which numbers accrue now? Can you explain why the placed number changes in this manner? (What happens when you move several plates?)

C) Form different numbers between 0,1 and 3 with always *four* plates. How many numbers can you form? (You always have to use all 4 plates! For this task you can also merely imagine the rubric or do drawings.)

D) Describe how you approached task C.

What do you think? Can you form more or less numbers between 0,1 and 3 with *five* plates? Describe and justify your opinion.

E) Chose a number of plates between 1 and 9. Form with different numbers with the same number of plates. (You can as well take the formed numbers from task A, B or C). Calculate the difference between the numbers and make a note of it. What do you notice?

H	Z	E	z	h	t
●			●		●

Problem solving task 1a: "Form and arrange numbers on the rubric" (grade 4)

Criteria grid for the self-assessment and assessment by other

	Observation criteria for the fulfillment of the subtask of a comprehensive task	Self-assessment	Assessment by others	
Z	I can place, read, and write down different numbers with 1 plate			basic
A	I can form and arrange at least 20 different numbers with 2 plates.			simple
B	I can explain alterations caused by shifting 1 plate in the rubric.			
C	I can form all possible numbers in a number domain (with 3 plates).			challenging
D	I can coherently explain my approach for the solution of the task C.			
E	I can divide numbers, which are formed with plates, by 9 and describe what I notice.			

Z: Access (foundation to deal with the task A to E)
A, B: Task for all children (basic requirement)
C, D, E: Task for individual children (extended requirement)

Problem solving task 1a: "Form and arrange numbers on the rubric" (grade 4)

Hints for the teachers:

	As to the criteria	competences	Support approach	
Z	Briefly introduce the task and clarify terms (demonstrate possibly also with or by pupils)	Know, recognize, describe* (problem capturing)	Narrow down the number dimension (For example leave hundred (H) and thousand (t) "empty" on the rubric; Apply practical and simpler version with money.	Simple
A	The minimal number can be adapted if needed (there is a total of 28 different numbers possible). Integrity of all the combinatorial possibilities is not (yet) a criteria (see subtasks C and D). What matters is the forming and arranging of the numbers.	Research and explore* (Develop and apply problem solving strategies)	Simplification: Check or reduce significant values (see Z); Moderate support through exchange between the pupils concerning possible problem solving strategies (mathematics-meeting). *Hints: Add and arrange selectively and strategically from small up to large or the other way around?* *PLEASE NOTE: Don't give these hints for a selectively and strategically approach too fast but encourage good pupils to find already here all solutions (possibly name the total number of 28 as motivation).*	Simple
B	Also accept drawings and calculations as reasons. If need be support with the formulation.	Operate and calculate* Argue and justify*	Let the operation be executed or be taken place only in the imagination. Give and put a small number. Simplification: Give small numbers in a reduced number dimension; enrichment; Extend the number dimension with additional significant values!	
C	Direct the observation focus on the strategical approach (concerted from the lowest to the largest possible number or vice versa or complement formed numbers; 23 solutions)	Mathematize and model* ("Generalization" of problem solving strategies)	Let the operation be taken place or draw solutions or let them be developed in abstract form with numbers lists. Simplification: Reduce the number range or the number of plates; enrichment: Larger number range or bigger number of plates.	Challenging
D	Also accept descriptions with number examples.	Interpret and reflect the results* (problem reflection)	Wording support with catchwords and repetition respectively visualization of terminologies concerning significant values.	Challenging

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E	Challenging extra task concerning the topic 10er system and 9er rest. Direct the focus of observation on the ability of concerted strategical research and accept simple descriptions (the differences always add up to ((9er numbers).	Operate and calculate* Research and explore *("Generalization" of problem solving strategies)*	Give hints that the subtask can only be solved by dividing with the residual.	

*=aspect of activity mathematics basic competence EDK (educational standards Switzerland)

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Problem solving task 2b: "Form and arrange amounts of money (5./6. class)

Task with basic idea and 5 subtasks

Z) Which coins and bank notes do you know? Count them.

Take for the following tasks only "decimal money" (1 cent coins, 10 cent coins, 1 euro coins, 10 euro bank notes, and 100 euro bank notes). Place different amounts of money with these pieces and name them.

A) Place different amounts of money with always *two* pieces. Write them down sorted by value. How many different amounts of money can you place?

B) Now place different amounts of money with always *three* pieces. How many different amounts of money can you place? (You always have to take three pieces!)

C) How many different amounts of money are there if you take from now on always *four* pieces? What do you think? Imagine this task and write down all amounts of money from the smallest possible amount through 40 euro sorted by value!

D) Describe how you approached task C.

Examine the range of money values from task C! What do you notice?

E) Calculate the difference between the amounts of money from task A. Examine as well the differences between the amounts of money form task B, C, and D which were placed with the same amount of pieces. What do you notice?

Problem solving task 2b: "Form and arrange amounts of money (grade 5/ 6)

Criteria grid for the self-assessment and assessment by other

	Observation criteria for the fulfillment of the subtask of a comprehensive task	Self-assessment	Assessment by others	
Z	I know the euro money and euro bank notes and can place and name different amounts of money with 1 cent coins, 10 cent coins, 1 euro coins, 10 euro bank notes, and 100 euro bank notes.			basic
A	I can form and arrange different amounts of money with 2 pieces.			simple
B	I can form and arrange every possible amount of money with 3 pieces.			
C	I can write down every possible amount of money up to 40 euro with 4 pieces.			challenging
D	I can coherently explain my approach for the solution of the task C.			
E	I can calculate differences and describe what I notice.			

Z: Access (foundation to deal with the task A to E)
A, B: Task for all children (basic requirement)
C, D, E: Task for individual children (extended requirement)

Problem solving task 2b: "Form and arrange amounts of money (grade 5/ 6)

Hints for the teachers:

	As to the criteria	**competences**	**Support approach**	
Z	Show euro money and clarify all terms for all coins and bank notes. Point to the limited input of decimal money value.	Know, recognize, describe* *(problem capturing)*	Repeat terms for money with game money and pictures.	Simple
A	Out of the 15 possible solutions most of the time all of them should be found. Yet integrity of the combinatory possibilities is not the central criteria (see subtask C and D). Important is here the competence of forming and arranging the money values.	Research and explore* *(Develop and apply problem solving strategies)*	Leave money values and/or draw. Develop an order system. Moderate support through exchange between the pupils concerning possible problem solving strategies (mathematics conference). *Hints: Add and arrange selectively and strategically from small up to large or the other way around?* *PLEASE NOTE: Don't give these hints for a selectively and strategically approach too fast but encourage good pupils to find already here all solutions (possibly name the total number of 28 as motivation).*	Simple
B	Out of the 25 possible solutions all should now be found.	Operate and calculate* Argue and justify*	Let the operation be executed or be only taken place in the imagination. Enrichment: Also give different euro bank notes and examine changes.	
C	Direct the observation focus on the strategical approach (concerted from the lowest possible amount of money of 00.04 € to the largest possible money value of 40 € or vice versa from 40 € to 4 cents; 35 solutions)	Mathematize and model* *("Generalization" of problem solving strategies)*	If need be let the operation be taken place or draw solutions; invite to develop lists in most abstract form as possible. Enrichment: Change to the alternative with the rubric or add different euro bank notes.	Challenging
D	Fitting and coherent description of the solution image from task C. Simple (pattern-) awareness concerning the relation between the number of used places (used money values) and the number of possible solutions (starting from 4	Interpret and reflect the results* *(problem reflection)*	Wording support with catchwords and repetition respectively visualization of terminologies concerning money (coins and bank notes).	Challenging

ASSISTME

	cents: 5,4,3,2,1;4,3,2,1;3,2,1;2,1;1)		
E	Challenging extra task concerning the topic 10er system and 9er rest. Direct the focus of observation on the ability of concerted strategical research and accept most simple descriptions (the differences always yield (9er numbers).	Operate and calculate* Research and explore *("Generalization" of problem solving strategies)*	Give hints that the subtask can only be solved by dividing with the residual.	

*=aspect of activity mathematics basic competence EDK (educational standards Switzerland)