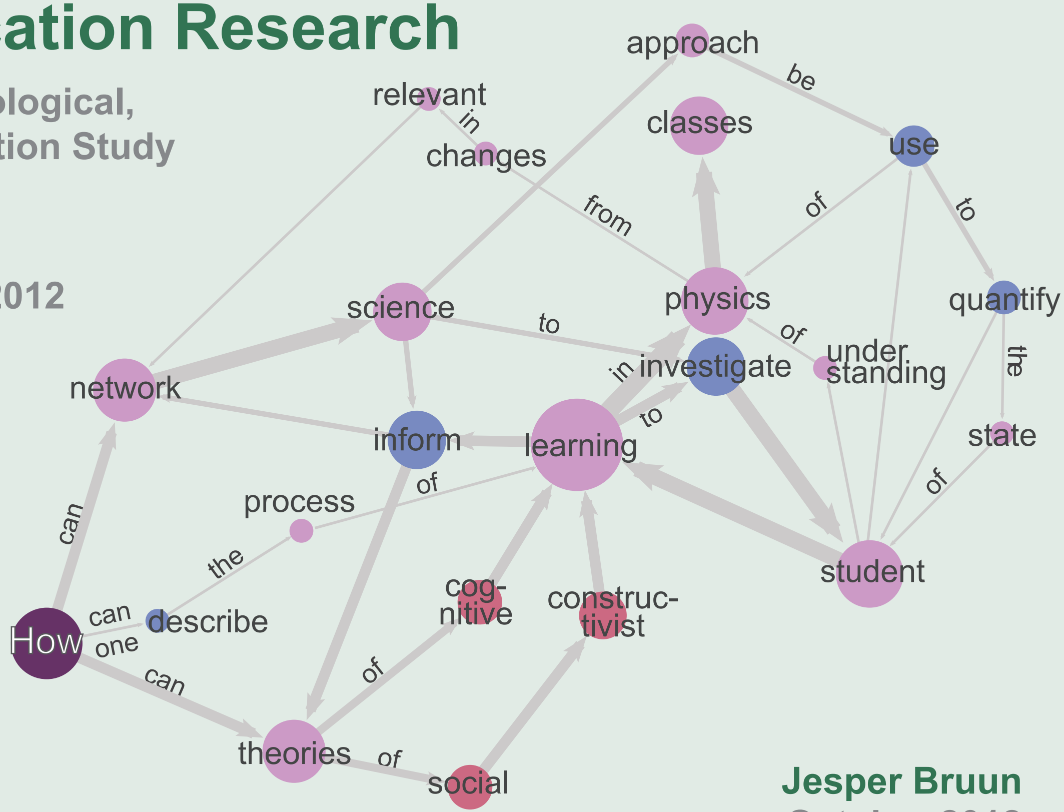
DEPARTMENT OF SCIENCE EDUCATION
UNIVERSITY OF COPENHAGEN

Networks in Physics Education Research

A Theoretical, Methodological, and Didactical Exploration Study

Doctoral Dissertation 2012



Jesper Bruun
October 2012

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Department of Science Education, University of Copenhagen

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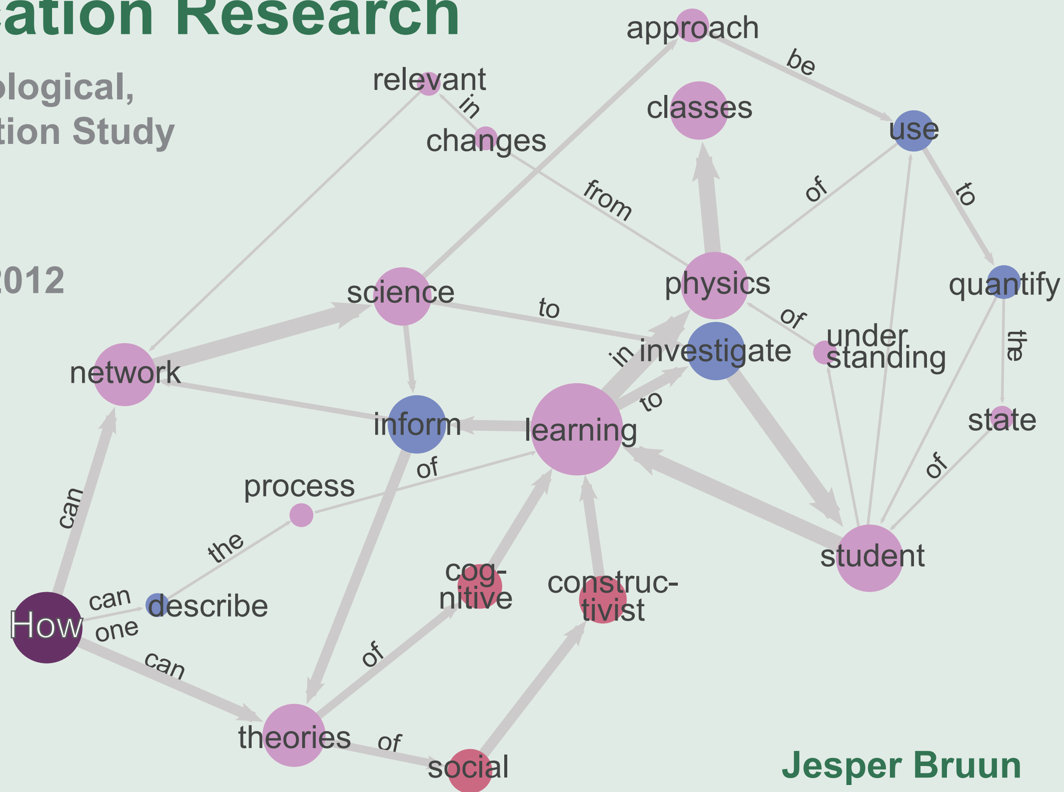

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October 2012

This book is an edited version of the dissertation that was submitted on October 1st 2012. Various changes have been made to page layout and formatting. I am thankful to Signe Skriver Hedegaard, Adrienne Traxler, and Peter Fallesen who had the time to help me with grammar, spelling, and language in the first two chapters. Also, figures have been changed to suit the new layout and format. This is an e-book color version of the thesis. For the black and white print version, please visit lulu.com

Published by the Department of Science Education,
University of Copenhagen, Denmark

E-version at ind.ku.dk/skriftserie

Printed at lulu.com. The dissertation can be bought through the marketplace at lulu.com.

Networks in physics education research: A theoretical, methodological and didactical explorative study, IND Skriftserie 28. ISSN: 1602-2149

Cover illustration: Network of research questions. Illustration by Jesper Bruun.

Please cite as: Bruun, J. (2012). *Networks in physics education research: A theoretical, methodological and didactical explorative study* (IND Skriftserie vol. 28). Copenhagen: Department of Science Education.

Contents

Acknowledgements

The Department of Science Education at The University of Copenhagen has been a great place to work with science education research. I have had the great privilege of working with almost everybody, and I have participated in meaningful and fruitful discussions about my own work and the work of others. The department as a whole, values diversity both in people, theoretical position, and methodology, and it is held together by the firm but loving hands of the highly competent technical and administrative personnel. Because of all these things, the department truly feels like an interdisciplinary work place. Here, I have been able to play with many different ideas about physics learning.

Although I acknowledge each department member for things that I believe have contributed positively to my work, I will confine myself to mentioning my supervisors explicitly. Jens Dolin was the supervisor of my Master's thesis and we continued to work together with this PhD project. I have always left our supervision sessions feeling enriched and more curious than before. This is also the case with my co-supervisor Kim Sneppen from the Niels Bohr Institute of Physics, who has been instrumental for my understanding of and work with networks.

Another category of people I wish to acknowledge are students and teachers, whom I have had the privilege to work with. Needless to say, teachers are gatekeepers to the students and I strongly believe that without their enthusiasm, the data, I have collected from students, would have been of poorer quality. In this respect, Morten Brydensholt as well as Ian Bearden have made this study possible. I also want to thank Morten for his technical work with Moodle, since quite literally, this made data collection possible.

Even if the teachers are enthusiastic, students do not need to be. However, in the cases of university students as well as upper secondary students, I am amazed at the tenacity with which many students have responded to my surveys and questions week after week. When meeting some of these students on occasion, they always show an interest in how the work is proceeding. I hope to illustrate that it went alright.

Working with networks was quite lonely, until I saw a poster about social networks in a physics learning center. This was at the National Association for Research in Science Teaching conference in Orlando and it led to a three month stay in Miami with the PER group at Florida International University. Again I want to express my thanks to the whole group for great collaboration, especially to Eric Brewe and Natan Samuels, who I also became my friends.

Finally, all the friends, family and everyone in between: Bjørn, Hjorterne, Kitt, Lars, Mantastic, Mor Birgit, Morbror Peter, Nighthawks, Peter Bror, Rebecca, Sacha, The Boys, and The Roleplayers. Thank you.

Summary

This thesis is about using complex networks to examine how physics students learn physics. It is an interdisciplinary and highly exploratory project between physics and science education, which results in a dissertation containing elements from network physics as well as constructivist learning theories. The premise of the thesis is that networks offer a way of quantifying learning situations, which can contribute to an understanding of what learning in physics entails. The thesis consists of a synopsis and four articles.

The synopsis develops the theoretical foundation for linking learning theory with network theory. The starting point is a tripartite division of the field of research in science teaching, which has a tension between measurements on learning situations, learning theoretical models, and didactical considerations. The goal of the second chapter of the synopsis is to make it probable that network theory can help to resolve some of this tension. The result of the second chapter is three broad research questions: The Theoretical Question, The Measurement Question, and the Didactical Question. These three questions are the recurring themes in the following articles.

The first and longest article describes a method to develop a network-based method which can capture socio-cultural aspects of learning. The starting point is open survey responses, where students in a Danish upper secondary physics class name, with whom they have worked in physics in the past week and give a short description of the work. The students' open response to this question is integrated with physics education research literature and socio-cultural learning theory to develop eleven different categories describing what students communicate about in physics. These categories are used in a new survey, where students can name, for example, whom they remember having communicated with regards to problem solving in physics. The students' are asked for replies weekly and their answers are used to create student interaction networks of class communication patterns within different categories. The networks are analyzed using network theoretical quantitative measures. Subsequently, the network analysis results are compared with students' reflections on what led them to name specific individuals and with the teaching plan for the class. From a network theoretical point of view, laboratory work is characterized by more ineffective communication than normal class communications. This leads to the hypothesis that a change in laboratory work should result in changes in the communication patterns measured by network theory.

The second article is a network theoretical article which examines how first-year university physics majors organize

themselves into groups. The data collection follows the same pattern as the data collection in the first article: Students are asked to name the individuals with whom they remember having communicated during the past week. They can choose between all students in the study and from the same eleven categories as in the previous study. The article shows that the network, which maps the academic interactions of students, seems to stabilize towards the end of the measurement period. This means that the students interact with numerous others at the beginning of their study, but they return to previous partners at the end. The article also develops and tests a measure of how students divide themselves in relation to gender, lab exercise classes, and grade. The measure is called the segregation. There is a significant but not large segregation according to gender and a significant and large segregation according to laboratory class number. There is no significant segregation with regards to grade.

The third article combines university student networks from the first nine weeks of their studies. Whereas the previous article analyzes the networks week-by-week, the links in this article are summed together to give a picture of how much individual students have communicated with each other over the course of the first nine weeks. The article uses categories developed in the first paper: problem solving, concept discussion and, in-class social communication. The last category describes non-physics related communication taking place at scheduled classes, lab exercises, and lectures. It turns out that the networks describing social communication outperform the two other categories when predicting a student's grade in later courses from their positional advantage in the current network.

Whereas the first three articles are concentrated on socio-cultural aspects of learning, in the fourth article I show that the network can also describe elements of students' cognitive state. The article argues that mental structures used to structure physics academic thinking can be captured by networks of words students use to describe how a physical system works. The article provides simple examples of these networks, made from high school students' responses to an open-ended problem concerning waves on a string.

Sammenfatning

Denne afhandling handler om hvordan man kan bruge netværk som en matematisk størrelse til at undersøge hvordan fysikstuderende lærer fysik. Det er et tværfagligt og i høj grad afsøgende projekt mellem fysik og undervisningsvidenskab, hvilket gør at afhandlingen indeholder både fysikfaglige elementer i form af netværksteori og undersvisningsfaglige elementer i form af læringsteoretiske analyser. Præmissen for afhandlingen er, at den kvantificering netværk tilbyder kan bidrage til en udvidet forståelse af hvad læring i fysik indebærer. Afhandlingen består af en kappe og fire artikler.

Kappen udvikler det teoretiske fundament for at koble læringsteori med netværksteori. Udgangspunktet er en tredeling af feltet for forskning i fysikundervisning, hvor der er en spænding mellem målinger på læringssituationer, læringsteoretiske modeller, og didaktisk brug. Målet med kappens første kapitel er at sandsynliggøre, at netværksteori kan bidrage til at løsne denne spænding. Kappen udmønter sig i tre brede forskningspørgsmål: Det Teoretiske, Målingsspørgsmålet, og Det Didaktiske spørgsmål. Disse tre spørgsmål er de gennemgående temaer i de følgende følgende artikler.

Den første og længste artikel beskriver en metode til at udvikle en netværksbaseret metode, der kan indfange sociokulturelle aspekter ved læring. Udgangspunktet er åbne surveybesvarelser, hvor studerende i en dansk gymnasieklasse navngiver, hvem de har arbejdet sammen med i fysik i den forgangne uge. De studerende giver ogsåen kort beskrivelse af, hvad de har arbejdet sammen om. De studerendes åbne svar på, hvad de arbejder sammen om, bruges sammen med fysikdidaktisk literatur og sociokulturel læringsteori til at udvikle elleve forskellige kategorier der beskriver, hvad studerende kommunikerer om i fysik. Kategorierne anvendes i et nyt survey, hvor de studerende nu kan svare om hvorvidt de kommunikerede sammen med bestemte navngivne andre om opgaveløsning. Kategorierne afprøves på samme klasse på et senere tidspunkt i deres studieforløb, og de studerendes ugentlige navngivninger inden for kategorierne bruges til at lave netværk af klassens kommunikationsmønstre. Netværkene analyseres ved brug af netværksteoretiske beregninger. Beregningerne kobles efterfølgende til både studerendes reflektioner over hvad der fik dem til at navngive bestemte personer og til læreplanerne for klassen. Ud fra et *netværksteoretisk* synspunkt er laboratoriearbejde karakteriseret ved mere ineffektiv kommunikation end normal klassekommunikation. Det leder til en hypotese om, at en ændring i laboratoriearbejdet skulle kunne måles som en ændring i kommunikationsmønstre.

Den anden artikel er en netværksteoretisk artikel som undersøger, hvordan førsteårsstuderende på fysikstudiet på Københavns Universitet organiserer sig i grupper. Dataindsamlingen følger samme mønster som den anden dataindsamling i første artikel: De studerende bliver ugentligt bedt om om at krydse de personer af, de kan huske at have kommunikeret med inden for den seneste uge. De kan vælge mellem alle studerende på studiet og de samme elleve kategorier som i det tidligere studie. Artiklen gør brug af én af de kategorier der blev udvilklet i den første arikel - kategorien der angiver, om to studerende har kommunikeret sammen om at løse en fysikopgave (opgaveløsning) - til at vise, at netværket af studerende ser ud til at blive mere fastfrosset henover måleperioden. Det vil sige at de studerende afprøver mange muligheder for at interagere i begyndelsen af deres studie, men de vender tilbage til udvalgte tidligere samarbejdspartnere senere i forløbet. Artiklen udvikler og afprøver også et mål for hvordan studerende opdeler sig i forhold til køn, regne- og laboratoriehold, og karakter. Der er en signifikant men substantielt ikke ret stor opdeling i forhold til køn og en signifikant og substantiel stor opdeling i forhold til regne- og laboratoriehold. Der er ikke en signifikant opdeling af de studerende i forhold til karakterer.

Den tredje artikel kombinerer universitetsstuderendes netværk fra de de første ni uger af deres studietid. Hvor den forrige artikel analyserer netværkene uge for uge, er forbindelserne i denne artikel lagt sammen for at give et billede af hvor meget de enkelte studerende kommunikerer med hinanden. Artiklen anvender *opgaveløsningskategorien* nævnt ovenfor og to andre kategorier, *begrebskommunikation* og *social kommunikation i forbindelse med forelæsninger, laboratoriearbejde, og/eller regnetimer*. Det viser sig, at netværkene der beskriver social kommunkation er bedst til at forudsige studerendes karakterer i de to kurser der følger direkte efter de første ni uger af deres studietid.

Hvor de første tre artikler har vægt på sociokulturelle aspekter ved læring, søger den fjerde artikel at godtgøre, at netværk også kan beskrive elementer af studerende kognitive tilstand. Artiklen argumenterer for at mentale strukturer der bruges til at strukturere fysikfaglig tankegang kan indfanges af netværk af de ord, som studerende bruger til at beskrive hvordan et fysisk system fungerer. Artiklen giver simple eksempler på disse netværk, lavet ud fra gymnasieelevers besvarelser af en løst formuleret opgave om snorbølger.

Project Overview

This PhD project explores how networks can be used in physics education research (PER) to connect measurements, didactics, and theory. More specifically, I have aimed at carefully connecting constructs from network science with constructs from learning theory (socio-cultural and cognitivist) and with constructs from didactics (competencies). This has allowed me to develop and employ quantitative measurement methods to create networks describing competencies and learning processes.

As mentioned in the summary, I have used these methods to describe learning processes in both secondary and tertiary level physics education. I see networks as a valuable link between these three overarching themes within the field of physics education research (PER), because they can act as complex quantifications of the structure of human relations. This thesis serves as an example of that.

The remainder of this chapter consists of an elaboration on my motivation for doing this project, followed by an overview of the process of writing this PhD thesis. Finally, I map connections between different parts of the dissertation as a network to help the reader navigate the dissertation.

Researcher motivation

Entering the field of science education and didactics with a background in physics, I have taken an interdisciplinary view on this thesis work. It has not been possible, nor do I think it is desirable, to neglect the way of thinking, the tools, and the reasoning skills from physics, or to neglect methods, practices, and theories in science education research. Since physics and science education research are (at least) two different disciplines, combining the two must be interdisciplinary at some level.

My rule of thumb in this work has been that the level of interdisciplinary necessary to answer my research questions has been reached when the disciplines have become so interwoven that removing one of them would cause the whole work to collapse. For example, the probabilistic measures developed and used in *Relations between network and other attributes* (p.59) were created in the field of network physics. The conclusions in that chapter are based on correlations between these measures and grades and cannot be done without them. On the other hand, the same conclusions are based on cognitivist and socio-cultural theories; they make little sense without these theories. Thus, the answers depend on both fields of research.

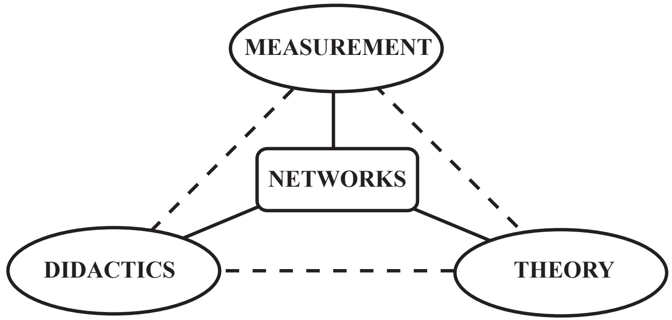

Figure 1: This work explores different ways in which networks can be used to connect measurement, didactics, and theories as all these apply to physics learning. The dotted lines indicate that, while other ways of connecting the three exist, this work deals explicitly with how networks can be used in this respect.

When I first started reading about learning theories, a divide was set up between cognitivistic and socio-cultural learning. The cognitivists were concerned with development of mental schemata, while the socio-culturalists primarily were concerned with the development of language and mediation of artifacts. Somehow, the different foci were seen as incommensurable theories. However, that notion never did make much sense to me.

In my master's thesis I argued that there were no any real differences between the two, and that they could be seen as merely views of learning at different scales (Bruun, 2008). Of course, one of the grand old men of cognitivism, Jean Piaget (1896-1980), did in his later work consider socio-cultural aspects, just like Lev Vygotsky (1896-1934), the grand old man of socio-culturalism, described detailed mechanisms for how the individual acquires knowledge (Vygotsky, 1978). Bruner (1997) describes the differences between the two:

> One, *Piaget*, studies thought in its nomothetic and explanatory manifestation, the other, *Vygotsky* its idiographic and interpretive expression (Bruner, 1997, p. 139, my emphasis).

In this view, the tension between Piaget and Vygotsky is between the general explanation and the detailed description. Bruner proceeds to say that as paradigms the two approaches may have grown incommensurable, but he maintains that both views are important to understand learning.

Lately especially work in the field of metaphor theory (Lakoff and Johnson, 1980; Roth and Lawless, 2002; Lakoff, 1993; Johnson, 1987) can be seen as bridging the gap between the two (Sinha and De Lopez, 2000); it connects the cultural aspects of metaphor and analogy as developing entities in their own rights with mental schemata. However, I do not see any real gap to be bridged. Instead, I see learning as a process happening in a complex system *'composed of interconnected parts that as a whole exhibit one or more properties [...] not obvious from the properties of the individual parts'* (Wikipedia). The interconnected parts constituting the system are both individual minds (as complex systems in themselves) *and* their relations. Part of my motivation for conducting this study is to test this argument and to find a way to model this system.

As a physicist, I view quantitative modeling as a key feature of natural scientific endeavor. At least in part, I view the field I am undertaking from a natural scientific perspective as opposed to restricting myself to humanistic and social sciences. This means that the models I want to create are of a quantitative nature. They will be heavily integrated with qualitative data, but the goal of the models is to produce hypotheses which can be tested quantitatively.

The tricky part is to be very careful to investigate and clarify what the models predict or try to quantify. What I am interested in is to find out how I can measure student responses to teaching-learning environments quantitatively. I want to investigate to what extend I can use these kinds of measurements as evidence that processes of learning have occurred. The predictions, explanations, and quantifications, which the models produce will influence my perspective on learning.

Making quantitative measurements of learning processes is a hard problem which I will try to solve using ideas from cognitivist learning theories, socio-cultural learning theories, and physics (specifically network physics). The key idea from physics is that it is possible to use quantitative macroscopic data to investigate microscopic entities and relations, or more generally, that it is possible to say something about what we cannot see based on what we can see.

We cannot see learning as a process, just like we cannot see pressure, energy, or heat. My argument is that we can see the effects of learning processes, simply because they are processes, and processes leave traces behind for those who care to find them. However, this is not to say that learning has not occurred if we cannot see the effects of learning processes.

If we become better at finding and measuring effects indicating that a learning process has occurred our certainty that learning has occurred will improve as well. Such certainty is based on evidence and one might suspect that different teaching methods will make students and teachers leave different traces of their learning processes. The tools I will use to investigate these traces come from cognitivist, socio-cultural, and network physics theories. Thus, what I am aiming at developing here is a framework for evidence-based models of teaching, learning, teaching methods, and teaching-learning environments.

I believe that evidence-based models of teaching can become useful by increasing the amount of data and the quality of that data. Just like a high quality picture takes up more space on a hard drive than a low quality picture, high quality data, whatever that might be, is likely to take up a lot of space in the research. However, just like the high quality picture shows more detail so should high quality data show more details about learning. I do not, for example, consider Force Concept Inventory (FCI) test scores or grades as high quality data in this respect. They may be reliable, but they do not tell us much about the processes of learning.

Increasing the amount and quality of data raises the question of ways to collect, manage, analyze, and interpret that data. I believe it to be desirable to automate as many of these processes as possible in order to meet these demands. Automation should make the handling of large amounts of high quality data manageable.

This raises the questions of what kinds of data can be processed automatically and what these kinds of data say about learning. A large part of the work with this thesis has been to find different ways of automating the processing of different kinds of data. An equally large part of the work has then been to relate the data, which could be automated, to learning in physics.

In sum, this thesis is based on the belief that it is a quality in itself within educational research to accumulate large amounts of data and process them to make quantitative explanations, predictions, and hypotheses. It is equally based on the belief that these predictions and hypotheses must be integrated with qualitative explanations, predictions, and hypotheses. Neither qualitative nor quantitative data are in themselves enough to understand how learning occurs or what processes lie at the root of learning.

Illustration of process

Figure 2 is an illustration of the different research processes leading to different products in this PhD work. The primary purpose of the figure is to show, that as a developing researcher, one goes through a lot of diverse and intertwining sequences and the whole process becomes more and more intense towards the end. Notice for instance that Year 1 has been allotted significantly less space on the downward facing timeline than Year 2, which in turn has a lot less space than Year 3. I will not attempt to explain all the abbreviations of the different activities. They merely serve to show that the activities were different.

I completed the majority of the required courses and attended most of the conferences in the beginning of the PhD project. Though not shown in Figure 2, the initial period was also characterized by a lot of reading and reflecting. Here both courses and conferences helped structure reading as well as thinking.

Especially reading about socio-cultural theory was important since data collection started in the beginning of Year 2 after the a pilot study. Here, the Learning in Practice (LIP) course which dealt with Cultural Historical Activity Theory (see e.g.Roth et al. 2009) helped structure my reflections about socio-cultural learning as I was reading Communities of Practice (Wenger, 1998) at the time.

The following period was a hectic mix of teaching, attending courses, designing the network survey, and collecting data. It resulted in a lot of products. The most important was the raw data used throughout the remainder of the PhD project. Here, I continuously got to present, write, and discuss my ideas with researchers from my own department, with other PhD students during courses, and with upper secondary as well as university teachers. These different ways of expressing thoughts about networks have continued throughout the study. In my view, they are essential to the development of this and further work.

One of the turning points in the process was the conference for the National Association of Research in Science Teaching in 2011 in Orlando. For the first time, I met other people who shared some of my thoughts on networks in science education research. This encounter resulted in a change of research environment[1] for three months at the physics education research group at Florida International University (FIU).

[1] The change of research environment is mandatory for PhD students at The University of Copenhagen

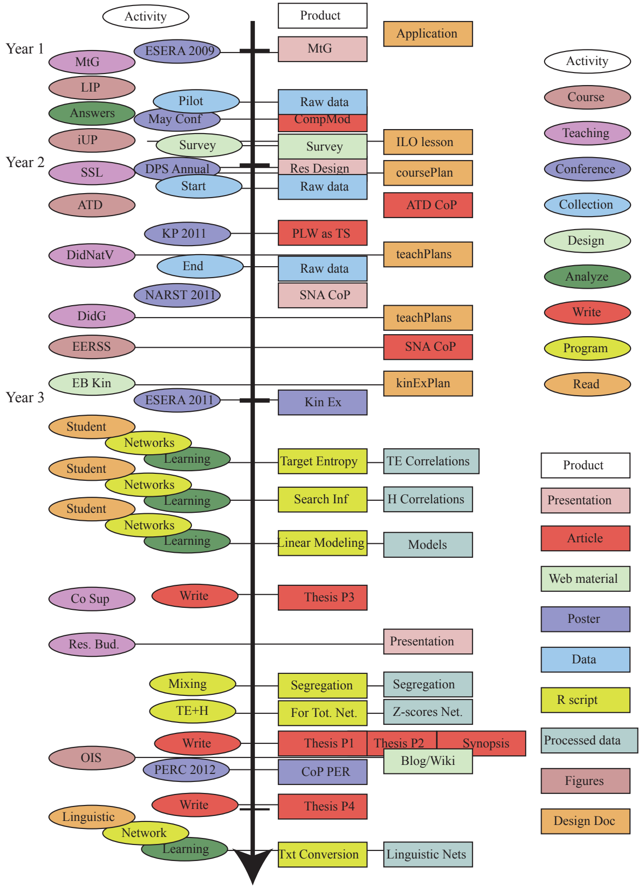

Figure 2: Illustration of processes and products on a timeline.

Working at FIU involved a lot of reading, programming, and analyzing to understand the kind of data I had collected. This is represented by the three *Student Network Learning* activity blobs towards the beginning of Year 3 in Figure 2. Network data is very difficult to get a grip on, because there are so many ways of combining data within the network as well as with external data. After many discussions, we settled on developing the network measures target entropy and search information as they apply to directed and weighted networks. The work eventually resulted in what I present as the third paper in the thesis.

Writing this paper also changed my view on the format that I wished to use when writing the dissertation. Before engaging in writing the first network paper, I had been working from the premise of writing a monograph. However, the paper made me realize that writing papers and a synopsis would provide me with a better focus.

This resulted in a hectic last period of the PhD project with many activities and products. I have indicated that I have not finished reading, programming, and analyzing the data. There are many more possibilities for activities and products.

Network map of the dissertation

The purpose of this section is to help the reader see some of the connections between different parts of the work, which I consider significant. Understanding the networks I have worked with during this PhD project in all their complexity is difficult. From my point of view, the thesis as a whole makes the most sense when regarded as a connected whole by the reader.

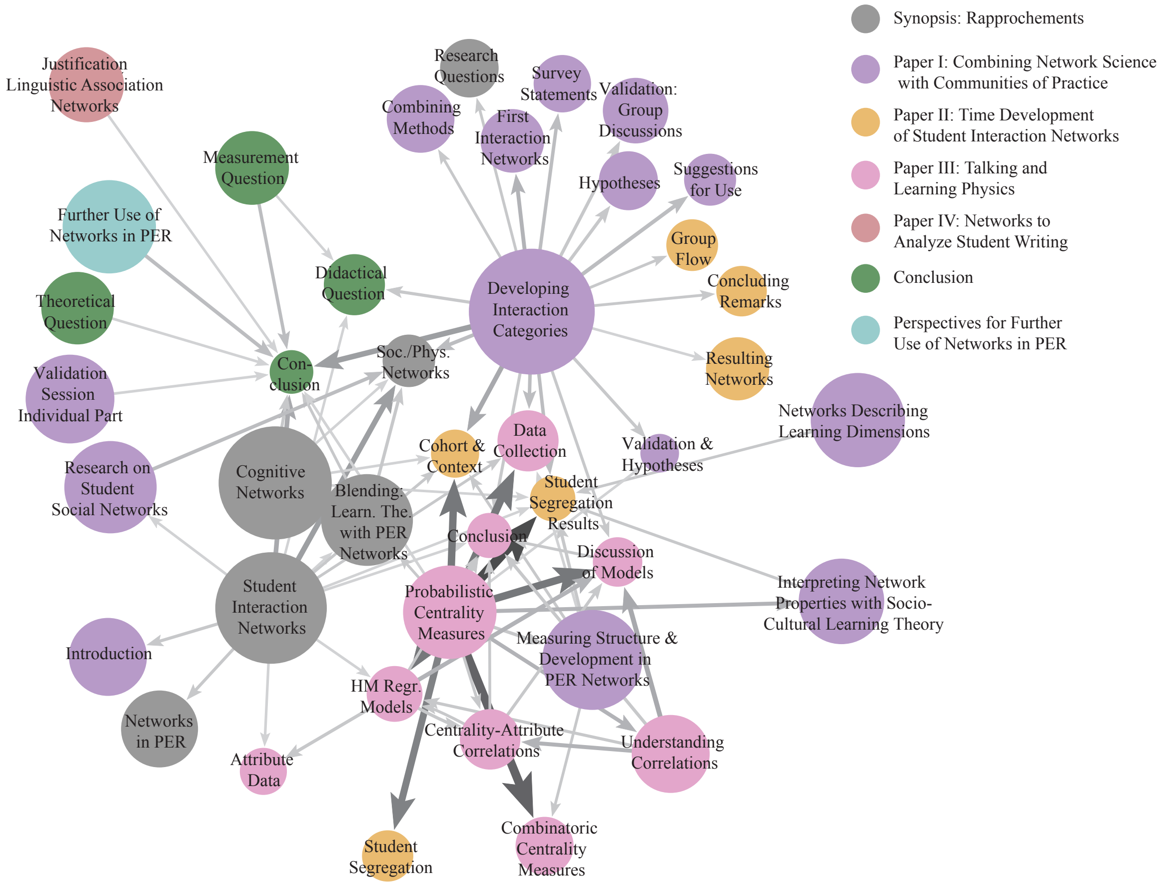

Figure 3: A network overview of the dissertation showing how the different parts are connected. See previous pages for a description.

This is why I have made a network map of the dissertation. The map of the dissertation is shown in Figure 3 on the facing page.

Although the map has been created using network methods, I will not go into the details of how it has been created or the detailed meaning of circles and arrows from a network perspective. This seems to be a research project in its own right and it is quite possible to derive meaning without it. However, a brief explanation of the color codes and an interpretation of circles and arrows are appropriate here.

The circles represent sections in chapters or papers in this dissertation. The color code shows where the circles belong. The sizes of the circles are based on the number of different words in the section. I interpret this as a marker that the big circles have more detailed arguments than the smaller ones. The arrows are based on how similar two sections are with regards to phrasing. The arrows generally go from bigger to smaller circles to indicate that the arguments developed in the detailed sections feed into the smaller sections. In general, I do not want to assign too much meaning to the direction of the arrows. What is important here is *where* the connections place the different circles on the map.

On the basis of this representation, the most important section in this work is *Developing Interaction Categories* in the first paper of the thesis. This section outlines most of the methodology permeating the whole thesis. Therefore, it feeds in to later sections in the same paper as well as other sections.

The learning-theoretical stand point of this thesis is developed in *Rapprochements: Networks, physics, and learning* (p.25). The central parts of this chapter are the developments of student interaction networks and cognitive networks as a blend between network theory and learning theory.

The upper left corner of the map shows a cluster centered around the green circles representing Conclusion (p.67). Again, the importance of *Developing Interaction Categories* is represented by the connection to the *Introduction* to the *Conclusion*. While a lot of sections feed into the sections in the Conclusion chapter, it is worth noting that the two circles representing Perspectives and the final paper are also located there, too. While it is natural to have some kind of connection between the conclusions and the perspective, the placement of the fourth paper is a reminder that this paper needs more work in order to become an integrated part of the dissertation. This issue will be further addressed in the summary for the paper.

The cluster to the lower right involves a lot of technical considerations, but the connection to the large section Interpreting Network Properties with Socio-Cultural Learning Theories could be seen as an indication that focus is always on how to interpret the quite complex network measures. However, the cluster gives an indication that this thesis is technically dense, something, which I believe, is necessary for a deep understanding of how learning and networks can be integrated.

Rapprochements: Networks, physics, and learning

This research is centered around physics education research (PER) and around the didactics of physics. While PER is a Northern American term, the didactics of physics is of European origin. While the two traditions have overlaps, there are differences. The starting points of PER center around individual student performance and issues, while didactics of physics in the French tradition focus on teaching situations as in Didactical Situations (Brousseau, 1997; Tiberghien et al., 2009). The German tradition has a philosophical aspect represented by the term Bildung - often translated to literacy (Dolin, 2002). Most of this thesis refers to the Northern American tradition, PER, because a lot of the research on networks relates mostly to this tradition.

PER grew out of a need for more physicists. In the beginning, researchers focused primarily on individual student performance (Forsman, 2011; Beichner, 2009). Since then, the field has grown to include motivation, self efficacy, policy, teacher training, retention/attrition, student epistemologies and interventions using a variety of methods, including various tests and other student output, (video) observations, and interviews. Lately, socio-cultural aspects have been taken into consideration by different PER groups.

Different sub-fields of PER can be broadly characterized as dealing with the development and testing of different teaching forms and methods, epistemological aspects of teachers' and students' beliefs and actions, and socio-cultural aspects of teachers' and students' agency and identity. This study lies within the intersection of the latter two. However, the end of the thesis presents the argument that networks are quite general in their application, so they might find their way into all forms of research.

Forsman (2011) lists around 80 PER groups around the world. This thesis is written as a PER project within a general science education department, and naturally the PER groups do not account for all research within or applicable to the physics teaching and education. However, they can be seen as hubs of research pertaining specifically to PER.

PER can be understood as a field within physics (Forsman, 2011, p. 12) and a field within educational research, and thus has to acknowledge standards from both fields. In doing so the PER field becomes interdisciplinary, and involves theoretical considerations and methods from sociology, the humanities, and science, specifically physics. Thus, PER is subjected to at least some of the standards from all of these major areas, depending on what view of interdisciplinarity we take.

At this point it might be fruitful to consider what kind of interdisciplinarity PER can be subjected to. One kind of interdisciplinarity is when one area is subjugated to the other. If we say that PER is an area within physics, the part that is physics has prominence and educational research may be a part of the PER package allowed only as an unwelcome passenger providing buzzword explanations to quantitative results. If on the other hand, PER is subject to educational research, physics becomes a case within this area. It might just as well be chemistry, biology, or history, and results from PER only make sense in light of results from other sub-fields within educational research. While some may believe that interdisciplinarity means that one field is in some way subject to another, the following paragraphs proceed to explore other possibilities.

The hierarchical coordinations of PER with physics and education research can be contrasted with a kind of interdisciplinarity seen in engineering. Here, different disciplines contribute to solutions on a footing determined by the problem at hand (Richter and Paretti, 2009). For example, when building a structure where both structural material properties and electrical circuits are important, these two engineering disciplines need to be coordinated to produce a solution. Still the two need not communicate extensively with each other about their epistemological and ontological foundations. In PER research an analogy could be the development of some physics problems (requiring for example the domain of quantum mechanics) and the analysis of how students solve these problems (requiring for example the domain of video observation). While both the problems and the analysis are important aspects of this kind of research, researchers need not consider how uncertainty principles in quantum mechanics affect their ability to transcribe student videos.

A third kind of interdisciplinarity is when the two fields are integrated. In science this happens when scientists from different disciplines work together to understand a phenomenon of interest to the involved disciplines. One example is nano science, where the common denominator is “things that are nanometer sized” ($\sim 1 - 100$nm). Other examples are planetary science and climate change. Here, researchers need to be clear on what premises lie behind their inferences in order to use others’ results in their work. A biologist in nano science needs to be certain that results from a colleague physicist build on data and warrants which are compatible with biologists’ if he is to use them properly in his research. In PER the question is more pressing since physics and educational research do not necessarily share views on what exists and how we produce knowledge.

One way of integrating two disciplines would be to view them as analogous to each other, and then investigating this analogy. Then blending the two (Podolefsky and Finkelstein, 2007) constitutes PER; that is, creating a space where some of the ideas from both fields survive and ideas which belong to none of them are created. It is beyond the scope of this thesis to perform this analysis, but this chapter does seek to lay the foundation of an integration between network physics, cognitivist, and socio-cultural learning theories.

This integration is vital for this thesis, since on the one hand from the perspective of educational research, networks are only mathematical structures with no relevant meaning. On the other hand from the perspective of physics, educational research offers very difficult data to analyze; it is not easy to set up and test successful quantitatively predictive models to investigate learning. A successful integration of physics and educational research could however lead to stronger results, since we would then gain the benefit of both the very full descriptions prominent in educational research and the hypothetico-predictive (Lawson, 2003) quantitative models which are prominent in physics.

Starting from how networks have been used in educational research (primarily PER), the following section focuses on two broad types of networks: student social networks and cognitive networks. The discussion draws mainly on work that has informed this project. As shown in *Illustration of process*, the process has been iterative, and including the many almost-but-not-quite relevant studies would create a more muddy picture than necessary.

Blending constructivist learning in physics with network theory (p.37) integrates constructivist learning theories with network physics. This may be seen as a step along the way towards understanding PER as a blend between physics and educational research. In this context, network studies represent examples of an understanding and *Challenges* (p.44) sums up up the challenges this understanding produces. Finally, *Research questions* (p.47) indicates which of the challenges this dissertation addresses.

Networks in education research: A focus on science and physics

Physics education research (PER) groups work with all aspects of physics education, but few articles within the field of PER use networks. The use of networks seems to be increasing in general in educational research and also in PER, but network articles seem to emerge independently in different sub-fields.

The majority of work in educational research utilizing network theory investigates social networks via social network analysis (SNA). In SNA researchers can represent people (for example students) as circles and connections between people as lines. Some people appear more central in these networks than others, because they are more connected. One of the major questions in SNA is then what this centrality means.

For example Enriquez (2010) investigated who students reported communicating with using different modes of communication, for example face to face contact, mobile phone, online posts, and online chatting. Asking about these different modes of communication produced networks with very different structures. For example, the face to face network resembled a star, whereas the mobile phone network had a much more complex structure. Enriquez concluded that centrality is fluid across communication modes, but she did not really tell us what the students communicated about. Thus mode of communication for Enriquez' study is more about what device the student used to communicate with, rather than what they communicated about.

Instead of focusing on the devices used to communicate, this study turn towards what students communicate about. The primary reason for this shift is that it is possible to use mobile phone data (Onnela et al., 2007; Eagle et al., 2009), online discussion boards (Dawson, 2008; Macfadyen and Dawson, 2010), emails (Kossinets and Watts, 2006) and even digitally based proximity data (Eagle et al., 2009) to create more accurate networks than self reported interactions. But it is not possible to use these data to find out what interactions in a learning context a student remembers.

One of the key ideas in this thesis is that it is possible to connect the interactions a student remembers in a physics learning context to learning processes and learning outcomes of physics instruction. *Developing network methodology* (p.51) describes a detailed design for making this connection. With such a connection, network representations of connections between students give insight into possible learning processes in a physics classroom (p.59) or in a cohort of physics students beginning at a university (p.59).

Networks can be many other things than networks of people. In general, networks consists of entities of interest and bonds/connections between entities. Linguistic networks (Masucci and Rodgers, 2009) are networks of words, and the relation between them can be that they are adjacent in a text. Epistemic networks (Shaffer et al., 2009; Rupp et al., 2009; Bodin, 2012) identify different cognitive categories in an observation situation and link them based on adjacency in time. This work makes use of networks of coding categories (for details, see Paper I) where two categories are connected if they are used to code the same piece of data, and correlation networks (Figure 14 on page 62), where two variables are connected if they are correlated to some level of significance. The problem in this context is to figure out what the entities and bonds mean for our understanding of physics learning processes.

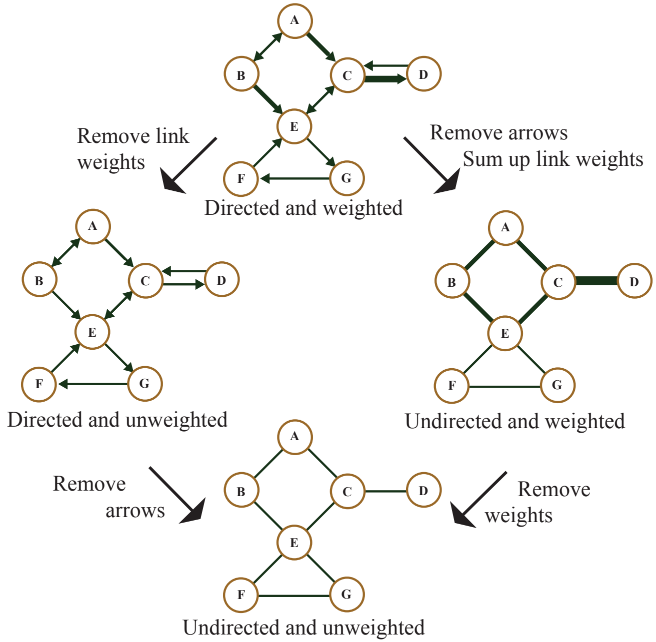

Figure 4: Four fundamental forms of networks (Costa et al., 2007), distinguishable by thickness of lines (link weights) and presence (or not) of a direction on the link. A network can also be composed of any combination of these four types.

In network theory, the entities are called nodes and the bonds are called links. We can distinguish between four types of networks (Costa et al., 2007) as illustrated on Figure 4. First consider the network, where there are arrows of different widths between the nodes. The arrow indicates a direction, for example that a student, A, has named another student, B. The width indicates the strength of the connection, for example if A has mentioned B three times in some study. This is called a *directed weighted network*. Consider now that the line between A and B signifies that they have taken the same three courses. Then it makes little sense to put a direction on the links. This is an *undirected weighted* network. Stripping the networks of weights then produces directed and undirected *unweighted networks*.

From an educational perspective, working with networks might seem very technical and not related to learning theories. This is why one of the main research questions in this work centers on how network science and learning science inform each other. The idea of coupling network and learning theory has already produced results which encourage further investigation.

The next sections review in turn physics educational studies using social networks involving students and using what can be called cognitive networks. The emphasis is on what the networks can tell us about issues pertaining to physics learning and education, and not on technical definitions from network theory. The sections below will not attempt to explain all the network terms used in the studies, but only those needed to understand the messages from the studies. Instead the terms relevant to this project will be explained in the chapters in which they appear.

Student interaction networks

If we only focus our attention on the students and how they interact with each other, we leave out interactions between students and teacher, students and family/friends, and interactions between students and other agents (e.g. computers)[2]. Thus, we only count interactions between students, and call these student interaction networks (SINs).

In social network analysis, SINs would be called social networks. However, social networks may indicate many types of relations, which student interaction networks do not. Social networks may involve connections between organizations, research papers, as well as different kinds of human interaction (Scott and Carrington, 2011). From this point of view student interaction networks are a subset of social networks.

Kossinets and Watts (2006) used time-stamped e-mail headers recorded over one academic year as a proxy for student interactions. In their study, they focus on what factors influence the creation of ties in SINs and they would probably see their networks as social networks of students set in the context of universities. Thus, their study is more a network study than a PER study. The total data set includes a whopping 22,611 graduate and undergraduate students.

Kossinets and Watts (2006) establish relationships between

[2]For example, in Actor Network Theory (Latour, 2007) non-living things may have agency as well. While the topic of agency is interesting in connection with networks, this study does not explicitly investigate how agency might be represented in networks.

students by noting that in the context they investigate, “ongoing social relationships produce spikes of e-mail exchange that can be observed and counted”. At any given point in time, the strength of a relation between two students in their network is approximated by number of e-mails sent between them in the last 60 days. They then assess the likelihood that two students make a connection based on different factors. Factors like gender, age, and year play less of a role compared to shared classes and the tie strength of mutual acquaintances. This means that if two people who do not know each other share a class or have a relatively close relationship with the same person, they are likely to make a connection.

This is an indication that the learning context and the creation of social and academic ties in that context are linked; being spatially close to another person for an extended period of time increases the probability of communicating. It seems reasonable to expect that what students would communicate with that other person about is somehow related to the learning context that they are part of. Extending our knowledge from “there was communication” to “what was the communication about” would allow us to probe what kind of processes facilitated the creation of the tie in that context.

Another interesting finding of Kossinets and Watts (2006), which can inform our position on SINs, is that while the overall structure of the network is stable, the connections of individual actors are not. This means that connections based on actions do not remain constant; student interactions change over time, as this study also confirms (see *Time development of student interaction networks*).

This is an extension of the fluidity argument put forth by Enriquez (2010); not only do student positions vary within modes of communication, but also in time. It is an important result for this study in two respects. First, we cannot assume that we measure the social network of students by making one measurement examining only one mode of communication. Second, social networks as they are lived/experienced by students are dynamic entities. One week, a student might be the center of the network, while he is the connecting link between two groups at other times.

Even if positions change from week to week, the position of a student in a network integrated over some time, can be related to academic success. *Relations between network and other attributes* (p.59) shows this for student grades as a marker for academic success, and a series of papers (Dawson, 2010; Dawson et al., 2010; Macfadyen and Dawson, 2010) have coupled student online discussion activity in courses with the likelihood of getting a good final grade.

To establish links between students, Dawson (2010) uses online discussion boards. If Student A posts a message to the forum, and Student B replies to that initial post, a relationship is established between Students A and B. Link weights are established on the basis of “increased levels of communication exchange between different actors [...] in the network” (Dawson, 2010). The networks of that study were established through discussion posts related to a specific unit of a chemistry course which blended online learning with traditional instruction.

One of the findings of the study from a network perspective is that students with a high number of out-going and incoming links (called the total degree) end up with a better grade than students with a low total degree. More precisely students in the 90th percentile grade group had a mean total degree of 5 ± 10, and students in the 10th percentile group had a mean total degree of 2 ± 3.

Moreover, high performing students also were more likely to include an instructor and high performing students tended to link to other high performing students. This is interesting compared to the findings in Paper II and Paper III in this thesis. These papers show no evidence that students coordinate themselves according to grade. The discrepancy may be explained by the cautionary note of Enriquez (2010): We might not capture the processes related to learning by measuring only one mode of communication.

Brewe et al. (2012) circumvent the problem of measuring modes of communication by asking students who they work with on homework with in a specific context: The physics learning center (PLC) where a diverse set of students meet to work on physics homework. The question neatly captures the context and renders the fluidity question irrelevant.

Linking the network representation to demographic data, Brewe et al. (2012) are able to predict the *centrality* of students in the resulting network. Via multiple hierarchical regression (see also Paper III), they found that days spent per week and whether or not the person was a physics major where the best predictors for centrality in the collaboration network. Gender, ethnicity, and introductory course type (modeling/traditional) were not.

In network theory *centralities* are measurable quantities, and in social network analysis centrality is often used as a proxy for power (Wasserman and Faust, 1994; Scott and Carrington, 2011). Centrality can be computed in many different ways (Wasserman and Faust, 1994), corresponding to different perspectives of what it means to be central. For Brewe et al. (2012) a central person not only has many connections; a node's centrality also depends on the centrality of the neighboring nodes. Brewe et al. (2012) quantify this using Bonacich's centrality (Bonacich, 1987), which depends both on the number of connections a student has and on the centrality of neighbors.

Power in SNA is understood as relational, meaning that powerful people have relations which make them appear central in the network (Scott and Carrington, 2011). For the purposes of studying physics learning, thinking in terms of positional advantage seems much more fruitful. For Brewe et al. (2012) a link means that a student thinks of another student as a collaborator when doing physics homework. Even if we do not know the details of the interactions leading to this link, we can assume that the central people in this network will have had more interactions about physics homework in the PLC than non-central people. If the interactions helped students with physics, this is advantageous, and then we can say that students who are central have a positional advantage. In this light, what Brewe et al. (2012) found was that the physics majors who spent many days per week in the PLC had positional advantage.

In a socio-cultural study of three students' increased participation (Wenger, 1998; Lave and Wenger, 1991; Bielaczyc, 1999) in a physics learning community, Goertzen et al. (2012) discuss what such a positional advantage may mean for students participating in a community of learners. The study has two data sets: One is a network survey where links are made between students based on students naming who they work with to learn physics. The other data set consists of three 60-minute interviews with each student to elicit the students' perceptions of the learning community and their part in it.

The researchers used diagrams of each student's self-reported connections to visualize how the student's number of ties changed during a period of time. Three such *ego networks* for each student showed that they started with zero to one connections, gained connections in the middle of the study, and ended up with more connections at the end of the study. The diagrams also showed that student ego networks became more connected, meaning that the students named by the subject student also named each other to a larger degree.

Detailed descriptions of how these students change their attitudes about learning, their identity, and their ties to other students in the physics classroom load meaning into the networks. For example Marta, one of the students, for whom learning involves discussing the meaning of physics concepts and explaining ideas to others. Thus, the meaning of a tie going from Marta to others is likely to be indicative of processes of discussing meanings of concepts.

It is tempting to say that successful students will have a larger amount of ties, but as the standard deviations reported by Dawson (2010) show, we have to be careful with interpreting the number of ties that way. While high performing students in that particular learning community on average have a significantly higher number of links than low performing students, the standard deviations are larger than the averages reported. This indicates that a high performing student might have a lower number of links than a low performing student. Thus we cannot characterize a student's positional advantage solely on the number of links. An analogous conclusion may be drawn in the case of the study of community of learners in physics (Goertzen et al., 2012), where one student, Sergio, is also successful and has very few links to others, and thus a small ego network. We cannot see how each of the three students' ego networks are connected to the rest of the network, and yet network connections may influence their academic success.

In Paper III, we show that measures that take the whole network into account when determining the centrality of a student (e.g., Bonacich's centrality), can aid in the prediction of future grades. Thus, it might be that even if Sergio does not end up having a large ego network and reports working mostly alone, he is still engaged with the learning community. He can benefit from discussions with others and contribute to other students' understanding.

One of the students investigated by Goertzen et al. (2012) said that the ties she made in her physics class were likely to be lasting, like family. Her statement emphasized that being a student at a university entailed both an academic part and a social part. These two parts where investigated by Forsman (2011) in the context of three Swedish engineering programs. With a focus on student retention, they asked students to write down the names of the fellow classmates they interacted with and what characterized the nature of the interaction. The characterization scale was from one (only social) to five (only academic), with three being both social and academic. They managed to distinguish between the academic and social networks, and thus quantified the existence of these two kinds of systems.

Other studies use SNA to analyze social networks in educational settings (e.g. Daly and Finnigan, 2010; Pitts and Spillane, 2009; Pustejovsky and Spillane, 2009), but they do not investigate learning processes in general or in physics in particular. The field of Computer Supported Collaborative Learning has made use of SNA in interpreting learning processes specifically involving the use of computers (e.g. Cho et al., 2007), which seems too limited for the purposes of this research. In fact, no publications seem to investigate how student networks of subject specific interactions develop over time. The next section investigates a more microscopic view of networks. The networks in the following section are all related to learning or ability to do physics as it is expressed in networks of student conceptual understanding. These networks are all related to cognition, which begets the term *cognitive networks*.

Cognitive networks

Cognitive networks aim at describing the cognitive state of a person and can be extended in many directions. Nodes in cognitive networks need not be written words. They may be actions that a researcher identifies and categorizes. Likewise, links in cognitive networks may be dependent on how students argue for their connections, or they may be based on adjacency of categories in a transcript. One may view concept maps (Novak, 2010) as a special case of this broader class of networks.

Researchers and teachers have long used concept maps as a way of accessing, developing, and assessing student knowledge. Novak (2010) describes them as hierarchical networks where concepts are nodes and a link between concepts signifies some connection between these concepts made by, in our case, the student.

A concept is 'a perceived regularity (or pattern) in events or objects, or records of events or objects, designated by label' *(Novak and Cañas, 2008).* A knowledge unit is a string of concepts tied together by lines linking words or linking phrases to form a proposition; a declarative statement which is either true or false.

As a teaching-learning tool, concept maps are used to make students organize concepts and connections between concepts. While the exact interplay between the learner's memory systems and developing concept maps is unknown, (Novak and Cañas, 2008) describe them as 'a kind of *template* or *scaffold* to help organize knowledge'.

A good concept map is generated as an answer to a focus question. From there overarching concepts diverge into less important or at least more isolated concepts, creating semantic units that can be read as sentences. Each semantic unit in a concept map is a proposition, here understood as a statement which has meaning in the context of the focus question. Maps with many such meaningful interconnections between concepts are considered good, while maps where concepts have been connected as strings represent 'poor understanding [...] or inadequate restructuring of the map' (Novak and Cañas, 2008). Thus, a student map is taken to express some level of the student's understanding of the focus question.

One might express two concerns with concept maps. First, if knowledge structures are not only propositional, concept maps do not necessarily reveal much about a student's understanding of physics. Johnson (1987) and diSessa (1993) both propose cognitive units important for learning which are not propositional. Johnson's image schemata are dynamic and changeable multimodal structures that we use to coordinate our actions. On a deeper level with regards to physics, diSessa's p-prims are knowledge units which coordinate a person's sense of mechanism with regards to mechanics. Thus, focusing only on how concepts relate in a concept map by prompting students for concepts may not reveal much of what they know.

The second concern is about the hierarchy of concept maps. diSessa (2002) argues that p-prims can be structured in *coordination classes*, which are highly hierarchical, and that this changes the way the p-prims are triggered and related. Moreover, change may involve stages where knowledge is not as hierarchically structured. By imposing a hierarchical structure on students, we might not get a fruitful map of their knowledge state; one which allows us to evaluate the students' knowledge in order to take some kind of action. To sum up this concern: How can we know from a good concept map if student knowledge is structured hierarchically or if this is an effect from the mapping procedure?

In a recent study Koponen and Pehkonen (2010) asked students training to become physics teachers to make connections (links) between concepts. The concepts were given to them by the researchers, thus producing a concept network. The concepts were taken from an expert version of the concept network, and the allowed links were either *"experimental procedures, which are operational definitions of a concept or laws or their demonstrations"* or *"modeling procedures, which can also be simple definitions or logical deductions"*. The nodes were not only concepts but also quantities, laws, and fundamental principles.

Using these two types of links lead to two different kinds of structure in the expert version of the network. First, the structure of the concept map revealed that concepts tend to be highly connected with each other, something which Koponen and Pehkonen (2010) interprets as a marker of coherent knowledge structure. Second, there are hidden hierarchies dividing the network into distinct levels much like a concept map.

For each node, i, in the expert network the importance, I_i, and the hierarchy, H_i, can be calculated (refer to Koponen and Pehkonen (2010) for details), producing vectors $I^{expert} = (I_1^{expert},..,I_N^{expert})$ and $H^{expert} = (H_1^{expert},..,H_N^{expert})$. Since the student networks consists of the same nodes, but with different connections and thus different I_i's and H_i's, they can define the dot product $I^{student} \cdot I^{expert}$. Normalized to the length of the expert vector, this is a projection of the student network importance values onto the expert network importance values. It is measuring the similarity between the two network measures.

This is a way of evaluating the student networks, since they measure the quality of knowledge as $Q = I \times H$, where I and H are the projected values. This allows researchers to compare different types of student networks with each other via the expert network. For example, Koponen and Pehkonen (2010) find that many of the student networks can be represented graphically like chains of weakly connected nodes. These networks signify low quality knowledge structures, while more web-like structures signify higher quality knowledge.

Koponen and Pehkonen (2010) show that careful interpretation of network motifs and structure allow researchers to quantify the coherence of student knowledge structures. They have made an effort to produce a correspondence between the structures in their network. Thereby, the structures are readily recognizable as knowledge structures pertaining to linking physics concepts via experimental and modeling procedures. This constrains the knowledge structures Koponen and Pehkonen (2010) examine to only "scientifically acceptable" ones, thus hiding from researchers information about other conceptions the students might have.

Moreover, the idea of projecting networks is also an important contribution. It allows us to compare different networks if we have calculated, for example, a measure of centrality for each node. However this similarity measure is dependent on a complete overlap of nodes in the two networks, thus again constraining students to use exactly the words teachers and researchers find acceptable. Thus from a point of view examining the development of language (Lemke, 1990; Ogborn et al., 1996; Roth, 1995), this is not the most informative approach.

Bodin (2012) uses a completely different way of establishing a cognitive network. She uses epistemic framing (Rupp et al., 2009; Shaffer et al., 2009) to code student interviews, and make networks of the codes based on adjacency of the codes in the transcript. The codes become nodes in an epistemic network of knowledge, skills, and beliefs.

According to Bodin (2012), epistemic elements represent knowledge, skills and beliefs. Epistemic framing describes the organization of epistemic elements. An epistemic frame is a construct theorized to contain information about knowing what, how, and with, thus forming an organizational principle for practices in a community of practice (Shaffer et al., 2009; Louca et al., 2004; Bing and Redish, 2009). Bodin's fundamental assumption is that epistemic frames can be validly represented by epistemic networks of coded transcripts. As such, epistemic networks represent knowledge networks in a more broad sense than for example concept maps or networks.

Groups of connected nodes can be captured by a particular algorithm for partitioning a network into groups, called Infomap (Rosvall and Bergstrom, 2008; Rosvall et al., 2009). Infomap bases groups on an information flow perspective on networks. In this view, links are conveyors of information between nodes. A group of nodes are grouped together in *modules* if information flows quickly between them. More important groups account for more of the information flow.

In this light, Bodin finds that modules labeled *Values*, *Difficulties*, and *Strategies* account for much of the information flow in the epistemic networks before a teaching intervention. After the intervention these categories have been replaced by *Model representation* and *Troubleshooting* as the most important modules. Also, Bodin is able to map how, for example, the *Difficulties* module *evolves* using so-called alluvial diagrams (Rosvall and Bergstrom, 2010). These diagrams illustrate, for example, that the nodes in Difficulties split up to be only part of the new modules Model representation and Troubleshooting.

Bodin's study shows that student responses can be used to produce networks which tell us something about student cognitive and affective structures. However, she uses interviews, which are time consuming and thus her sample is small (six students). In this study, Paper IV discusses how written responses to conceptual questions may be converted to linguistic networks representing aspects of student thinking. The price is that such questions might not elicit any affective factors which can be converted into nodes and links.

Inspired by Shaffer et al. (2009), Bruun (2011) shows an analysis of a video recording of students and uses the software ELAN to annotate student words, gestures and actions. The analysis shows that the network representation can be used to visualize how students use different modes of representation when communicating about physics. The categories representing nodes are very coarse grained. For each student, Bruun only considers speech, gesture, and other actions. With three participating students, this only yields nine nodes, and only outlines the structure of the interaction. However, it does show that gestures are part of how the students communicate.

Another but still related form of representation to consider are networks that map human writing. While these *linguistic networks* seem unexplored in an educational context, other research suggests that linguistic networks may reveal structures concealed in plain text (Masucci and Rodgers, 2009). One possible use of these kinds of networks is for analyzing student writing about physics.

The rationale for using *student* writing as an indicator for their cognitive state is that writing permeates our culture. One way of asking people to describe their view of something is to ask them to write about it. In physics classes writing is mostly confined to reports or to argue for your reasoning when doing a test. Asking students to write how they would go about solving might produce well structured texts revealing how they think about a specific physics problem.

Recent studies(Masucci and Rodgers, 2009; Masucci et al., 2011; Masucci and Rodgers, 2006) have investigated writing as linguistic networks. One way is to use text mining techniques (Feldman and Sanger, 2007) to reduce words to their stem form, remove common words (for example “and”, “a”, “for”) and connect the resulting words in a network if they are adjacent in the transformed text.

These studies are not related to physics education research, but Paper IV discusses a technique derived from this line of thinking to investigate how linguistic networks can be interpreted as markers for students’ cognitive states. Such networks may complement other markers like grades and FCI test scores. The networks are made from student written answers to openly formulated problems in physics.

The idea is that the way a student expresses himself/herself about physics is indicative of how they perceive concepts and their connections in physics, in a way that concept maps, concept networks, and epistemic networks do not capture. The present work positions linguistic networks in the discussion of how networks can be used to capture student understanding and use of physics in a quantifiable manner. This is an important discussion if networks are to be used for this purpose, which is why one of the research questions will center around this discussion.

Blending constructivist learning in physics with network theory

Constructivism is the underlying idea in work with both social and cognitive networks. From a constructivist view point, a person learns by integrating new knowledge with pre-existing knowledge. Classically, two views on how this happen have competed since the 1920's.

In the cognitivist or Piagetian tradition, the learning process starts with the student making an observation (Lawson, 2003, p. 7). The observation could be the task to draw a force diagram for a box on a slab. The current mental structure, or schema, in his/her long term memory system uses the observation to initiate some kind of behavior (ibid). This could be to draw a force diagram, with the box represented as a rectangle and arrows representing forces. Say that there is a teacher who can verify the drawing. For the student the expected outcome of his/her behavior could be that the teachers acknowledges the drawing. Then we would say that the observation had been assimilated into the knowledge structure of the student. He/she would know only that he/she could also solve this kind of problem. If the teacher indicates that it is wrong, the student is in a state of disequilibrium. To make sense of the observation and solve the task, he or she must make room for rearranging the knowledge structures. This would be to accommodate new knowledge.

In the competing socio-cultural or Vygotskian tradition, learning involves first and foremost language and signs (Vygotsky, 1978). Learning happens in two steps. First there is a social interaction in which language plays a key role in mediating knowledge about how the task should be solved. This would in many cases involve a textbook or a teacher presentation about force diagrams. Then the student needs to internalize the drawing of force diagrams in order to solve the task. Students might scaffold each other to solve the problem if the problem lies within their zone of proximal development.

For Vygotsky, learning happens by using signs and artifacts in social interactions, and knowledge as a higher mental function is then primarily developed in relation to others. In the cognitivist tradition, learning happens when an individual adapts mental schemata to their observations, and thus knowledge is represented by mental schemata.

Knowledge is a bit misleading from this author's standpoint, and one could argue that developing competencies might be a better concept for describing learning.

As defined by Dolin et al. (2003) scientific competency is:

> The ability and will to act, alone and with others, using scientific curiosity, knowledge, skills, strategies, and meta-knowledge to create meaning and autonomy, and participate in decision making in situations where it is relevant. (Dolin et al., 2003, author translation)

For the purposes of this thesis, consider this changed version as a description of physics competency:

> The ability and will to act, alone and with others, using curiosity, knowledge, skills, strategies, and meta-knowledge *as these apply to physics, in order*

> *to negotiate meaning, to develop a distinct identity within the field of physics,* and to participate in relevant decision making situations.

Keeping a focus on competencies acknowledges that knowledge is both individual and shared, but also relates knowledge to other aspects of learning, thus emphasizing that a focus on knowledge alone does not give us the full picture of learning in physics. Like Dolin et al., the physics competency stresses that *curiosity* and *strategies* are things that can be learned, just as *knowledge* and *skill*. It recognizes, like Novak and Cañas (2008), that learning also entails affective aspects, so not only ability but also will to act is an important part of learning. The removal of *scientific* serves to emphasize that this work is about physics learning and results from here may or may not apply to other fields. *Creating meaning and autonomy* in the original definition derive from the German Bildung tradition (Klafki, 1996), and it is linked to a way of being in the world or in this case to act within the field of physics. However, in the view presented here, any one person does not create meaning and autonomy. Instead, persons negotiate meaning and develop identities - notions which are derived from, among others, Wenger (1998).

This description of competencies is an important of the study, and one of the research questions addresses the connection between teaching practices, competencies and changes in networks. This relates to the didactics of physics, since the question addresses how networks can help investigate how students respond to the learning environment, for example teaching and learning activities. Learning as a change of competencies should be recognizable as changes in networks.

In this view, learning is the integration of new ways of being curious, of new knowledge, skills, strategies, and metaknowledge with the existing, which changes a person's ability and will to act, while being aware of both oneself and one's surroundings; the context. For now it remains a postulate that this view can integrate both cognitivistic and socio-cultural learning theory. The following section elaborates on this idea, thus further relating cognitivism, socio-cultural theory and network theory.

Blending learning mechanisms: Socio-cultural theory and constructivism

If the definition of learning postulated above actually does integrate cognitivistic with socio-cultural views learning, then we should make sure that the mechanisms for learning are compatible. Remembering that Bruner (1997) warns us that they may have grown incommensurable, the learning mechanisms of Piaget and Vygotsky do not exclude each other. For example, Sinha and De Lopez (2000) find that even young children integrate 'socially normative knowledge [...] with their capacity for schematizing spatial relations.' Further they argue that this 'presupposes a cognitive representational capacity, which [...] is based in the ability to "abstract" schematization from immediate perceptual content.'

In the context of learning physics, schemata, cognitive structures which shape among other things student actions, predictions, affect, and strategies, may be viewed as a working model for how to act in given situations involving physics.

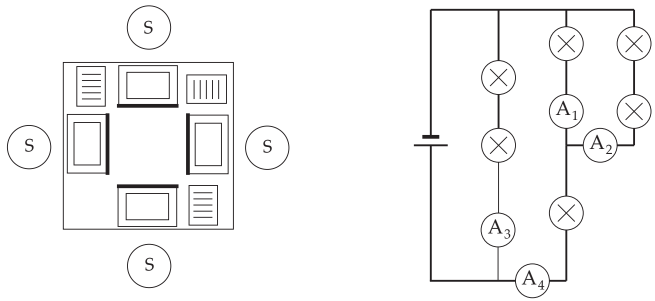

Figure 5: Student setup and the circuit that the students discussed (Bruun, 2011). In the circuit, circles with X's are light bulbs with equal resistance. Students rank the read-out of each ampere-meter in the circuit. Each student has a worksheet and a lap-top computer. A piece of software installed on the computer allows students to test their ranking.

These can be incorporated in a socio-cultural understanding of learning, where sign and language affect and are affected by these schemata. Learning in this sense cannot be static, but must entail change. It can be change of both signs, language and individual schemata.

As an example of this Bruun (2011) investigates how four students negotiate meaning (Wenger, 1998) while doing a task in an upper secondary physics class on electricity. In the study, the *praxeologies* of the Anthropological Theory of the Didactical (Bosch and Gascón, 2006) are used as proxies for schemata. In short, a praxeology consists of a perceived task, a technique for solving the task, and a theoretical layer which explains the use of the technique and reasoning behind using the technique.

In the study, the four students are to rank the read-out of ampere-meters in a circuit with resistors with known resistance connected in a combination of serial and parallel connections. See Figure 5. One student, Catherine, expresses a working model of electricity, in which she follows the current by following the lines in the circuit. In this model, the path with the fewest light bulbs is the path with least resistance. Another student, James, has a different working model, which prompts him to identify and mark the parts of the circuit which are serial and parallel. He then uses the appropriate formula to calculate the resistances and is able to compare quantitatively which ampere-meter has the greater read-out.

The students discuss which model is right, and James quickly convinces Catherine that his answer is correct. He does this without any of the students trying to find out if her answer is correct using the computer. Over the course of about five minutes, there is discussion where Catherine repeatedly asks James about his thinking, and where James explains it using gestures, drawings and words. Catherine changes her view to be in accordance with James', and in the end she feels confident that she understands the situation, and proceeds to explain it to a second student, Elin. Notably both Elin and the fourth student, Idun, have been practically absent in the preceding discussion.

This example highlights that to understand what kind of learning is taking place, we need to consider both what is going on between the students and what is going on within the students. As *a group* the students change the coordination of competencies: James explains his thinking to Catherine, who then acts to explain the physics reasoning to the Elin. As *a person*, Catherine changes her thinking about that particular problem, something which might signify a change in the coordination of her internal schemata. But all this happens as a complex interplay between students, internally in students and in the students' interaction with artifacts.

This view of learning is different from both socio-cultural learning and traditional cognitivism. The internal change in Catherine could be explained as a schematic accommodation, but it is bound to the socio-cultural context of the learning situation. Thus, the view of learning is a blend in the sense that (Podolefsky and Finkelstein, 2007) use it, because corresponding ideas in two domains of knowledge (here the two learning mechanisms) are projected into a new domain (Figure 6). In this new domain, internalization is enriched and thus changed to include accommodation, and the assimilation/accommodation is enriched to incorporate the interpersonal scaffolding. The new concepts are something else than both of the "old" concepts, and we can now describe learning as the tension between shared and individual competencies.

While Podolefsky and Finkelstein use blending to teach electro-magnetic theory via analogies (Gentner and Colhoun, 2010), this section shows how blending can support an integrated view of the mechanisms of learning. The view entails a complex tension between the individual and its surroundings, but no attempts at explaining how these theoretical considerations might be studied, verified, changed via empirical observations or rejected have been made yet.

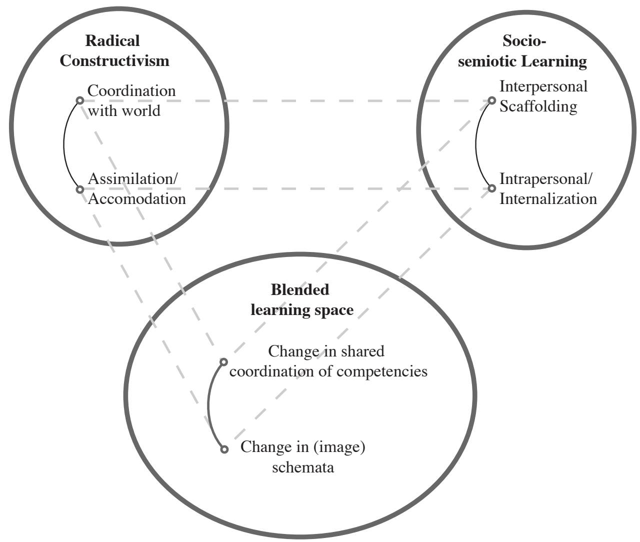

Figure 6: Blending learning mechanisms. In this thesis the view is put forth that classical cognitivism, exemplified by radical constructivism (Von Glasersfeld, 1996) does not concern itself with interpersonal relations, while socio-cultural learning as exemplified by Vygotsky (1978) does not concern itself with the details of how individuals minds change.

Since networks offer unique mathematical and visual ways of investigating complex patterns and have already been successfully used to inform physics education research, they may be expected to be useful to further an understanding of learning. Networks might be useful for connecting cognitive and socio-cultural networks in an operational manner, and this will be part of one of the research questions. To render such a connection probable, the next step is to integrate the current understanding of learning with network theory.

This amounts to more than using networks as a methodological piece of equipment. For the purposes of physics education research, nodes and links need to be infused with meaning, but within network theory nodes and links are connected to other concepts. So if we want to use networks, we have to consider more aspects of the theory.

Blending the blend: Educational networks in physics

The task of this section is to link network theoretical concepts to the learning mechanisms derived above. This is more than a mere operationalization of learning concepts using the network toolbox. The area of networks has its own theoretical constructs, some overlapping with the constructs we have already seen. Blending these with the already blended learning space yields a PER understanding of three different kinds of networks: student interaction networks, cognitive networks, and epistemic/discussion networks (Figure 7).

Before this, consider briefly two conceptions of network theory. Borgatti and Lopez-Kidwell (2011) distinguish between *theories of networks* and *network theory.* For Borgatti and Lopez-Kidwell the first case concerns models of how networks form, but the meaning extends to apply also to how structure is measured and how communities are identified. Network theory could concern the advantages of *social capital,* where one's position in the network allows one to gain access to more or less resources than others. A discussion about social capital is beyond the scope of this thesis, but especially Paper III identifies ties between network position and advantages not measured by networks.

This section focuses on how structural aspects of networks and the ideas of communities can be related to learning. As highlighted both by Borgatti and Lopez-Kidwelland by Rosvall et al. (2009), network theory can view the links between nodes in a network as conveyors of flow as Bodin (2012) did, but also as a kind of girders supporting the architecture of the network (Borgatti and Lopez-Kidwell, 2011). If for example a link signified "someone with which a person could act", then no information might flow. This tension could be called "flow versus relational", borrowing from Rosvall et al. (2009).

Blending network theory with learning theories yields a third kind of tie, as put forth by McCormick et al. (2011). Here a link signifies a process of transformation; learning is neither just the flow of information nor to act with someone. In a participatory framework like that of Wenger (1998) - and consistent with the definition of learning in this work - a link should be designed to signify the construction of knowledge, and development of skills, strategies, curiosity, and the shaping of identities.

Focusing now on Figure 7, consider first blending structural changes in networks with the two types of changes in the blended learning space. An important concept in both theories

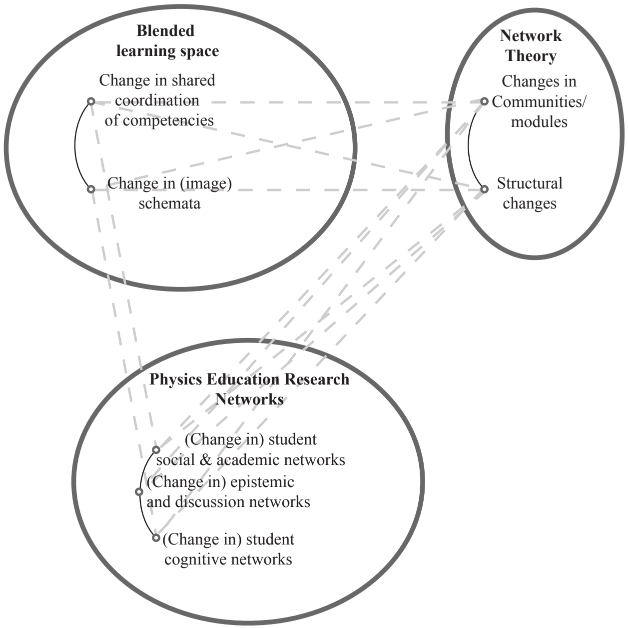

Figure 7: Blending network theory with learning theory as conceptualized in the previous section (Figure 6) involves projecting network concepts to multiple other concepts in the new blended domain (physics education research Networks).

of networks and network theory is centrality, and we need to relate this to learning. In general, a node's centrality measures how important the node is in the network or advantageous its position is. There are many centrality measures, and ones found most fruitful in this work are the ones where a node's centrality is calculated on the basis of the rest of the network. Bonacich's centrality is one, and Paper III describes three others in detail.

Structural changes in the network will consequently change these centralities, as is the case in Bodin's work, where the centralities of nodes are used to calculate the modules describing student epistemic frames. In Koponen and Pehkonen (2010) the centralities of nodes determine the overall quality of knowledge, and changes in centralities would then change the overall quality of knowledge. In Goertzen et al. (2012), the degrees of students and their ego networks change as they participate more and more in the learning community. Thus, structural changes as captured by centrality measures happen in networks describing both communities of practice and cognitive development. This is why in Figure 7 there are lines from structural changes in networks to cognitive, epistemic, discussion, academic, and social networks.

The structure of networks can also be characterized by the presence and abundance of different motifs in the network (Costa et al., 2007). Clustering is a measure designed for such a characterization. The (global) clustering coefficient of a network is the number of closed triplets (three nodes A, B, C with links A-B, B-C, A-C) divided by the total number of connected triplets (three nodes A, B, C with a *minimum* of two links between them, for example A-B, and B-C).

Another measure of the structure of a network is the degree distribution, measuring the frequency with which you find a node with k connections. Figure 8 shows the total degree dis-

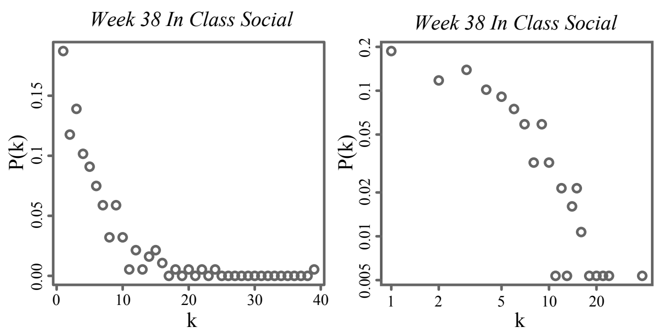

Figure 8: The degree distribution of in class social discussions from week 38 of the study described in Paper II shown both with normal scales and with double logarithmic scales.

tribution for a week in one of the studies in this work. Many real life networks including linguistic networks (Masucci and Rodgers, 2009) obey a power law or a power law with a cutoff (Newman et al., 2001). This seems to be true for this particular network. This means that there are many students with few connections (few reported interactions, in this case), and a few students with many connections. In a learning context we might consider using the few students to spread ideas, or we might consider working to change the distribution so that more students have many links.

Likewise the degree distribution of cognitive networks may be related to the cognitive state of the student, for example a power law might indicate that some concepts have higher priority than others, although the work of Bodin and Paper III suggests that other methods for investigating this might be more fruitful. Degree distributions are mentioned here mainly as an example of how change in overall network structure could be related to learning and because they are often used to characterize networks in network papers.

While structural changes in the network as measured with centrality and whole network measures inform our position on networks in physics education research, the idea of developing communities in networks may also be captivating. Network literature is ripe with algorithms for partitioning a network into communities, modules, or groups (Lancichinetti et al., 2009), but there is no consensus of what defines a community. Perhaps PER can provide data, which may aid theories of networks to this end.

In PER it seems that only Forsman (2011) investigated briefly the community structure of a network of students. As he relates to retention and not learning, we have little guidance as to how network communities or modules may be blended with for example communities of practice or to cognitive states of students. Thus it remains open how network communities relate to groups of students studying physics.

With the work of Bodin (2012) however, we do have a first step of how modules of cognitive (epistemic) elements might be grouped to inform a position of schematic change. If Bodin's modules represent cognitive structures like p-prims, image schemata and other types of schemata related perhaps to competencies in the sense defined on page 37, then her study shows how teaching and learning activities can change these structures in the context of physics. Then this is an example of how learning can be captured via networks.

As pointed out by Koponen and Pehkonen (2010), viewing networks drawn in different ways may point to different features, so there is also a visual side of network theory, which we should not neglect. In this dissertation this is exemplified by the snapshots of networks depicted in Paper II and also online (see Paper I). The visual representation is one of the strengths for researchers working with networks, and looking for changes over time in a cognitive or student interaction network could give valuable insights into learning processes as they unfold in a classroom and in students.

This means that the PER Networks in Figure 7 include this form as representation on equal footing with numbers, tables and graphs. Brewe et al. (2012) uses a circular form to show where the most connections are, Koponen and Pehkonen (2010) uses a dendrogram to show hierarchies, and most of this work uses a forced based drawing algorithm (Kamada and Kawai, 1989; Csardi and Nepusz, 2006) to show the overall structure of the network.

Most of the studies on educational networks do not consider the actual changes in these networks over time; only Goertzen et al. (2012) and Bodin (2012) do that. While the stationary state of a network can say something about a student's cognitive ability or about possible student interactions, in the context of learning, we will always be concerned with changes, and thus changes in networks over time seem to be the most interesting development we can strive for. To get there however, we need to overcome several challenges. The following section outlines some of these challenges, thus allowing research questions to be formulated.

Challenges

The challenges outlined in this section fall in three categories: Challenges related to theoretical issues, challenges related to measuring networks, and finally challenges related to didactical implementation of network theory. The challenges are outlined in turn.

This chapter has focused on taking up the challenge of relating network theories to learning theories. The scope has been narrowed down to learning mechanisms. Theories of learning involve many other aspects, as the wide array of subfields in PER testifies. The detailed role of language as represented in networks has not been investigated, and this has been pointed out by many (classically Ogborn et al., 1996; Lemke, 1990) as an important factor when learning science. Further, gestures and other actions like interactions with software, pen & paper and other forms of representation (Roth, 1995; Roth and Lawless, 2002)are crucial for understanding how physics is learned. Also, capturing identity issues might be challenging in a network representation, something which epistemic networks might be useful for. In reverse, given a network (cognitive, epistemic, or interaction), it is a continuous challenge to derive meaning from its structure, from individual node positions, and from any development of the network. For example, to what extent does a node in a student social network represent a student? To what extent does a node in a cognitive network represent a part of a mental structure in a student?

The challenges of measuring networks involve establishing reliable and valid connections between nodes and even to define what nodes are. Any understanding of the composition and structure of the network (i.e. the state of the network) must be intimately connected to the way the network data was collected. The understanding students have of a network survey question (what prompts the student to mark someone as a collaborator) hold valuable information about the learning processes experienced by that student. Likewise, a student's understanding of what it means to create a concept network in a particular subject in physics influences the way in which the map is created. So one part of the measurement challenge is to collect data in a way which makes student understandings transparent. A second part is then to link the network representation to this understanding as Koponen and Pehkonen (2010) does.

In terms of interaction networks there is a fundamental problem with people rating the quality of their connection with others as good/bad or more or less influential. It is simply a matter of the truthfulness of the rating; people who answer surveys (including network surveys) notoriously impose whatever picture of themselves they believe the researcher should have.

On the other hand there is a fundamental problem with objective measures because they do not measure the quality of the connection. In a phone call network, what did they talk about? In an e-mail network, what was the e-mail about? In a discussion forum, what learning processes can we connect to the fact that one student answers another student's post?

The third challenge is related to how networks can be used further in didactical research design. This is in many ways the most central question, because if we cannot use networks, why bother investigating them? Note again that most of the studies utilizing networks do not investigate how they change. But learning physics is a process, so the usefulness of networks must be tied to how they relate to this process. To be useful in understanding physics learning, networks must yield information about processes of learning physics, be they cognitive, socio-cultural or a blend. They should act as a quantitative gauge of competencies in physics, and they should in some way be related to teaching practices and learning outcomes.

These are harsh challenges. It is easy to get lost in a fascination of networks, but we must stress that if networks are to gain position in physics education research, they should at least show the promise of overcoming these challenges. And in essence, that is what this dissertation is about.

Before formulating these challenges as research questions, I want to make a short digression. I have written at length about social network analysis, networks as they are used in physics, and networks as they are understood in PER. In the next short section I want call attention to the relation between social network analysis and network physics as fields.

On the relation between the physics of networks, social network analysis, and the networks in this work

There is a divide between the literature in physics dealing with social networks and the corresponding literature in sociology. This illustrated in Fig 3.3 in Scott and Carrington (2011), where it is clear that physicists cite physicists and sociologists cite sociologists. This partition into broad groups citing each other can be seen as an indicator of some enmity between physicists and sociologists dealing with networks.

This is further backed up by Chapter 3, where physicists are portrayed as imperialists ignoring all the good work the sociologists have done. After having discovered SNA, "*these physicists, new to social network analysis, did not read previous literature; they acted as if our 60 years of effort amounted to nothing [...] They simply claimed research topics that had always been part of social network analysis and made them topics in physics*" (Ibid, p. 28). The two groups have developed very different ways of analyzing and interpreting networks, although some things remain the same. The book calls for fruitful collaboration between the two, and this work does consider the work of both groups.

As a further note, the understanding of networks in this thesis is fundamentally different from the assumptions of SNA. As pointed out in *Blending the blend: Educational networks in physics* above and in Paper I, linkages may not allow flow of information between actors nor conceptualize lasting patterns of relations among actors (Borgatti and Lopez-Kidwell, 2011; Brewe et al., 2012; Wasserman and Faust, 1994). Links between students in this work signify processes of transformation (McCormick et al., 2011), consistent with the constructivist view on learning presented above. As such, I hesitate to see network positions as indicative of power, but would rather like to see them as positions of advantage in the case of humans and positions of importance in cognitive networks.

Research questions

The previous sections have elaborated on the connections between network theory, learning theories, didactics, and quantitative measurements. One of the themes has been of a more theoretical nature, with a focus on how network science, cognitivist, and social constructivist learning theories can inform each other. A second theme centered on how these learning theories can be operationalized in a network approach for studying learning. This theme has to do with how network science can be used to measure quantities relevant for learning. Finally, a third theme touches on the use of networks in didactical research investigating the processes of learning. This theme is of a didactical nature, addressing how changes in networks correspond to for example changes in competencies. The previous section outlined some of the challenges to each theme. The following three sections specify the challenges to theory, measurement, and didactical use in three overarching research questions. Each overarching research question is followed by sub questions to give more precise directions. Since the work presented in this thesis is by nature exploratory and the challenges are considerable, the research questions are broad. Very sharp research questions belong to well-developed research fields and traditions. As a field of research, educational networks is far from well developed.

The Theoretical Question

Q1: How can the science of networks and theories of social constructivist and cognitive learning inform each other to investigate student learning in physics classes?

- Q1a: What parts of social constructivist theories and cognitive theories can be represented in networks?
- Q1b: What analogies between the science of networks and learning theories are fruitful for an understanding of what it means to learn physics?
- Q1c: How do variations in network structure, dynamics, and agency contribute to a social constructivist and cognitive learning perspectives?

The Measurement Question

Q2: How can a network science approach be used to quantify the state of student understanding and use of physics?

- Q2a: What methods for collecting data can be developed and how can the data be represented?

- Q2b: How can student levels of competency in physics be understood in the context of network representations?

The Didactical Question

Q3: How can one describe the process of learning of physics from changes in relevant networks?

- Q3a: What types of networks can be defined to yield information about processes of learning physics.
- Q3b: How can the development of students' physics competencies in relation to their network practices be assessed?
- Q3c: How can the effect of teaching practices on student learning outcomes (competencies and knowledge) be investigated using knowledge about changes in networks?

Comments

The intention is to engage with the question of what it means to 'learn something' in physics from the perspective of a physicist as well as from that of an educational researcher. The methods used in both disciplines are weaved together in the following sense: The students in this study are subject to conceptual questions and questionnaires as is one of the standard methods in physics education research. A mixture of theory from physics and didactics is then used to create categories, which can serve as nodes and links in network representations of social interactions and cognitive networks. Once obtained, these networks are analyzed using the methods from the physics of networks and social network analysis (SNA). From these quantitative analyses, the theory and method must be constantly modified to integrate physics and physics education research in a meaningful way. The thesis papers show how one can engage with the question of learning from a physics point of view as well as from an educational research viewpoint

The chapters following this one are overviews of the thesis papers. The actual papers are included as appendices. Each overview consists of an introduction where the paper is framed in terms of what research questions it addresses and the paper's status with regards to publication. This is followed a short overview of the main results of the paper, which also puts the work in a more general context. Finally, the epilogue of each chapter addresses issues of validity and reliability. The central issue for this thesis work is *validity*, while it does not prioritize reliability as much. The thesis as a whole is to be seen as a proof of existence, rather than a continuation of a well-established field. However, for each of the papers, the epilogue sketches how reliability could be improved.

The logic of the order of papers is as follows. After the theoretical considerations of this chapter, *Developing network methodology* (p.51) outlines how Paper I addresses the problem of finding a method to establish theoretically informed link categories based on a particular student context. The outline of Paper II (p.55) shows that the categories developed for the context of Paper I can be used meaningfully in another but

	Chapter 2: Rapprochements	Paper I	Paper II	Paper III	Paper IV
Theoretical Question (Q1)	Q1a-c	Q1a,Q1c	Q1c	Q1a-b	Q1a-b
Measurement Question (Q2)	-	Q2a	Q2a	Q2b	Q2a-b
Didactical Question (Q3)	-	Q3c	Q3a-c	Q3a	Q3b

Table 1: The chapters and papers in this dissertation that help answering the research questions. Each paper is outlined in a different chapter following this chapter.

related context; the shift is from an upper secondary physics class to first year in a university physics program. Both Paper I and Paper II investigate week-by-week variations in student interaction networks. Paper III, outlined in Relations between network and other attributes (p.59), then assumes that summing up links from each week of a seven week period gives a valid network of a students positional advantage during the first semester. The final paper presented in Paper IV, which is outlined in Analyzing student writings in physics (p.63), is an example of how one could work with cognitive networks. It investigates the possibility of converting student writing to linguistic networks (Masucci and Rodgers, 2006), which are then analyzed using network theory.

The papers do not follow the order of the research questions. Instead, each paper addresses different parts of the main themes of this thesis as represented by the sub questions. Table 1 shows which sub questions each chapter addresses. sums up how the different research questions have been answered, and finally outlines how networks can be integrated in future physics education research.

Developing network methodology

This chapter is an introduction followed by an outline of and an epilogue to the first paper of the dissertation: *Combining methods to analyze student relations: Network science and communities of practice.* The actual paper is included in Appendix I.

The paper addresses how socio-constructivist and competency based learning theoretical constructs can be represented as links in networks. As such it addresses the theoretical sub question Q1a. It also addresses the theoretical sub question Q1c and the didactical sub question Q3c, because it investigates how networks vary in time, and relates this to both learning theory and didactical use. Finally, it is an answer to the measurement sub question Q2a, because it describes a method for collecting and representing network data.

The paper needs further editing to be ready for submission, but the intention is to submit a version of it to International Journal of Research and Method in Education (IJRME).[3]

[3] At the time of publication, it is still uncertain in what form and where the paper will be submitted. Please cite as: Bruun, J. (2012). Combining network science with communities of practice to analyze learning processes: Interpretation, structure, and development of student networks. *Networks in physics education research: A Theoretical, Methodological, and Didactical Explorative Study* (IND skriftserie vol. 28). Copenhagen: Department of Science Education.

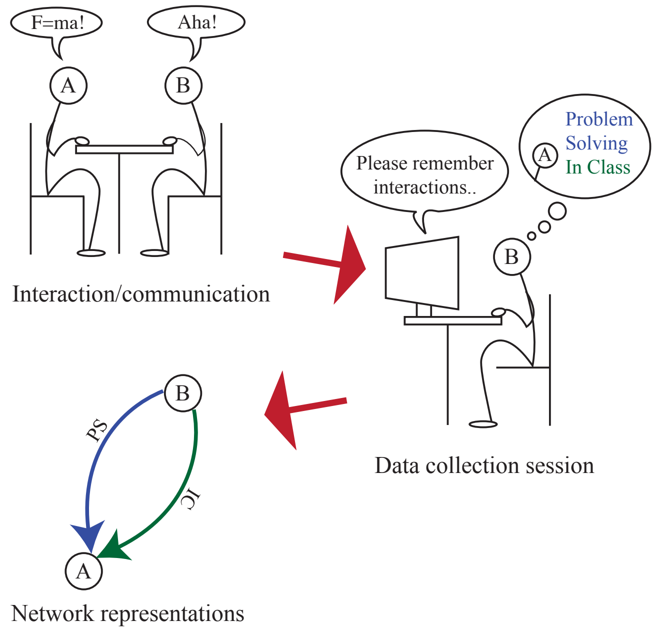

Figure 9: How networks are generated from student self reports in this study. In the bottom, the circles with the letters A and B are called nodes, and the arrows are called (directed) links.

Outline of Paper I

The paper contributes to the field of physics education research by creating, testing, and validating a method for generating networks to describe social learning processes in physics classrooms.

Figure 9 on the preceding page depicts how this study uses student self reports to generate different types of student interaction networks. Two students, A and B, communicate about how to solve a problem in physics during class. Student A says something which B remembers later on. In a data collection session that occurs after the interaction, B sits in front of a computer and answers an online survey where he can check different kinds of interactions. B remembers A for the interaction about *problem solving* (PS) *in-class* (IC). In network representations of these choices, one link from B to A is generated in a network that describes PS student interactions and another link in the network that describes IC student interactions. The primary purpose of this study is to investigate how educational researchers can develop an understanding of these links between students.

The paper shows the development of nine other interaction categories than PS and IC. The categories were created by analyzing student written answers to open-ended surveys using physics education research literature and socio-cultural learning theory. The categories were formulated as statements which students could use to characterize interaction they reported having with other students. These statements were administered as surveys on several different occasions during a semester, producing networks of student interactions.

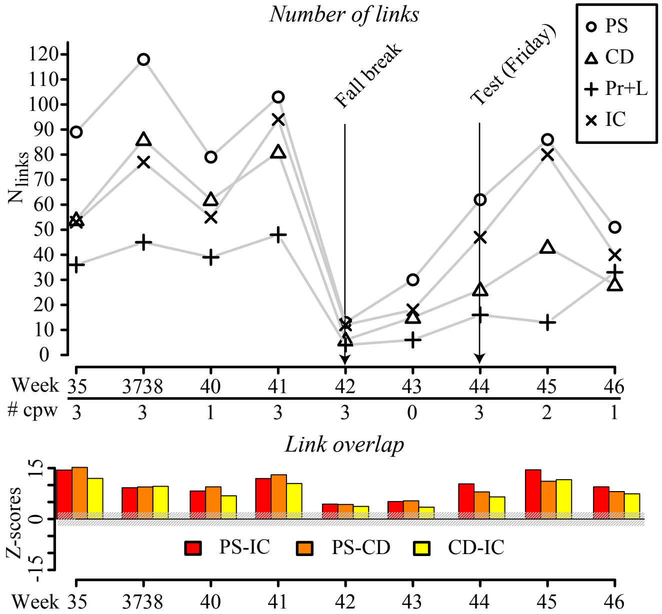

Figure 10: The number of links in the *problem solving*, *in-class*, *concept discussion*, and *language and presentation* networks. The number below each week is number of teaching units (1.5 hours) in the preceding week. Thus, week 39 had one teaching unit, even if the number 1 occurs beneath week 40. The Z-scores at the bottom show the overlap between links of three categories compared with 1000 random versions of these networks. The significant region is marked by the shaded grey area. In a week, networks have a high degree of overlapping links, meaning students tend to name the same people in these categories.

The networks were analyzed using network theoretical measures. Figure 10 on the preceding page shows the result of one of three different analyses. It shows that the activity with regards to physics interaction in class as measured by the number of links, N_l, varies each week in correspondence with the number of teaching modules the students had in the preceding week. Notice that problem solving is the most prominent of the physics categories, outranking for example concept discussion.

This observation is used to generate the hypothesis: *If student learning focused on activities emphasizing novel (from the perspective of the student) use of other representational forms than those pertaining to use of formulae, then the concept discussion network would show more activity (more links) than the problem solving network.* This and two other hypotheses based on two other observations are generated on the basis analyses of student written answers that were given in a validation session held after the networks were collected.

The hypotheses are not answered in the paper. The purpose of the paper is to show how a method for generating student interaction networks has been created and that the resulting networks can be used to generate meaningful hypotheses.

Epilogue

The paper is almost entirely focused on generating networks[4] that correspond to actual interaction patterns between students. Thus, the primary focus of the paper is validity. It is meant as a proof of existence, and therefore does not concern itself explicitly with reliability issues. For example, given the same student answers, other researchers might have reached other coding categories or other classifications of the answers due to the same codes. Adding more researchers and testing for intercoder reliability would be a simple way around this issue.

Given the networks, many of the calculations are completely reproducible. Only the randomized calculations would yield somewhat different Z-scores each time. This issue though is resolvable by increasing the number of iterations on which to base the Z-scores. This can be solved with increased computational power.

Finally, the paper makes no assumptions as to how general the categories or resulting student interaction networks are. However, the categories were tested by using them in a different context: First year university students taking an introductory physics course. This is described in the following chapter.

[4] The networks can be found online here: http/www.jbruun.org/om/studentNetworks

Time development of student interaction networks

This chapter is an introduction followed by an outline of and an epilogue to the second paper of the dissertation: *Time Development of Early Social Networks: Link analysis and group dynamics.* The actual paper is included in Appendix 2.

The paper is based on university students' answers to the previously developed network survey (see the preceding chapter). The students attended an introductory course in physics at the University of Copenhagen. It is not a physics education research paper but a physics paper. However, it can still be used in answers to the research questions. The paper helps answering the theoretical sub question Q1c, because it yields results about variations in network structure and show how to couple student interaction networks to external variables (grade, gender, and laboratory exercise class) relevant to social constructivist an cognitive learning theories. It is also an answer to the measurement question Q2a, because it shows that meaningful data can be collected using the previously developed survey, and it shows different ways of representing and analyzing the data. Finally, it is an answer to all the sub questions in the didactical question, since (1) student interaction networks are shown to yield information about the processes by which students organize themselves to learn physics, (2) student group formation is coupled to a (crude) proxy for their physics competencies (grade), and finally (3) the effects of how the introductory course is managed are visible in the variations in group structure in student interaction networks.

The paper will be submitted to Physical Review E, because this journal "is interdisciplinary in scope" and focuses on among other things on "many-body phenomena". In the context of physics, this paper is about many-body phenomena. [5]

Outline of Paper II

This paper is meant as a contribution to the field of network physics. The primary contribution is to supply, display, and analyze a set of interaction networks which describe the early

[5] At the time of publication, this paper is undergoing revisions of language and title as well as minor additions of analyses. To cite the paper as it appears here use: Bruun, J. (2012). Time development of Early Social Networks: Link analysis and group dynamics. *Networks in physics education research: A Theoretical, Methodological, and Didactical Explorative Study* (IND skriftserie vol. 28). Copenhagen: Department of Science Education.

evolution of a (social) network. The analyses uses both existing network research methods and develops a measure for student segregation. For physicists studying networks one of the interesting findings of the paper is that the network seems to freeze over the course of measurement. Figure 11(a) shows that the fraction of completely new links decreases over time, and the function looks exponential. This may have impact on computer models of network evolution.

The paper may contribute to educational research on first year university students. First, the alluvial diagram in the top of Figure 12 shows that student groups change between weeks. This is indicated by the streamlines between the blocks representing the groups. Towards the end of the measurement period, groups seem to be homogeneously composed according to grade. This is indicated by the overall green colors. Alluvial diagrams make use of the PageRank centrality measure. It is a measure of node's relative importance in the network developed by Page et al. (1999). In the next chapter, Paper III describes how PageRank may apply to physics education research.

The results shown in the alluvial diagram are interesting when one takes into account that the head teacher of the laboratory exercises had students rotate groups (of three) each week. The alluvial diagram can be interpreted as an indicator that this initiative may have contributed to a homogenous distribution of student grades in groups. This homogeneous distribution is also apparent in Figure 11(b), which shows that students never segregate significantly according to grade.

However, Figure 11(b) shows that students segregate according to laboratory class and to a lesser extent gender.

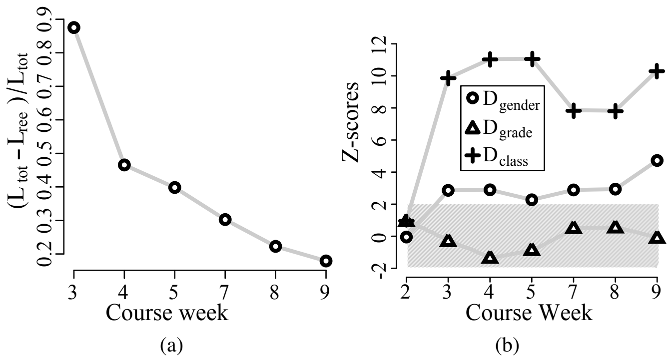

Figure 11: Freezing and segregation in the interaction networks. (a) The fraction of completely new links relative to the total number of links, $\frac{L_{tot}-L_{ree}}{L_{tot}}$, seems to decrease exponentially. The number of unique links for all weeks is 1214, which is about 4-6% of the total number of possible links in a directed network with 140-160. This means the the decrease in completely new links is not due to a saturation of network links. (b) Segregation Z-scores for gender, grade, and lab class for each week. The shaded area indicates the non significant region.

This emphasizes that laboratory and problem solving exercise classes in this physics course are important structures affecting how students behave. Thus, teaching-learning activities as they unfold at these exercises classes may be very important for students' participation in communities of physics learners.

Finally, notice the network maps beneath the alluvial diagram in Figure 12. These maps show between-group com-

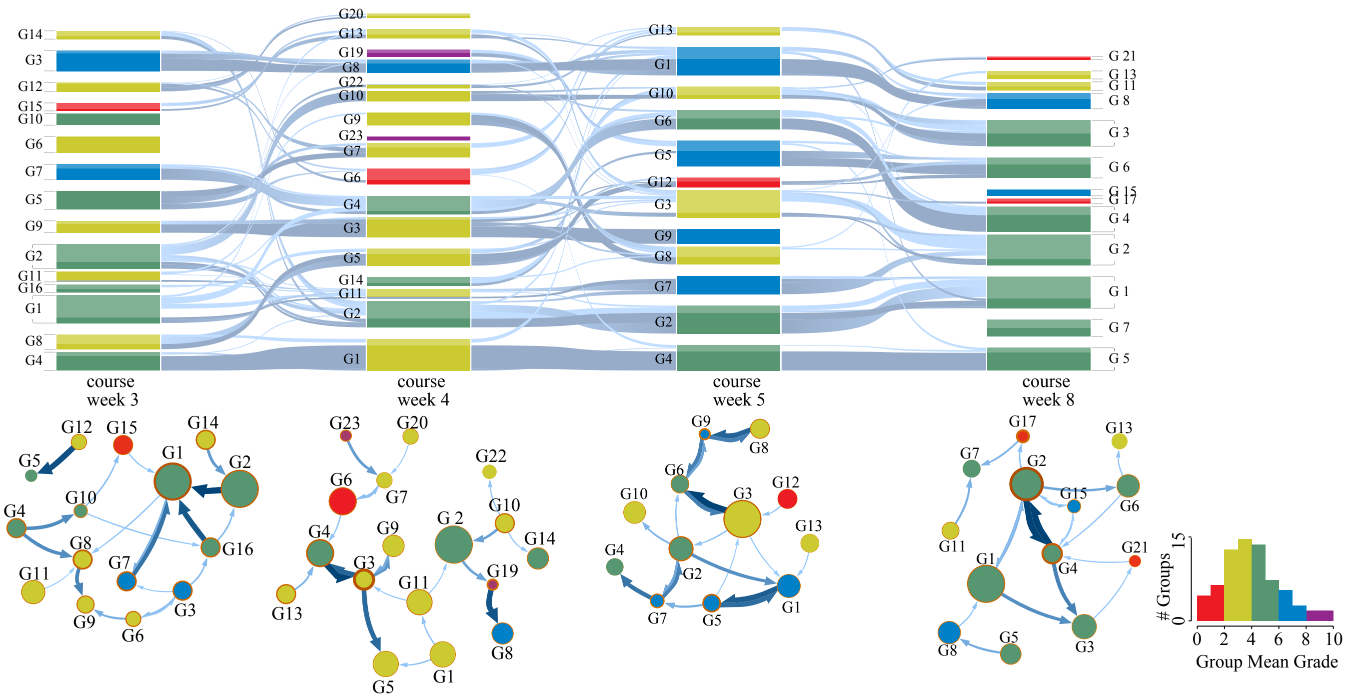

Figure 12: *Top:* Alluvial diagram for groupings of the four networks displayed in the paper's Figure 2. The height of a block representing a group is proportional to the accumulated PageRank (Page et al., 1999) of the group, and the lighter colors in each group indicate how much PageRank is insignificantly clustered (Rosvall and Bergstrom, 2010). The thickness of the grey streamlines between groups in different weeks indicate the initial and final PageRank the nodes making the transition from one group to another. *Bottom:* Flow maps of the same groups shown in the alluvial diagram. Node sizes are proportional to the number of students in the group. Arrows are proportional to the information flow between groups as calculated by Infomap. The total number of groups each week is 28, 28, 22, and 22, respectively. Groups are color coded according to their mean grade. The histogram shows the distribution of grades for all groups in all networks.

munication patterns for each week. The direction of an arrow indicates that a group points to another. For example in the map of course week three, four groups point at group G1. This means that many students in other groups remember having communicated with students from this group. If these interactions signify learning processes, then this group could be a target for a particular (new) method of solving problems. A successful strategy would be one in which the method would diffuse through the network to other groups through actions involving negotiations of meaning centered on understanding physics.

Epilogue

Some of the validity of the survey instrument is lost by using it in a new context without analyzing a validation session as in Paper I. However, some of it must be preserved, since it produces meaningful results in the context of this particular university course. At least the *problem solving* category seems to measure interactions between students engaging with problem solving and laboratory exercises.

With regards to reliability the results in this paper are reproducible given these particular student interaction networks and the method. Infomap is a reliable algorithm, meaning that for the networks examined it makes the same partitioning almost every time. This is a strength of this particular grouping algorithm compared with others (**?**). However, the bootstrap procedure used to generate the alluvial diagram shows that small variations in the network would produce somewhat different groups (a wording which is not used in the paper). In this researcher's view this is actually not a reliability issue, but more one of validity: Are the groups found by a grouping algorithm really the groups that exist? With regards to the segregation scores, the results are as reliable as the grouping. Again there will be a slight variation in the Z-scores, but with 10^4 calculations there is no significant variation.

The purpose of the next paper is to develop an understanding of the meaning of links through an analysis of different categories. Also, it relates grade, seen as a crude external proxy measure for cognitive ability, to socio-cultural learning theory. The paper uses student interaction networks that have been summed up over time. Thus, the student interaction networks presented in the next chapter are social networks with weighted links that have been established incrementally through time.

Relations between network and other attributes

This chapter is an introduction followed by an outline of and an epilogue to the third paper of the dissertation: *Talking and learning physics: Predicting future grades from network measures and FCI pre-test scores.* The actual paper is included as Appendix 3.

The paper presented in this section bridges individual student performance with their engagement in a community of physics students. It addresses the Theoretical sub question Q1a by representing proxies for student cognitive abilities as node attributes and student engagement as network centrality scores. This analogy provides a basis for integrating cognitivistic with socio-cultural learning theories, where learning physics is both tied to social interactions and cognitive aspects. This is why the paper is also linked to the Theoretical sub question Q2b. This is very much tied to this dissertations view of physics learning as a change in physics competency as defined in *Rapprochements: Networks, physics, and learning* (p.25). As such the networks in this paper yield information about the processes of learning physics, and thus the paper is also an answer to the Didactical sub question Q3a.

The paper has been reviewed by Physical Review Special Topics - Physics Education Research (PRST-PER). The reviews make it clear that while paper appears very technically dense and needs significant revisions, it has the potential to make an important contribution to the field. In our revision of the paper, we will address the issues identified in the review process. [6]

Outline of Paper III

This paper contributes to the field of physics education research by using social and academic network data as a novel way of relating cognitive and sociocultural aspects of learning. In the paper links are understood as representing learning processes as described previously. It examines three different student interaction networks from an introductory physics course and these networks relations to student grades in two subsequent courses. The first student interaction network is the *problem solving* network, which is generated as the sum of the problem solving networks examined in Paper II. To be more precise, in the problem solving network a link of strength w_{ij} is created from student i to student j, if student i has named student j w_{ij} times during the measurement period. The *concept discussion* and *in-class social* communication networks are created in the same manner. Figure 13 on page 61 shows

[6] After revisions, the paper has been published by PRST-PER. It can be located here: http://link.aps.org/doi/10.1103/PhysRevSTPER.9.020109

the in-class social network. The link strengths are indicated both by thickness and gradient; thicker and more dark links have a higher weight.

The grades of students are indicated by the legend at the bottom left, and the force-based layout of the network shows that the peripheral nodes seem more likely to be characterized by lower grades, while the center nodes seem more likely to characterized by higher grades. Quantifying this perceived pattern, the paper develops target entropy as a measure of centrality for individual students in a directed network, and in Figure 13 the sizes of nodes are proportional target entropies. It turns out that there is a highly significant correlation ($p < 0.001$) between the network position as measured by target entropy and the *sum of grades* for the two subsequent courses. In Figure 14 on page 62, this is shown as a line between the node T - belonging to the in-class social communication network - and the sum of grades (SoG) - in the middle of the network. The number on the line, 0.38, indicates the strength of the correlation, and the number in the orange circle, 97, indicates that the correlation has 97 degrees of freedom.

Figure 14 shows all the highly significant correlations between attributes and network position measures for the three student interaction networks. This type of network is called a correlation network. From this researcher's point of view, the most important finding of this paper is that (1) the part of the correlation network describing problem solving centrality correlations can be overlaid on the top part of the in-class social centrality correlations, while (2) the concept discussion network centrality can be overlaid the bottom part of the in-class social network position correlations. This means that the in-class social network seems to capture aspects of problem solving as well as concept discussion interactions. The paper suggests that the process of learning physics entails purely social interactions as well as academic interactions, in the sense that the social interactions facilitate academic interactions.

Epilogue

The paper allows for an interpretation of the three link categories problem solving, concept discussion, and in-class social communication as proxies for learning processes. However, it does not reestablish the tight connection between theory, student answers and survey categories developed in Paper I. This is a validity issue; we can only surmise that students modify ideas and practices, since no direct evidence - for example, from a validation session (Paper I) - to support interpretation.

The results are reliable in the sense that they are reproducible given the data and method. Bootstrapping relies on random sampling of the networks. However, the bootstrap models have been performed on the data a number of times during the analysis, producing the same results.

The paper uses Force Concept Inventory data and grades as proxies for student understanding of mechanics. While it produced meaningful results, these do not represent all important aspects of student competencies in physics. The next paper presents the argument that student writings about their understanding of a physical system can be represented in networks of their writings. Further, this would represent an important part of their competency with regards to doing physics.

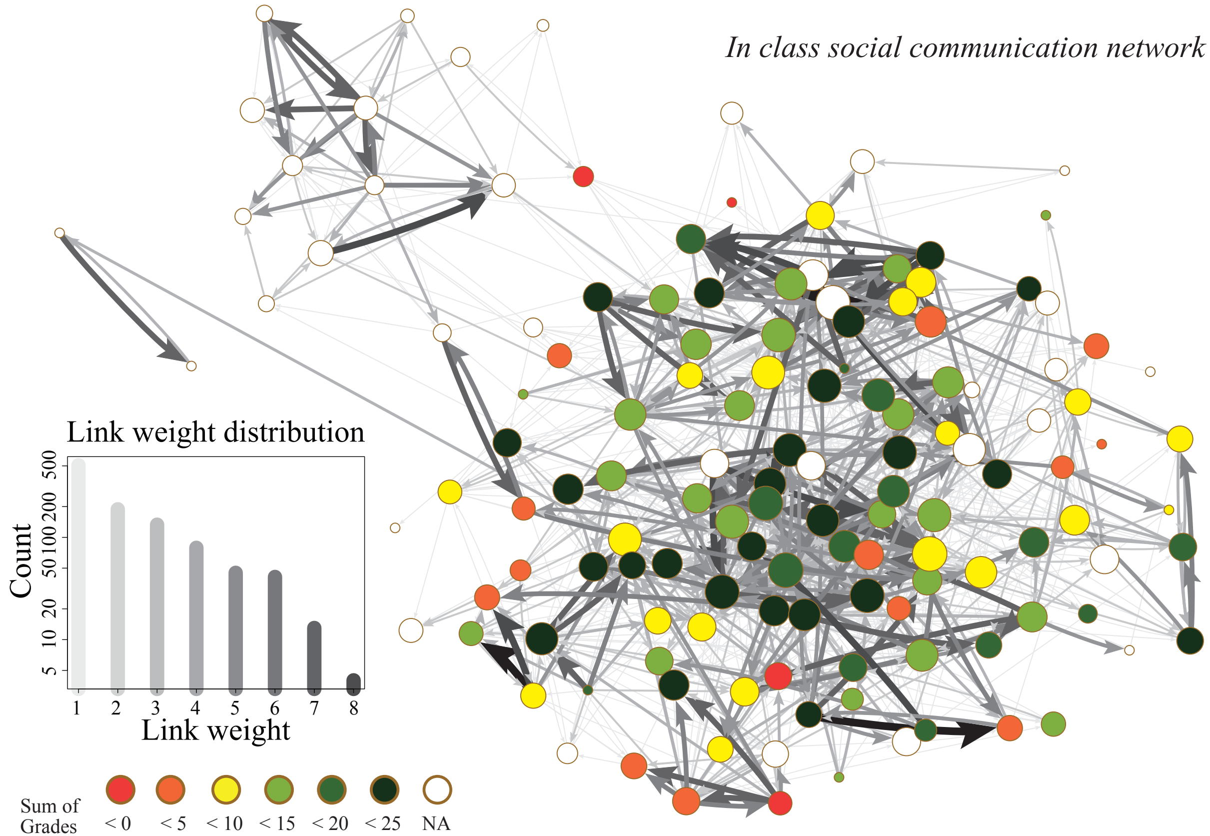

Figure 13: Sociogram of the in-class social network with an inset of the link weight distribution. Links are coded according to strength, with light grey representing weak links and darker grey representing increasingly stronger links. Nodes are coded according to the sum of grades as indicated at the left bottom. The size of nodes are proportional to target entropy. The lay-out has been determined by the Kamada-Kawai force-based algorithm (Kamada and Kawai, 1989) also used previously.

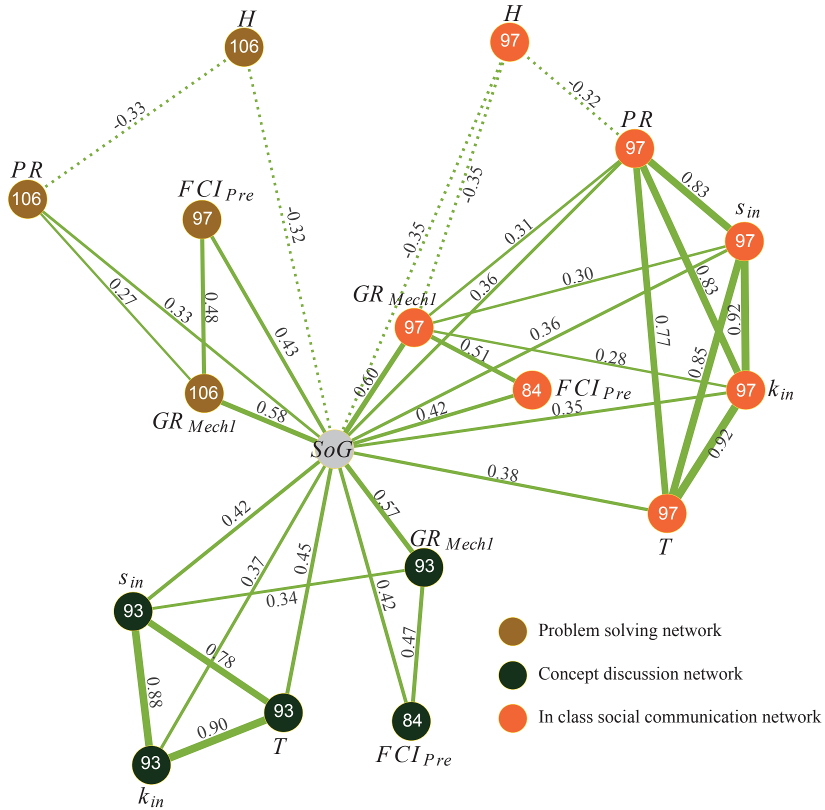

Figure 14: Correlation network of measures that correlate to a highly significant level with SoG (p<0.001). Correlation coefficients are included on the links and the number of degrees of freedom are shown in the nodes.

Analyzing student writings in physics

This chapter is an introduction followed by an outline of and an epilogue to the third paper of the dissertation: *Representing cognitive schemata with networks of student free text answers to conceptual problems: Justification and first steps towards a method.* The actual paper is included as Appendix 4.

As the only paper in the thesis, this one deals explicitly with cognitive aspects of student competencies. It argues that student writings can be represented in networks, called linguistic association networks, describing how students coordinate mental schemata to associate physics concepts. Thus, it addresses the theoretical sub questions Q1a and Q1b, because *linguistic association networks* function as analogies for mental structure: Nodes represent concepts and links ways of coordinating concepts. It addresses the measurement question by developing a design for capturing linguistic association networks and by arguing that structural network measures, like target entropy and search information, may inform researchers about student competencies with regards to conceptual and phenomenological forms of representation. Finally, it addresses the didactical sub question Q3b, because it shows developments of students' linguistic association networks over a period of time.

The version of the paper submitted with this dissertation is an early first draft. Major analyses lie ahead to substantiate the arguments put forth in the paper. The task is to integrate the results of network analyses with cognitive schema theory to relate linguistic association networks or a related concept to existing physics education research. [7]

Outline of Paper IV

The central argument of this paper is divided into two parts. First, if pre-linguistic structures - like image schemata and to some extent phenomenological primitives - structure human understanding and are represented in written language, then written language in turn should hold some of that structure. Second, networks of written language allow for a study of language structure, so networks of written language may encapsulate some of the structure of human understanding.

The article describes a method that has students produce written outputs. From this output, linguistic networks are gen-

[7] To cite this paper, use: Bruun, J. (2012). *Representing cognitive schemata with networks of student free text answers to conceptual problems: Justification and first steps towards a method.* Networks in physics education research: A Theoretical, Methodological, and Didactical Explorative Study (IND skriftserie vol. 28). Copenhagen: Department of Science Education.

erated and analyzed. Student writings are collected using online open-ended conceptual physics questions. These writings are then treated as linguistic data that can be systematically altered by what one could call *linguistic reduction*. While Figure 15 explains the data collection process, linguistic reductions discussed in detail in the paper[8]. Linguistic reduction may include one or more of the following: Reducing all words to their stem form (for example, *waves* is converted to *wave,* and *added* to *add)*, generating and implementing synonyms, and removing common words (like *that* and *is).* A particular linguistic reduction generates a linguistic network based on the adjacency of the remaining words. As an example, consider the phrase *waves will meet* (Figure 15) A linguistic reduction that reduces waves to wave would generate three nodes and two connections (*wave*→*will*, *will*→*meet*). A linguistic reduction that also removes the word *will,* would generate two nodes and one connection (*wave* →*meet).*

In Paper IV, eight written answers to conceptual physics questions by two different students are converted to linguistic association networks. For each written answer, words are reduced to stem form. Keeping every word as nodes and basing links on adjacency word adjacency generates eight linguistic networks. Eight different networks are generated by removing frequent Danish words from the answers and using the remaining words as the basis of linguistic networks. The result eight pairs of linguistic networks. Each pair is based on the same text but processed with a different linguistic reduction.

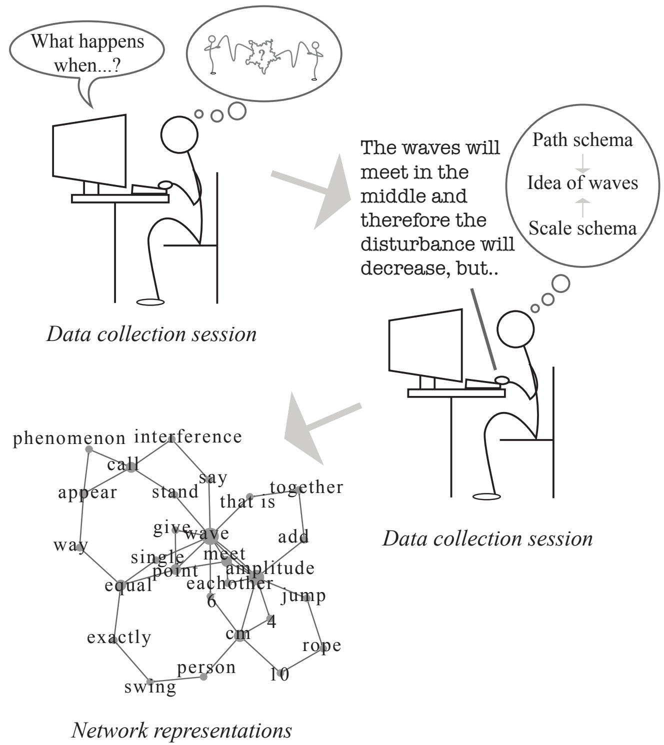

Figure 15: Students read an open-ended question and answers it online. They are asked not to discuss their answer with others. The hypothesized mechanism is that while students are writing image schemata are activated and used, resulting in a written text. The text is converted to a network based on the methods described in the paper.

[8]In the paper, the process is not termed linguistic reduction. It is described under the heading: Generating linguistic networks and investigating them with cognitive linguistics.

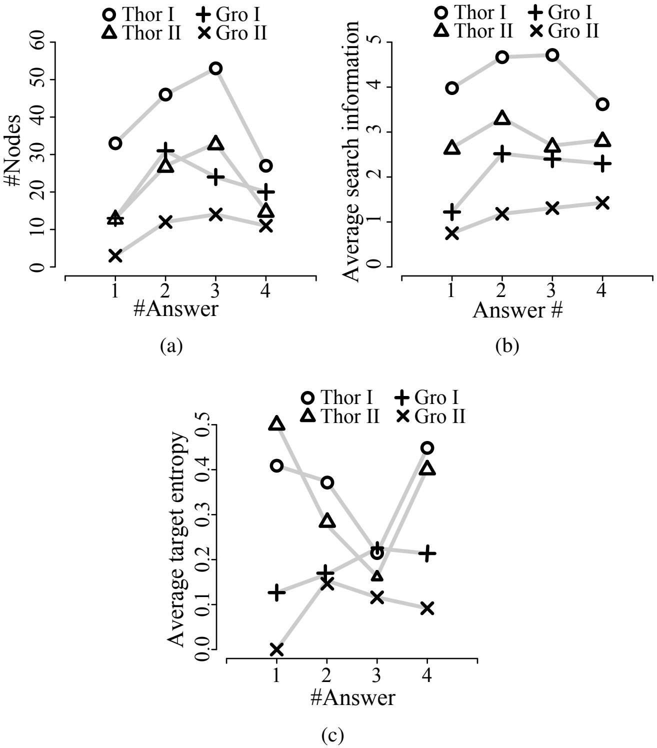

Figure 16: Three figures comparing different structural aspects of networks of two students' writings. The writings are from four separate occasions. For each student, two different linguistic reductions are analyzed. The difference between e.g. Thor I and Thor II is that frequent words have been removed as nodes from the networks. In each network, search information (b) and target entropy (c) have been calculated for each node and averaged for each network.

These networks are then analyzed with network analysis. Figure 16 shows the result of calculating three networks measures: the number of nodes (unique words or entities) in the networks, the average search information, and the average target entropy (used in Paper I on student interaction networks). The search information, target entropy, and the number of entities characterize student associations. The paper argues that these properties might be connected to student competencies in future research.

As the figure shows, removing selected words have profound impact on the network structure. While the first type of network is a linguistic network, the paper introduces the term *linguistic association networks* to describe the type of network where frequent words are removed. The paper suggests that linguistic association networks can be used to find traces of different kinds of image schemata and these image schemata's connection to physics concepts. This should be done (1) by identifying and keeping words that serve as traces for specific image schemata (even if these words are frequent) or (2) by acknowledging frequent words as signifiers for processes of association, in which case the network analysis should include different link types. The latter would be described as moving frequent words to links.

Epilogue

Even if the paper is mostly an outline, there is one pressing reliability issue to discuss. One could argue that linguistic association networks are too dependent on how students chooses to use words on a particular day. Thus, it would be difficult to derive lasting knowledge structures from them. At the same time this is a validity issue, since it cannot be taken for granted that the networks actually measure competencies in physics.

One way of addressing these two issues would be to relate linguistic association networks to other ways of evaluating student competencies. For example, the analyses in this study can be compared with student grades and with a student positions in student interaction networks, since the writings were collected along with the student naming data of Paper I. If structure in linguistic association networks can be reliably coupled to other ways of evaluating competency, then the reliability of student wordings is not an issue for the applicability of linguistic association networks.

The promise of this approach from a research perspective is that the networks are easy to generate (students simply need to write). Once a method is developed it can be applied to many pieces of writing quickly. This allows a big corpus to be analyzed quickly to see patterns in student understanding as represented the networks. These patterns might then in turn inform researchers about student understanding of physics.

Conclusion

The present dissertation has described a PhD project of a highly exploratory nature. The primary task was to conceptualize a field within physics education research that integrates network science with constructivist learning theories. As such, the dissertation has delineated and laid first building blocks, rather than continuing the work of other researchers in a well-established field. Only a sparse research literature on the subject supported any claims about physics learning as represented in networks. It was a risky endeavor. There were no guarantees that any of the research initiatives in the project would yield meaningful results.

As an exploratory enterprise, this work investigated two kinds of systems: Systems of social interactions and systems of cognitive structure. In system of social interactions, students were represented in networks as nodes and different kinds of interaction processes were represented as links. A central finding was that these *student interaction networks* comprise one possibility for formatively assessing the development of students' competencies. In this work, student interaction networks were used (1) to predict future grades from network positional advantage, (2) to find that a students structured their interactions about problem solving activities according to laboratory class and gender rather than grade, and (3) in the development of a novel research method that integrates qualitative and quantitative methods to investigate student interactions in physics education.

Student written answers to conceptual physics questions where investigated as systems of cognitive structure. The students' answers were represented as *linguistic association networks*. In these networks nodes can represent either words from physics or words that signify image schemata. Links between nodes signify word adjacency in the original text.

Section introduced a competency based understanding learning. In the context of this work it involves elements from both cognitivism and social constructivism. This means that neither student interaction nor linguistic association networks as separate entities will capture students' competencies in physics. They need to be brought together in a combined understanding.

The rapprochements between learning theories and network theory in Rapprochements: Networks, physics, and learning (p. 25) led to three overarching research questions: The theoretical, the measurement, and the didactical questions. Each of these has its own sub questions to give more precise directions to this research project. This chapter sums up how the present dissertation has answered each of the overarching questions and the sub questions.

Answers to the theoretical question

The first overarching research question deals with the problem of aligning concepts from constructivist learning theories (for example, engagement, mental schemata, and competencies) with concepts from network science (for example nodes, links, and community structure). The question reads:

> How can the science of networks and theories of social constructivist and cognitive learning inform each other to investigate student learning in physics classes?
>
> - What parts of social constructivist theories and cognitive theories can be represented in networks?
> - Which analogies between the science of networks and learning theories are fruitful for an understanding of what it means to learn physics?
> - How do variations in network structure, dynamics, and agency contribute to a social constructivist and cognitive learning perspectives?

On the one hand, network science has informed learning theory by bridging cognitivist and socio-cultural aspects of physics learning in a quantifiable way. This has revealed patterns relevant to learning physics on a cognitive as well as on a social level. These patterns are not recognizable without a network model of the environment in which learning takes place. Thus, network science may in the future serve to test, refine and develop theory, although such an attempt has not been made here.

On the other hand, Paper II suggests that network science can also make use of concepts from learning theories. In network science conflicting ideas about the nature of communities has resulted in the development of a variety of algorithms that partition networks into groups. The ideas about the meaning of nodes and links in different kinds of networks, which are developed in (physics) educational research, may provide researchers in network science with a different understanding of the "philosophy" behind grouping algorithms. In particular, Paper II shows how the Infomap grouping algorithm can be combined with information about gender, grades and laboratory class to characterize groups of collaborating students. Gender, grades and classroom dynamics as concepts are treated extensively in learning theories.

This study has shown a correspondence between students as represented by connections and students as represented by grades and gender. Using the correspondences *node* $\leftrightarrow$ *student* and *link* $\leftrightarrow$ *interaction* as a basis for an analogy, a community found by a grouping algorithm might be interpreted as a community of practice characterized by forms of mutual engagement, joint enterprises and shared practices. Further, the target entropy measure might capture learning opportunities in a network, and hide measure might capture the efficiency of student communication. However, the analogy needs to be further investigated using a combination of qualitative and quantitative methods. Finally, for a single student in the network, target entropy, hide, and PageRank (measures of a node's positional advantage in the network) would correspond to a student's participation in the learning environment. For student interaction networks, this analogy between network and learning theory seems to be fruitful.

The variations in student interaction network structure over time show how students and perhaps communities of practice respond to a changing teaching environment. For example, the students in this study shifted between groups the first couple of weeks of their studies, but after some weeks the groups seemed to stabilize. Also, some students become peripheral to the corresponding student interaction networks, and in some cases end up not represented in the networks. Many of these students end up not passing the course. Also, These variations might indicate how communities of practice form and dissolve in the context of for example higher physics education. In the case of secondary physics education, variation in interaction patterns seem more coupled to teaching and already established patterns.

Within the cognitive realm, linguistic association networks might be seen as samples of students' conceptual ecologies. If this is true then variations in network structure over the course of a teaching period can indicate learning in physics. Nodes in linguistic association networks represent concepts in the sense that they consist of a label and a set of connections to other connected labels. The links represent associations but not necessarily propositions or causal relationships. Thus they allow us to find out how many and what concepts a student activates when answering a particular conceptual question in a particular context, and also how the student connects different concepts to explain the problem. Language plays an important part in these networks. The key analogy proposed here is between structures in linguistic association networks and image schemata; general image schemata help coordinate physics concepts, and to some degree this coordination is visible in linguistic association networks.

Significant changes in student linguistic association networks might be coupled to accommodation of a difficult concept over time. An increase of physics words is a crude indicator of language development, but the network allows for a more sophisticated analysis: If a concept takes over the position in the network of another concept, then this might correspond to a student participating more and more in a physics learning community, thus using for example "mass" instead of "weight" because of his/her alignment with the community.

Answers to the measurement question

The second overarching research question points to the task of finding ways of measuring student competencies in a quantitative way using network theory. It is implicit that these approaches need to validly portray learning processes. The question reads:

> How can a network science approach be used to quantify the state of student understanding and use of physics?
>
> - What methods for collecting data can be developed and how can the data be represented?

- How can student levels of competency in physics be understood in the context of network representations?

The key finding here is that asking students with whom they communicate about physics, is a viable way of collecting data describing interactions in physics. Moreover,different interaction categories like *problem solving* and *concept discussion* measure different aspects of student interaction with regards to physics. These categories can be developed using mixed research methods in an iterative design that involves both student feedback and theoretical considerations. The categories serve as different kinds of links in *student interaction networks*.

The data was collected by allotting time for students to complete an online survey each week for an extended period of time. A straightforward conversion of the data describing whom students remembered having communicated with, makes it possible to represent student interaction networks graphically using force-based layout algorithms. Researchers can use these graphical representations to follow the development of student interaction networks on a week-by-week basis.

The development of the survey was instrumental for collecting valid student interaction network data. Even so, it cannot be stressed enough that teachers, instructors and students all needed to support the collection of these kinds of data. The teacher and instructors were vital for coordination and student engagement in data collection.

On the level of knowledge, linguistic association networks were used to investigate student writings. Students wrote answers to conceptual questions on several occasions thus forming a corpus of interconnected terms, which these students associated with explaining physics. Text mining techniques were used to prepare the corpus for network analysis by reducing verbs and nouns to their stem form, by correcting spelling mistakes and by using synonyms to reduce the amount of different words. In linguistic association networks words are represented by nodes and links between words are based on adjacency in the text.

Linguistic association networks as well as student interaction networks were represented graphically using force-based algorithms, which give an overview of network structures. Network theory offers many different measures to characterize node positions, for example PageRank, target entropy and hide. These measures have been represented in networks as the size of nodes (the diameter). Demographic data on students have been indicated by color and shape of nodes in student interaction networks, and likewise word types has been illustrated with color and shape in linguistic association networks.

Note that linguistic association networks and student interaction networks are not the only outcomes of this work. The development of the survey instrument for collecting student interaction data relied heavily on the use of networks of the coding categories used to analyze student feedback on different versions of the survey. These *category networks* not only highlight the relative frequency of codes used, but also their connections.

To discuss how competencies may be understood in the context of network representations, consider the definition of physics competency (p.):

> The ability and will to act, alone and with others, using curiosity, knowledge, skills, strategies, and meta-knowledge *as these apply to physics, in order to negotiate meaning, to develop a distinct identity within the field of physics,* and to participate in relevant decision making situations.

Together, student interaction networks and linguistic association networks quantify interesting elements of this competency. It must be stressed, though, that this study does not explicitly show, how the two types of networks may be integrated. This is a task for future work.

Student interaction networks quantify the ability and will to act with others in order to negotiate meaning and to develop a distinct identity within the field of physics. Specifically, target entropy, hide, and PageRank as calculated on accumulated student interaction networks of university student interactions relating to *problem solving*, *concept discussion*, and *in class social interaction* can be seen as quantifiable measures of this part of the description of competency in physics. To make this leap, nodes should be interpreted as students and links as processes of transformation.

Linguistic association networks quantify the ability to work alone using knowledge and skills as these apply to physics. While knowledge is represented in the networks as the number of concepts and how they are structured in agglomerations, skill is represented by target entropy as a measure of order in the network and search information as a measure of the networks efficiency. Thus, as students become better at physics, the corresponding linguistic association networks should show an increase in the number of meaningful physics terms, an increase of target entropy signifying that many concepts are linked in many different ways and decrease of search information signifying that the student has easy access to relevant concepts relative to the size of the network.

Answers to the didactical question

The third and last overarching research question revolves around how networks can be put to use in a didactical manner. This is highly relevant, since evaluating teaching and learning practices is at the heart of physics education research. The question reads:

> How can one describe the process of learning physics from changes in relevant networks?
>
> - What types of networks can be defined to yield information about processes of learning physics?
> - How can the development of students' physics competencies in relation to their network practices be assessed?
> - How can the effect of teaching practices on student learning outcomes (improved competencies) be investigated using knowledge about changes in networks?

The key finding for this question is that student interaction networks show changes that describe how students react to changes in the learning environment. Secondary physics student interaction networks showed correspondences between network activity (measured as the number of links) and teaching frequency but also between target entropy and hide of the network and specific teaching activities, like laboratory experiments. University student interaction networks showed the effects of the didactical choice of predefining student work groups within laboratory exercise classes and changing these work groups each week: As a whole, student groups as found by Infomap changed on a week-by-week basis, but seemed to settle towards the end of the data collection. This reflects that students did work together in the predefined work groups, but also that they returned to earlier collaborators.

Student interaction networks comprise one possibility for formatively assessing developments in students' competencies in physics, for example by continuously measuring students' positions in the networks. However, since they rely on student self reports, a cautionary note is in order: Researcher and teachers took great effort to engage students and to make clear that how students answered the survey had no impact on their course grades. Thus, the study has not used these networks for summative assessment. Investigating this possibility would demand further research.

In principle, linguistic association networks collected over time can be used to assess the development of a single student's or of a collection of students' use of conceptual and phenomenological forms of representation. However, much is dependent on student familiarity with the particular format used to elicit answers and with expressing themselves about physics in writing. Like all other ways of expressing one self, this particular way needs to be learned.

With regards to the links between teaching practices and student learning outcomes, the answer is tentative but straightforward. Given some kind of variation in the teaching-learning environment, changes in student interaction networks give insight into how students reacted to the variation. For example, when a secondary school physics class did laboratory exercises, target entropy and search information spiked, indicating high predictability and low communication efficiency in the corresponding student interaction networks. This in turn produced the hypothesis, that students would not have improved their competencies a lot with this kind of activity.

This work has not explicitly investigated the effects different kinds of teaching activities might have on linguistic association networks. But a setup where students answer conceptual questions online on a frequent basis could be coupled with controlled variation of teaching activities. This might reveal changes in students' competencies in physics as expressed by linguistic association networks.

Reliability and validity issues

Reliability and validity issues have been discussed in epilogue sections of Chapters -. As described in the beginning of this chapter, this project has been an exploratory conceptualization of an emerging field. The papers all have a proof-of-concept/proof-of-existence quality; the question was rather if this kind of work could be done validly than if it could be done reliably.

Be that as it may, the quantitative aspects of the project are reproducible: Given the same data and the same methods of analyses, other researchers should arrive at the same results. Random sampling is part of the calculations, so one has to be careful to include enough iterations that the results do not vary much in new calculations.

The study has reliability issues at the points where this researcher has made choices that affect the analyses. Here, another researcher might have reached different results, even if the raw data and the method was held constant. For example, the thematic analysis employed in Paper I (outlined on page 51), might have yielded different coding categories or relative importance of existing codes in the hands of other researchers.

In the sense that validity and reliability (and generalizability, which is not discussed much in this work) are interconnected, reliability issues have implications for how the link categories developed in this work can be understood to represent interactions in the real world. However, the methods have produced meaningful results. Thus, even if the understanding of the links may not completely or in a detailed way correspond to real world interactions, it captures elements of it.

Prospects for further use of network analysis in physics education research

The project has positioned networks as a way of connecting didactics, learning theories and measurements as applied to physics education. This dissertation has shown different ways of using networks to see the effects of didactical choices, to inform learning theories, and when finding methods for collecting and analyzing complex data to describe learning processes. This chapter suggests how networks may be used in further developments in physics education research that connects didactics, theory, and measurement.

Instead of positioning networks as an entity, consider Figure 17 where networks are integrated into links between the three themes. The purpose of the figure is to indicate the major work of this thesis: Moving networks from being something external to physics education research into being a valuable part of the field's different research methodologies. The question now is how different parts of physics education research may use networks

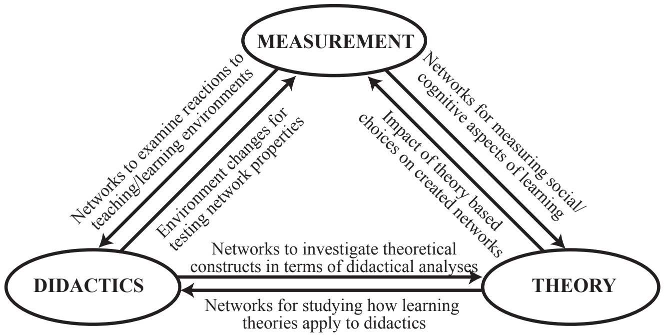

Figure 17: How networks can be integrated in research and didactical considerations connecting the three main themes: Measurements, didactics, and theory in the context of physics education research. The arrows indicate that one theme informs the other using networks. Text accompanying an arrow correspond to one of the directions future research using networks could take.

Each of the links in Figure 17 have labels that are meant to invite the reader to think about which directions future research could go. The following paragraphs outline some of these possibilities. The list of possible directions is not meant in any way to be exhaustive, and neither are the general themes serving as nodes in Figure 17. Labels and themes merely serve as starting points for ideas, some of which are presented here.

Networks to examine reactions to/impact of teaching-learning environments. Designing a teaching-learning activity can be seen as an iterative process. For a developer of teaching-learning activities, it is interesting to know not only if the students achieved the learning goals but also how students acted while learning. Student interaction networks as examined in Papers I-III can be used to pry into student interactions on a week-by-week basis. Depending on what the developer wants to know about, self-reports and/or more objective measures (online posts, for example) may be desirable. Student interaction networks show the developer how students communicate and/or perceive to have communicated. If teaching was supposed to encourage discussion of concepts it would be nice if this actually showed up in the corresponding student interaction networks describing concept discussion. Also, identifying central and non-central students in different areas could become important tools for both developers and teachers.

If student writings can be analyzed in a way derived from the outline in *Analyzing student writings in physics* (p. 63), then developers may use this as one tool for investigating the impact of teaching activities and/or teaching environments on students' use representational forms pertaining to, in this case, writing.

Teaching/learning environment changes for testing network properties. A direction of methodological research would be to see how a type of network that is relevant to educational research responds to different teaching environments. An example of this would be to use this study's network survey at other universities or with other cohorts at this university. There would be a continuous tension between understanding network properties from the student cohort and understanding the student cohort from network properties.

This tension can probably not be resolved completely, but continuously comparing with other methods for describing a cohort might aid such an endeavor. One example of a study for testing the impact of teaching-learning environment changes on network properties would be to change the way a laboratory exercise is taught. Hypothesis 2 developed in Paper I stated: *Incorporating accountability between groups doing an experiment, will increase communication efficiency and learning opportunities as measured by search information and target entropy respectively. This will result in a better and more lasting understanding of the subject matter.* While the aim of that paper was to develop but not test hypotheses, a study could be undertaken to see if there was a difference in search information and target entropy for networks describing different kinds of laboratory exercises, for example between traditional laboratory work and work that incorporates accountability between laboratory groups.

Networks for measuring social/cognitive aspects of learning. The major theoretical work of this thesis has been to develop the notion of student interaction networks and of linguistic association networks. While these can of course be continuously refined and developed according to learning theoretical considerations, there are other possibilities. For example:

- Analyses of video recordings to find shared models between students. Here, nodes could be student gestures, words, and actions as proposed by Bruun (2011). To find meaningful models though, such a study should have more detail than Bruun (2011), so one could see which kinds of gestures, words, and actions follow each other.

- Analyses of interaction patterns between students and different artifacts, for example computers, a pen and paper task, or museum exhibits. Again, this would require some detail in the analysis.

Also, if network measurements are to have an impact on theory, the reliability of some of the network representations have to be further established. For example the category networks in Paper I are based on coding made only by this researcher. In future work more than one researcher should code to make it possible to test for inter-coder reliability.

Impact of theory based choices on generated networks. As noted in the description of Paper IV, which discusses linguistic association networks, there is a lot of work in aligning cognitive schema theory as applied to physics learning with network operationalization. One of the reasons were that removing a word consistently from student answer had not only profound impact on the network, it also had deep theoretical considerations. For example "wave" is not considered a common word in most contexts, but in a description of a physical system involving a wave on a string, "wave" becomes quite common. Removing it from the texts might be justifiable if we want to reduce the complexity of the network. But then the network may loose information that is valuable in characterizing student knowledge structures as represented in these networks. Conversely, the word "in" is rather common, and in text mining analyses, it would normally be removed from the corpus to reduce complexity. However, in an image schematic approach to understand how students reach associations, such a word might be very important as an indicator of for example a container metaphor. Removing it from the network describing these associations may have an undesirable impact on the interpretation of the network, if the point is to see how the container metaphor is used to structure physics concepts.

This is just one particular kind of network in which theoretical choices impact on generation and subsequent analyses. In the case of student interaction networks, using another theoretical framework than Communities of Practice would probably have emphasized other kinds of interactions, just like different foci within a theoretical framework would have led to other networks.

Networks to investigate theoretical constructs in terms of didactical analyses. This is one of two connections in Figure 17, which this work has not investigated. This direction of research would use networks as a link between *theories* of learning and *planning* for teaching and learning. One possible direction is to model the possible learning outcomes of a student by viewing a teaching-learning environment as a network of activities. On the program level, the activities could be courses in a program and the theoretical construct to model would be a students learning trajectory in this program. On a teaching sequence level, teaching and learning activities could be labeled with different forms of representation, and the modeled construct could be some aspect of knowledge. However, these thoughts are on an abstract level, and need further development.

Networks for studying how learning theories apply to didactics. This is the second type of connections in Figure 17 on page 75 not investigated by this work. The study of Pitts and Spillane (2009) exemplifies how theories about leadership can be represented in network surveys. A possible link to didactics could be to investigate how a teacher's position in such a network related the teacher's approaches to teaching. In another but related direction, one could investigate how networks of stakeholders within education policy affect teaching practices and the implementation of curriculum change. Thus, this direction seems to involve political as well as educational issues.

This delineation shows that methods involving networks of different kinds hold an enormous potential for informing very diverse fields within physics education research. It should also show that it is reasonable to expect that networks also holds potential for science educational research and educational research in general. Hopefully, future research will fulfill and expand upon this potential.

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Appendix 1 - Combining methods to analyze student relations: Network science and communities of practice

Combining network science with communities of practice to analyze learning processes: Interpretation, structure, and development of student networks

Jesper Bruun

Quantifying social learning processes may aid in the design of teaching and learning situations by making hypotheses about changes in social networks in a learning context testable. However the validity of such quantifications necessitates a fruitful understanding of how nodes, links and network dynamics can be understood in terms of social networks in education. Here, such an understanding is developed using student responses, research literature and the theoretical framework of Communities of Practice (CoP). This understanding is then used to design and implement a mixed methods research design, which seeks to integrate theoretical constructs from network theory with CoP and student perceptions of different types of interactions. The results of this design are first a series of networks describing the development of student interaction over a period of time. Second, an analysis of a validation session with the students lead to hypothesis, which could be tested using the kinds of networks developed in the paper. The method is developed for research designs aiming at integrating learning theories with social network analysis.

Introduction

In science education many studies consider the design and implementation of teaching and learning situations, for example, in science teaching (Cobb et al., 2003; Tiberghien et al., 2009; Tiberghien, 1994). Some of these studies are based some idea of how students can use each other to learn in the context of a classroom (Roth and Lawless, 2002; Osborne et al., 2004; Lemke, 1990). However, the structure of social learning processes as they unfold in the classroom are often hidden to the researcher, which makes it difficult to explain how teaching methods work and why they work like they do in the context of a classroom. Documenting such learning processes may help when designing new teaching and learning situations.

Recently, researchers have used social network analysis (SNA) to investigate student relations in learning environments (Enriquez, 2010; Dawson, 2008, 2010; Brewe et al., 2012; Goertzen et al., 2012). Social networks reveal patterns of engagement in communities of learners, which can then be related to other factors important for learning. For example, students' sense of community (Dawson, 2008), student use of on-line (Dawson, 2010) and other resources for communications (Enriquez, 2010), and student participation in physics communities of learners at the tertiary level (Brewe et al., 2012; Goertzen et al., 2012). Particularly, Goertzen et al. (2012) asked students in a physics course on mechanics who they talked with to learn physics. They did this three times during the course to create social networks, and supplemented this with in-depth interviews with two selected students. The interviews revealed how the students felt they became part of a physics learning community, and this was supported by the network data, which revealed an increase in the number of connections to other students. The studies reveal that SNA can be a way of uncovering student interactions in connection with learning, although Enriquez (2010) does caution us that students' importance in the network can change with the mode of communication considered.

None of these studies have investigated how these social educational networks change over time according to the learning process. For example, Dawson (2008) extracted forum logs where students answer each others posts and used the accumulated posts as a basis for creating social networks. Enriquez (2010) looked at different modes of communication, for example, face to face conversations, e-mails and mobile phone networks from student self reports, but still without the time aspect. Brewe et al. (2012) relied on self reports on who students work with on physics homework at a physics learning center to generate a single network of student connections. Goertzen et al. (2012) study the development of self reported connections between students learning physics to investigate how their personal (or *ego*) networks change over time. However, Goertzen et al. did not study the whole network. Thus, this particular aspect of social networks in educational research does not seem to have been investigated much.

The time development of students' social networks may show how a classroom of students react to teaching and environment. Data which describes who the individual students communicated with and what they communicated about can be used to create networks. Such networks can show how much the class communicates about, for example, relevant concepts or particular problems, and they may also reveal group structures within the class. If a particular teaching method aims at facilitating more student interactions of a particular kind, then network analysis can be used as a tool to investigate how this happens.

This article's contribution is a mixed methods research design (Johnson and Onwuegbuzie, 2004) applied to a physics learning context. The design is similar to that of Pitts and Spillane (2009), which can provide a quantitative interpretation of the structure and development of different types of social networks in education. Different categories of interaction determine different types of networks, for example, interactions concerning *problem solving* lead to a different network than interactions concerning *concept discussions*. Knowing how to interpret these two categories, provides means to interpret the corresponding network structure and the development of this structure over time.

In this paper, categories of interaction are developed iteratively using student answers, research about physics as a discipline, and learning dimensions from communities of practice. The result is a set of interaction categories which may describe different aspects of the negotiation of meaning and other dimensions of learning (Wenger, 1998). Through a process of analysis, validation and re-validation, the categories seem to have become robust in terms of how they are perceived by students. If the argument holds and the categories give a valid interpretation of student interactions in terms of a community of practice view, then the measures from network analysis of the corresponding networks may be used to "add precision to words, pictures, and narrative" (Johnson and Onwuegbuzie, 2004, p. 16). In other words, the quantitative part of network analysis can support the qualitative findings of on-line open questionnaires and vice versa.

A clearer understanding of how research in students' social networks has already contributed with findings in educational research, is useful as a premise for the argument. This is the topic of the next section, which develops an understanding of how nodes, links and network dynamics can be understood in social networks in education. This understanding is then used to design and implement a mixed methods research design, which incorporates in detail how students perceive different types of interactions. The results of this design are first a series of networks describing the development of student interaction over a period of time. Second, from an analysis of a validation session with the students, a set of hypotheses which are tested and used to generate new research questions for further mixed studies of student learning processes as the progress in a classroom context. The following section describes these developments for the case at hand using calculation methods from network science.

Research in students' social networks

Networks of student interactions can be generated in many ways. To begin the discussion, consider Figure 1 which depicts how this study uses student self reports to generate different kinds of networks. This is meant merely as a point of departure, to discuss how other studies have collected and then analyzed data.

Imagine two students, A and B, communicating about how to solve a problem in physics during class. Student A says something which B remembers later on. At a later time in a data collection session, B sits in front of a computer and answers an online survey where he can check different kinds of interaction. B remembers A for the interaction about problem solving (PS) in class (IC). In the network representation of the data, a link from B to A is generated in a network of all the PS interactions and another link in the network describing IC interactions. The primary purpose of this study is to investigate how educational researchers can develop an understanding of these links between students.

In network terminology the representation of people in a network are called nodes and the arrows between them are called (directed) links (Costa et al., 2007). In general, nodes *represent* entities of interest, for example, students, and links are connections between these entities. Two nodes can be connected, if one student names another as a friend or if one student sends an e-mail to the other.What the network means and how its structure can be interpreted is dependent on the underlying idea of what nodes and links are.

Recent studies applying social network analysis in education have used different kinds of technology to establish networks of student interactions and bonds. Dawson (2008); Macfadyen and Dawson (2010); Dawson et al. (2010); Dawson (2010) used discussion from posts to an online forum to establish links between students. These studies mirror network studies outside of the educational field, such as Kossinets and Watts (2006) and Eagle et al. (2009), who use measurable behavior, like e-mail correspondence and digital proximity measurements, to establish links between nodes.

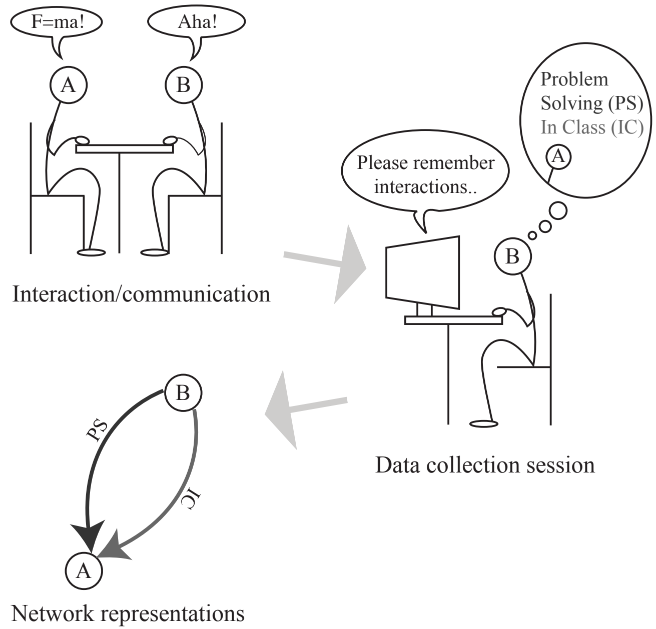


Figure 1: How networks are generated from student self reports in this study. On the right, the circles with the letters A and B are called nodes, and the arrows are called (directed) links.

Most network studies in education rely on students' self reports, asking students who they work with or know either socially or academically (Brewe et al., 2009; Goertzen et al., 2012; Forsman et al.; Enriquez, 2010; Brewe et al., 2012). The danger of self reported interactions is that they can be biased in number of ways. One bias is for over- or underreporting links if this seems to put the reporter in a more favorable social position (Liljeros et al., 2001; Liben-Nowell, 2005). Another bias is for fatigue effects, where respondents get exhausted from answering the survey or they only give the "names they believe satisfies the request for information" (Pustejovsky and Spillane, 2009). Section discusses how these biases relate to this study. Finally, there is a bias for how respondents understand the question or questions (Pustejovsky and Spillane, 2009). Building a framework to analyze how respondents understand the survey prompts is one the primary concerns of this paper.

Though biased, networks with different categories have offered researchers some information about student perceptions of interactions. Forsman (2011) asked university students to report who they interacted with from the university. Furthermore, he asked them to distinguish between social and academic ties allowing him to gain information about how a particular student perceived a particular tie to another student.

This study continues this line of work in two ways. First, the aim of the study is to elaborate on the academic category by creating more detailed categories using student feedback, physics learning theory, and a communities of practice perspective. Second, it extends the data collection process by asking students to name collaborators at several data collection events during a semester.

In many studies utilizing networks in educational research, the resulting networks are analyzed quantitatively using social network analysis (Wasserman and Faust, 1994; Scott and Carrington, 2011), after which a qualitative framework is used for interpretation (Enriquez, 2010; Brewe et al., 2012; Goertzen et al., 2012). This study suggests a continuous iterative approach where student feedback is used to inform the questions, and afterwards new student feedback is used to generate an interpretation of the resulting networks. Finally, students are confronted with the new interpretations. This approach is analogous to the work of Pitts and Spillane (2009) on leadership at schools who used interviews and think aloud responses to validate their School Staff Social Network Questionnaire.

McCormick et al. (2011) used a different approach to networks in education. They asked teachers and teacher educators to draw networks as they perceived them. Their initial thoughts were to merge these drawings into network representations, but they did not find this conversion to something quantifiable very productive (McCormick et al., 2011, p. 87). One reason was that the drawings included institutions and abstract concepts, and thus it was not possible to find coherent definitions of what nodes in these networks meant. Another reason was that it was not clear what kind of link the researchers should infer from the lines between entities participants had drawn. Thus, with these drawings they could not standardize what nodes and links represented.

This lead them to another way of working with network analysis. Instead of quantifying, they employed concepts from social network analysis on which they pinned interview data. They discussed at length what the meaning of nodes, links, and properties derived on the basis of these, could be in networks. They used drawings and interviews as examples throughout their work.

One of McCormick et al. (2011) key arguments is that a link in a network of learners does not signify merely flow of information or a stable relation between students as represented by nodes. When a link is related to learning, it signifies a transformation; an idea which is modified, a change in practice, or of meaning making (McCormick et al., 2011, p.115). This insight is important for this study, since we cannot assume that a *problem solving* link from B to A signifies that they are friends or work together often with regards to problem solving. Neither can we assume that information has been transferred along the links as though they were pipelines. What we know is that student B has had some kind of memorable interaction with A, which may include for example, checking if the answer is correct, which formulae to use and how to use them, or how to go about solving problems of the kind they were solving.

Getting the correct answer to a problem can be seen as information transfer; it is difficult to argue that a student idea has been transformed. Finding the right formula and using it the correct way may not modify a student's ideas about physics in general, but it is possible that it signifies a change in problem solving practice. If the interaction in some way changed the way student B and/or A go about solving problems, then it would definitely be a change in praxis and possibly in the way they make meaning of some area of physics.

The purpose of this study is not to investigate the details of these interactions from an observer point of view. Rather, it is to investigate how students view the interactions. Thus, in this study, links are viewed as possible learning processes and by having different types of links it seeks to separate different kinds of learning processes.

Having an interpretation of how students view interactions puts us in a position to interpret measures of the structure of a network and of the development of networks. The following section reviews such network measures based on two networks from the first part of this study. This will make it possible to interpret these measures from a learning perspective, which is done in Section .

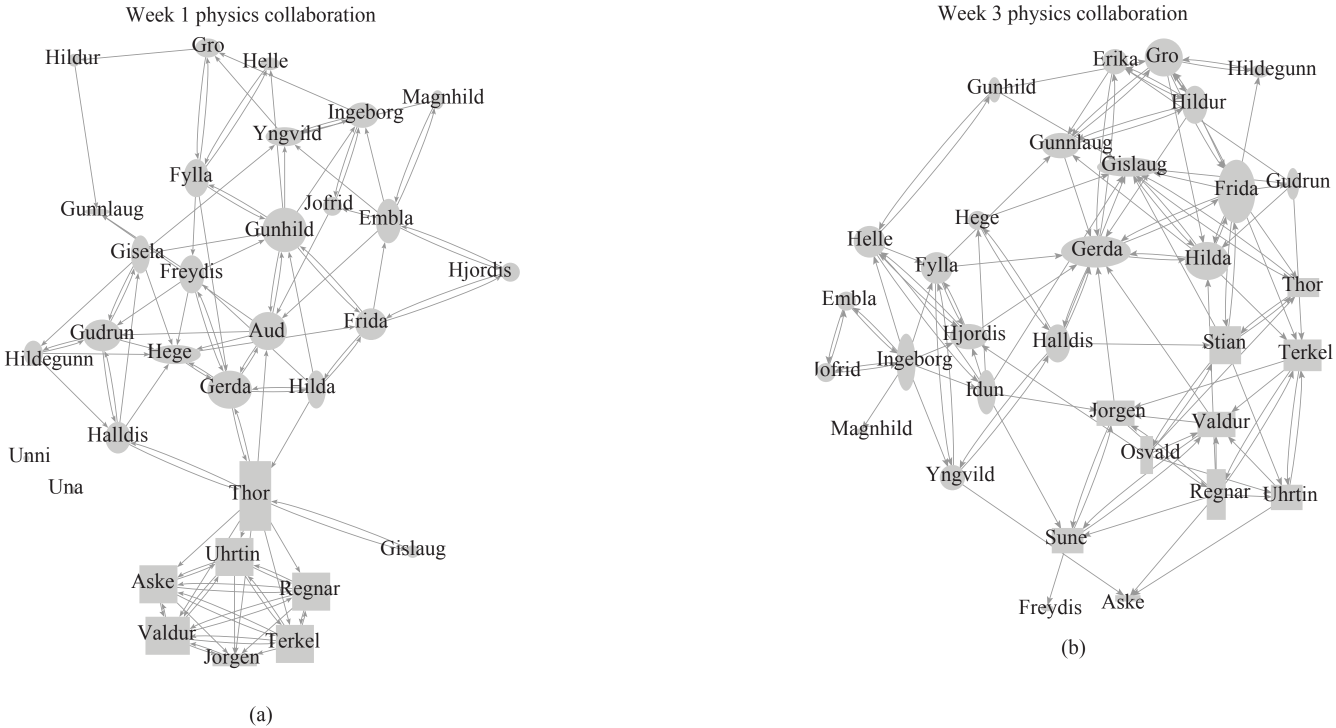
Measuring structure and development in educational networks

In SNA studies the power or influence of a person is equated with some centrality measure (Borgatti and Lopez-Kidwell, 2011). Power is to be understood broadly as the ability to influence the opinions and actions of others (Knoke, 2011). Centrality as a measurable quantity gauges a person's positional advantage with regards to the network.

A focus on individual nodes in a network may point out influential students in the corresponding classroom, but it does not characterize the whole network. Other measures gauge how difficult or easy it is to navigate a network or how structured information flow is in the network Rosvall et al. (2005); Sneppen et al. (2005). Such measures characterize the whole network, and not only individual nodes.

If these structural numbers and changes in them can be interpreted meaningfully with regards to learning, they can be used to characterize learning situations involving a classroom of students. The premise of this study is that such quantitative characterizations may aid in the understanding of learning situations, when coupled with other ways of researching and understanding learning. Thus, although I now proceed to explain these how these measures account for structure in a network, the point is that they will be related to learning.

To begin answering these questions, Figure 2 shows two net-



N=32	N_l=111	N=33	N_l=118
ΔL=19	$Z_{L_\cap}$=1.5	$L_{w3\cap w1}$=19	$Z_{L_\cap}$=1.5
T=0.038	Z_T=−20	T=0.043	Z_T=−5.9
S_A=4.77	Z_{S_A}=4.6	S_A=5.54	Z_{S_A}=6.8
S_H=5.22	Z_{S_H}=12	S_H=5.16	Z_{S_H}=3.6

Figure 2: Two networks from week 1 (a) and week 3 (b) of the pilot study, and values of network characterization measures (see text). Boys are represented with rectangular nodes, girls with ellipses. The width of a node is proportional to the number of people naming the corresponding student. The height of a node is proportional to the number of people being named by the student.

works. The nodes are students and the links are self reported interactions between students. The students were asked in four consecutive weeks to name the students from class which they could remember having communicated with about physics in the preceding week. The two networks are from week 1 and 3 of this part of the study. A directed link is made from student A to student B, if A indicated student B on a roster containing all names.

The number of nodes, N, is not constant , and neither are the names on the nodes. This reflects two things: sometimes students are not at school for various reasons (e.g.. sickness), and at the time of measurement, students could browse classes and choose where they wanted to be. The number of links, N_l, can be interpreted as a measure of the activity level; it is the number of times students name each other. This number may vary from week to week in different categories due to different teaching situations and simply differences in the amount of time spent in the physics class room. In Figure 2, nodes (students) are elliptical (girls) and rectangular (boys), and links (namings) are shown as arrows.

As a first connection to learning theory, consider teaching which offers few opportunities for cooperation. Students will have few opportunities to learn from and scaffold each other. Few opportunities would be reflected in fewer links in the corresponding network, if a link signifies one or more interactions.

The structure of the network - how the students link - might support learning more or less, even if the number of links is roughly the same. Looking at the networks of self reported student academic interactions in Figure 2, Week 1 (Figure 2 (a)) seems very different from the Week 3 (Figure 2 (b)), even if the number of students and links are roughly the same. But in Week 1, the boys (rectangular nodes) are isolated and only connected to the rest of the network through Thor. If Gerda had a good idea about solving a problem, and we imagine that the idea in some form could travel along the links to reach Terkel, it would have to pass through Thor. Whether or not Terkel would benefit from the idea would depend on Thor's understanding of the idea, if Thor was able to convey the idea, or even if Thor interacted with Terkel after he interacted with Gerda. In network from Week 3, an idea traveling on links have many other possibilities for reaching Terkel from Gerda. It can go through Jorgen, Valdur, Hilda and Frida. It may still be subject to some of the same problems as before, but there would be a larger probability that some form of the idea would reach Terkel. Terkel would then have the opportunity to construct his own version of the idea. In this sense, the structure of networks can support learning differently, although if the number of links and number of students are roughly the same.

Network theory offers different ways of characterizing network structure and dynamics with numbers (Costa et al., 2007). A simple measure of the dynamics would be to ask how many links, L, persist over time. For example, the links between Thor and Gislaug are present both in Week 1 and Week 3. It turns out that for the networks shown in Figure 2, 19 of the links in Week 1, L_{w_1}, are also present as links in Week 3, L_{w_3}. Using the $\cap$-sign to mark the number of intersecting links between the two weeks, $L_{w_1 \cap w_3} = 19$.

To find out whether or not a number of intersecting links are significant, one can compare $L_{w_1 \cap w_3}$ with the same measure in randomized versions of the same networks. To randomize the Week 1 network in Figure 2, take a pair of nodes with a link between them, for example, Gro and Hildur. Then take another pair with a link between them but no links to or from the other two nodes, for example, Embla and Aud. Now, rewire the link so Gro now points to Aud and Embla points to Hildur. Gro and Embla both point to the same amount of people as before, while Aud and Hildur has the same number of people pointing at them as before. Doing this at random and enough times each student ends up being connected to the same amount of people, but the names of their connections are randomized. After randomizing both networks, the number of intersecting links can be computed again. Randomizing the networks a number of times the Z-score $Z = (L_{w_1 \cap w_3} - \left\langle L_{w_1 \cap w_3} \right\rangle_r)/\sigma_r$ measure how many standard deviations the the measured number of intersecting links is from random the number of intersections between randomized versions of the networks. For $\mid Z \mid > 1.96$ the difference from random is said to be significant (Maslov et al., 2004).

Besides the number of links and the the number of intersecting links, this study makes use of three different measures for network structure. Two of them, called the hide, H, and the access, A, measure how much information is needed on average to find a node (A) or to be found as a node in a network (H), so they are measures of communication efficiency (Rosvall et al., 2005; Sneppen et al., 2005). The higher H and A scores the more it difficult is to be located as nodes or to locate nodes. The third, the target entropy, T, measures the predictability of network communication in the network(Sneppen et al., 2005). The two following paragraphs explains the details of each of these measures for the interested reader.

To start investigating the structure of a network, consider the following. Say that you start at Thor (in the Week 1 network), and you walk through the network following the links. If your "destination" is Gudrun, go through Aud and Halldis takes two steps. These paths are called the *shortest paths* or *geodesics*. [1] . Since Thor has 10 outgoing connections, and 2 of them will send you on a right path, the probability of choosing one of these paths at random is $\frac{1}{5}$. From Aud the probability is $\frac{1}{5}$, and from Halldis $\frac{1}{4}$ of choosing Gudrun. The total probability of choosing right is $\frac{1}{5}(\frac{1}{5} + \frac{1}{4}) = \frac{9}{100}$. Taking $-\log_2(\frac{9}{100}) = 3.47$, you get the average number of questions you would need to ask an hypothetical omniscient person to reach Gudrun from Thor [2]. This is called the search information, S_{ij} from i to j (Sneppen et al., 2005). If you want to measure the average accessibility, A_{Thor}, from Thor to the rest of the network, simply average the S_{ij}'s over the nodes reachable from Thor; $A_{Thor} = \frac{1}{N_{reach}} \sum_l S_{Thor,l}$ (Sneppen et al., 2005; Bruun and Brewe, 2013). For a whole network of i nodes, $A = \frac{1}{N} \sum_i A_i$. We could also ask the reverse question, how easy is it to reach Thor? In that case, we get the hide; $H_{Thor} = \frac{1}{N_{reach}} \sum_l S_{l,\mathrm{Thor}}$, and $H = \frac{1}{N} \sum_i H_i$. In non-directed networks, $H = A = S$ (Sneppen et al., 2005), but in directed networks, this is not the case as seen in Figure 2 on page 7.[3] You are able to access the network from a node with outgoing connections, but if the node has no

[1]Other paths from Thor to Gudrun exist but these are usually neglected in calculations because they would demand too much computation.

[2]The similarity between $\log_2(p)$ and number of questions is explained in detail in the Supplemental Material to Bruun and Brewe's paper (2013).

[3]Also, Rosvall et al. (2005) find that in non-directed and connected networks, $s = \frac{S}{\log_2(N)}$ is constant for a given network structure, and we take $a = \frac{A}{\log_2(N)}$ and $h = \frac{H}{\log_2(N)}$ to mean the same.

incoming connections, the node is not accessible from the network.

Next, imagine that messages originate at nodes and travel on links. If a message reaches a node, it is passed on through one of its links. Thus nodes with a lot of incoming links will shoot out many messages. Target entropy, T_i, measures the predictability of message traffic [4] to a node i (Sneppen et al., 2005; Bruun and Brewe, 2013). If node i has only one incoming connection, it is certain where the next message comes from, and $T_i = 0$. If it has two connections which in turn receive an equal amount of messages, then the probability of getting a message from either node is $\frac{1}{2}$, and $T_i = -\frac{1}{2}\log_2(\frac{1}{2}) - \frac{1}{2}\log_2(\frac{1}{2}) = 1$. But if one of i's connections has more messages going through it (for example, because it belongs to a more connected part of the network), then the probabilities are skewed, and $0 < T_i < 1$. Here, $T = \frac{1}{N_s}\sum_i T_i$ characterize the overall predictability of networks.

Recently target entropy and hide have been used to predict student grades (Bruun and Brewe, 2013). Each node has a T, A, and H score, and for the university students in Bruun and Brewe's study, target entropy in a network at one time was positively correlated with grades received on subsequent courses. Hide was negatively correlated with future grades. Thus, these two measures seem to predict grades, although Bruun and Brewe (2013) did not infer causality.

Finally, the networks have been plotted using the Kamada-Kawai plotting algorithm (Kamada and Kawai, 1989). This algorithm or algorithms like it - called force based algorithms - are widely used in network literature since it gives a picture of what nodes are close to each other. The algorithm treats the network as a physical system of connected particles. In this analogy, links are springs with an ideal distance proportional to the shortest path between two nodes. In a given layout, each node is separated by a distance and for most nodes this distance will be different from the ideal distance. In analogy to a physical system, this means that each spring contributes to the combined energy of the system. The algorithm calculates the coordinates of nodes which minimize this combined energy.

This means that if a group of nodes like the boys in Figure 2 share a lot of connections with each other, they are likely to be placed close to each other in the layout. This is why forced based algorithms like Kamada-Kawai is sometimes as a rough indicator of group structure.

This section has only hinted at the relation between network measurements and learning theories. To make this relation, the next section develops an understanding of what links and nodes resemble, and use this understanding to interpret target entropy, access, hide, and number of (intersecting) links in terms of constructivist and socio-cultural learning theories.

Interpreting network properties with socio-cultural learning theory

A self-reported link, where a student A has mentioned a student, B, can have different qualities. It could be about leadership (Pitts and Spillane, 2009; Pustejovsky and Spillane, 2009) if the interaction held some sort of asymmetry, for example, if A came to B for advice. It could also be a discussion of conceptual nature, which A remembers. It could be collaboration on a problem set, where A and B scaffolded (Vygotsky, 1978) each other, or some other interaction requiring mutual engagement

[4] Network literature uses the word traffic. In this paper, words like interaction, activity, or naming would be more appropriate

Measure	Symbol	Description	Reference
Number of nodes and links	N, N_{links}	The total number of different names/namings in a network	Wasserman and Faust (1994); Costa et al. (2007)
Number of intersecting links	$L_{g_i \cap g_j}$	The number of links present in both network g_i and g_j	Wasserman and Faust (1994)
Number of different links	$\Delta L_{g_i g_j}$	The number of links present in g_i but not in g_j. Note that in general: $\Delta L_{g_i g_j} \neq \Delta L_{g_j g_i}$	Wasserman and Faust (1994)
target entropy	T	The predictability of messages in a network. A high T means that on average it is difficult to predict where the next message to a given node would come from.	Sneppen et al. (2005); Bruun and Brewe (2013)
Total Network access	S_A	Starting at node i, A_i is the average number of questions you would need to ask to locate another node reachable through existing paths (geodesics). S_A is the average of this quantity over all nodes i in the network.	Sneppen et al. (2005); Bruun and Brewe (2013)
Total Network hide	S_H	Ending at node i, H_i is the average number of questions you would need to ask to locate this node through existing paths (geodesics) in the network. S_H is the average of this quantity over all nodes i in the network.	Sneppen et al. (2005); Bruun and Brewe (2013),
Plot lay-out		The lay-out of a graph plotted with a force based algorithm gives a rough indicator of network structure	Kamada and Kawai (1989)

Table 1: The above measures can be used to understand different structural aspects of a network. Section explains the measures in more detail.

(Wenger, 1998). Whatever the quality, we can be sure that from A's perspective, something happened which triggered the link.

It is also worth to consider, what aspects of a student is represented by a node in a network where students name who they remember having interacted with. McCormick et al. (2011) distinguishes between nodes as entities and nodes as relationships. Expertise in their view can be something derived from the individual or it can be something derived from the relationships to and from that node (McCormick et al., 2011, p.113-114). If there is no other information about the nodes than the students named, nodes must be relationships. Further, since the links are based on remembered interactions, we are investigating the dynamics *on* the network (Watts, 2003) where *"people do something.. and they are influenced by in this decision by their neighbors"* (McCormick et al., 2011, p. 89). The network does not yield exact information about what students have done, only that they have done something, and thus acted.

McCormick et al. (2011) argue that links are processes of transformation, specifically "interactions in which transactions are central" (McCormick et al., 2011, p. 145), rather than just channels through which information may flow. These processes are seen as as "opportunities for meaning-making" (McCormick et al., 2011, p. 145). This is in accordance with constructivist learning theories (Vygotsky, 1978; Wenger, 1998; Von Glasersfeld, 1996; Gentner and Colhoun, 2010), in the sense that knowledge is constructed from pre-existing knowledge; transformation requires that something exists which can be transformed. Furthermore Wenger (1998) argues that learning happens in a process of mutual engagement. If the links represent processes that have occurred, the network may describes some aspect of the dynamics of a class of students, just like nodes might represent aspects of student agency.

The details of these aspects depend on the reasons students have for naming each other. For example, if a network is built around namings based on communications regarding problem solving, then the network as a whole describes dynamics regarding some generalized idea of problem solving. But we need to identify student understandings of problem solving to interpret the network structure. In this way establishing and maintaining an understanding of what a single link means may further the understanding of what the structure of links between nodes means.

The link intersection from week to week may tell us about changes in interaction patterns over time. From a didactical view point it is interesting to know if changes in teaching activities also changes who students interact with or what students interact about. If students always work with the same people, the number of intersecting links from week to week would be high, and it might signify the existence of perhaps several closed communities of practice which may not share repertoire, enterprise, or practice (Wenger, 1998).

In the pilot study students were asked to list their interactions regarding physics and their social interactions. The question as to how alike the two networks are can also be answered with the intersection measure. For example, in week 3 the physics interaction network ($N_l = 118$) and the social interaction network ($N_l = 340$) share 39 links, which is 40 standard deviation below the intersections between the random counterparts. Thus, the categories do not elicit the same kinds of responses.

In order to interpret access, hide, and target entropy, assume that a link represents an interaction between students where an idea or practice has been either created or modified. An *idea* could be for example, a model, a concept, or a general line of thought, a *practice* how to solve a problem, how to represent knowledge graphically, or how to talk about concepts. The key assumption is that the idea or practice can spread through links, because links are processes, but that the ideas are not unaltered by the spreading.

We cannot know if a particular link, for example, the one between Hilda and Thor on Figure 2 on page 7 (left), signifies an idea or practice spread from Hilda to Thor, from Thor to Hilda, was developed in collaboration, or if Hilda remembers Thor for other reasons. However, Bruun and Brewe (2013) did find a significant negative correlation between individual students' H and grades[5]. That is, if a student is generally well hidden (has a high H) in the network of student interactions, the tendency is that he or she will not get as good grades.

A and H might be related to learning in the sense that they measure the effectiveness of communication in the classroom. If the network structure allows ideas to spread and reach students easily, then we may suspect that many students get to work a lot with the ideas and practices of others. That is, low values of A and H would signify more opportunities for learning than high values would.

A low value of T would signify that many students get to modify their ideas through a limited number of persons. In terms of learning, a high value of T directly means that a student has had interactions with many students who in turn has had many interactions with other students. Thus, to use metaphors for learning, there has been many opportunities for acquiring knowledge or participating (Sfard, 1998), and knowledge/practices have been shared through a lot of students. With a low value of T, students generally do not engage with many different students, and thus do not get to discuss their ideas, concepts, and models with a lot of other students. Again, Bruun and Brewe (2013) lends support to the benefit of high T values for individual students[6].

Finally, looking at drawings of the networks allows to indicate how group structures can change (or not). If teaching focuses on different students working with each other in different weeks, then this should be visible in the networks because different people would be closer together in different weeks. Linking other student attributes to the network, for example, by coloring nodes according to gender, will yield information about how much students mix with regards to that attribute in this particular network. Coloring with respect to grades, for example, might tell us if students in general segregate, so that students with good grades work together and students with bad grades tend to work together.

Neither of network concepts discussed above have been developed sufficiently in educational research, so great care has

[5]The article uses three naming categories to be developed for this study, namely the problem solving, conceptual discussion, and in class communication categories. The negative correlation is between hide in the problem solving network and grades in physics and math.

[6]This time the correlation is positive between T in the concept discussion network and grades.

to be taken when interpreting the results. In this study, these interpretations are based on an iterative probing of students via surveys and whole class sessions, which is described in the following section.

Mixing methods in data collection and creation

The design of this study is in some ways analogous to that of Pitts and Spillane (2009), and lies within the mixed methods paradigm as described by Johnson and Onwuegbuzie (2004). In short, both this study and Pitts and Spillane (2009) are concerned with the validity of the questionnaires used to quantify human relations in networks. This study's mixed methods design is outlined in Figure 3 and the details of the process and arguments for specific design decisions follow this short description.

To create the first interaction networks and to elicit student thoughts about collaboration in physics this study used student written responses to an open-ended survey. (See Figure 4). Following Johnson and Onwuegbuzie (2004) Mixed Methods Process Model (MMPM), the purpose of this part of the study was to elicit instances of what a network relation means for students. Thus, students where prompted for the qualitative nature of the interactions they remembered as well as for names of whom they remembered having had interactions with. The data collection was analyzed using open coding to reduce the complexity, and emerging patterns were identified by displaying the codes and their interconnections as a net-

Figure 4: Screenshot of part of the first survey. The title reads *Who do you talk to about physics?*. The text on the left reads: *Check everyone you remember collaborating with within the last week*. In the text to the left of the crossed out names reads: *Check here, if you have worked with any of these classmates!* The final text reads: *Give a short description of what you did and where you were*.

work of interrelated categories. This mix of qualitative (coding) and quantitative (network representation) focused mainly on capturing as much of what students meant with an interaction while keeping researcher bias in the interpretation of categories to a minimum.

In the next step in Figure 3, the analyzed data, Wenger's (1998) communities of practice framework , and Physics & Science Education Research literature (Roth, 1995; Dolin et al., 2001; Jacobsen, 2008; Lemke, 1990; Ogborn et al., 1996) informed the categories extracted from student answers to produce a set of final categories. This corresponds to a second iteration in the MMPM, where the interpretation of data leads to a new research question and purpose of research.

The final categories were worded as statements and administered to the same class in ten weeks during the course of

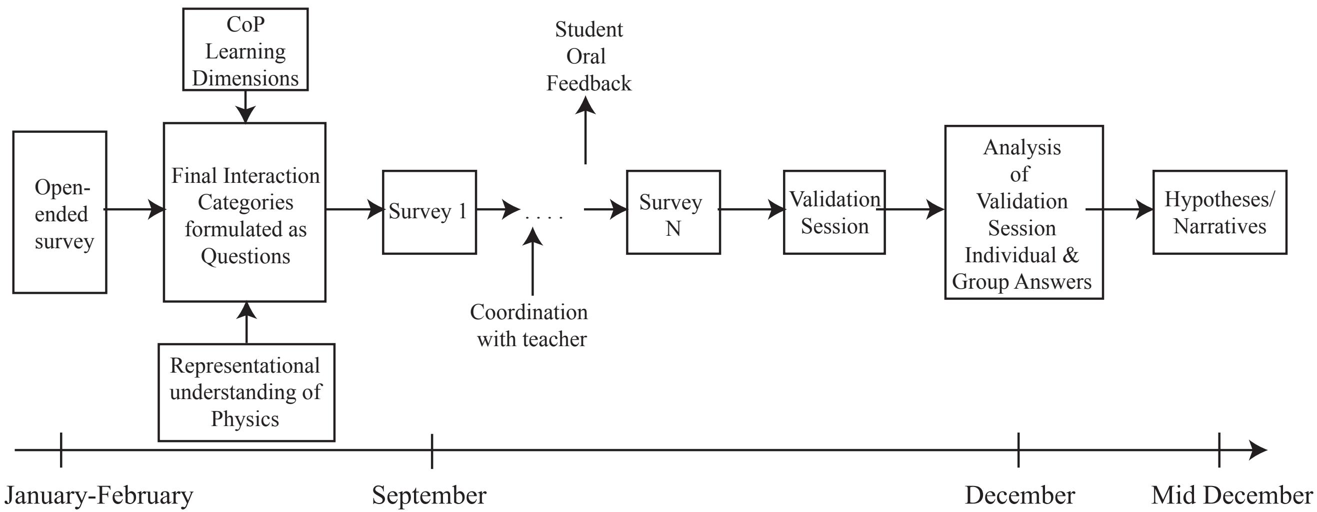


Figure 3: Research design. In each of the survey measurements and the validation session, the researcher was present to answer student questions. All sessions were coordinated with the teacher.

a semester. Each survey session was coordinated with the teacher to fit the teaching plan, and the researcher was present throughout most of the sessions to answer clarifying questions.

After the answering sessions, the teacher and researcher set up a validation session as part of the teaching plan. 24 students participated were present at the validation session. First students answered an individual online survey asking them what they meant when they indicated interactions in a given category. Subsequently, they discussed their answers in groups of 3-5 writing down the differences and likenesses of their answers.

The analysis of student answers where done in much the same way as the initial answers, and so this corresponds to a second iteration of the MMPM. After this iteration, the study allowed for the construction of hypotheses about the collected network data The data analysis involved calculations on networks and transforming network data to represent network development. Finally this was used to create hypotheses which can be tested in new studies. This required the use of both network data, of teaching plans, and the continuous use of the analyses of student responses.

Notice that apart from the box “Final interaction categories formulated as questions”, all other boxes involve sessions where students where given time in class to work with the questions from this study. This was possible because the class teacher was convinced that students would benefit from participating in the study and that the study would benefit from the teachers cooperation. During the whole period outlined in Figure 3, the teacher and researcher coordinated extensively to fit sessions with teaching plans.

The coordination and attitude of the teacher also seemed to convince students that the study was worthwhile for them to participate. During the entire research period, students engaged actively with the researcher to understand the questions and seemingly did their best to answer them as honestly they could.

First interaction networks and descriptions of interactions

The purpose of this section is to explain how categories and questions were designed from student answers to network surveys. From initial student answers, categories were developed using a representational understanding of physics learning (Dolin et al., 2001) and a Communities of Practice (CoP) learning dimensions (Wenger, 1998, pp. 231-236) perspective to further develop categories.

The subjects of the study were a class of upper secondary students engaged in a mid-level physics course. The teacher was an experienced physics and mathematics teacher. Initially the researcher gave a presentation of the study and was present during most of the naming sessions to answer questions from the students. In addition to the communication questions, students answered conceptual physics questions with free text answers to explain their thinking. The conceptual questions are not treated in this study.

The box describes the students, the teacher, and the setup. Using an online learning management platform[7], students used laptop computers and stationary computers located in the classroom to answer two online questions. They were asked to name with whom they remembered having communicated physics with during the last week (Figure 4). Also, they were asked to name with whom they remembered having communicated with socially (Not shown, but analogous to Figure 4). In addition, they were asked to elaborate on what they had communicated about.

For the two questions, the headlines were: *“Who do you talk with about physics?”* (Figure 4) and *“Who do you talk with socially?”*. These headlines were elaborated in the instructional texts. In the instructional text accompanying the physics question, students were told to list who they had *collaborated* with in physics during the last week. In the text accompanying the social question, students were told to list who they had *talked* with.

The researcher was present as was the teacher and both answered student questions about when to name some one. Stu-

[7] www.moodle.org

Category	Label	Example
AC	Affective towards collaborator	"I *really like working with* Gunnlaug because we work in the same way"
AS	At School	"We were *at school* working with...", "We sat *in class* working on...", "We worked on the problem set *we had been given*"
AtS	Affective towards Subject	"I did physics with Gunhild [..], we talked about how much we suck at physics"
C	Specific Concept	"We did a lab exercise on the *density of water*"
CH	Chat (online)	"Regnar and I *talked physics on Facebook*, where we helped each other with homework. "
CU	Conceptual Understanding	"We *discussed* how to *illustrate a serial connection."*
DP	Doing Physics (unspecified/field)	*"Looked at something with* potential difference. ", "Me and Thor worked together in one of the classes"
E	Experiments	"We did a *lab exercise* on the density of water"
G	Collaboration in General	"*Generally,* I work with the person sitting next to me."
H	Helped/Was Helped	"*I explained* series and parallel connections *to Yngvild*"
HWA	Home Work Assignment	"*.. but we also worked together on a hand-in assignment and on a regular homework assignment and on a Moodle quiz or two on electricity*."
I	Involving Other Students	"We did a video recording, where we had to show an experiment [...] *and then make questions for our classmates to see if they had gotten all the information.* "
LMS	Worked in LMS platform	"We worked with potential differences in *Moodle*. ", "We did the *Moodle* quiz. "
NA	Not Available	"I wasn't at school last week"
NAS	Not At School	
Pr	Discussed how to present	Besides, Jorgen helped me with a *presentation about voltage and current*.
PS	Problem Solving	"I helped Gerda with some problems about resistors."
RT	Repetition/Preparation in Connection with Test	"We practiced *using formulae for a physics test*"
SC	Specific Computer Program	"We used *Logger Pro* to record observations."
SE	Something Else	"Were in group with Erika"
ST	Specific Task	"Halldis and I [...] found out how we could simulate an ampere meter and volt meter in a 'simulated' circuit".
TI	Teacher Initiated	
TP	Teacher Presentation	"We listened to the *teacher going through* something at the blackboard."

Table 2: Categories, labels and examples of answers.

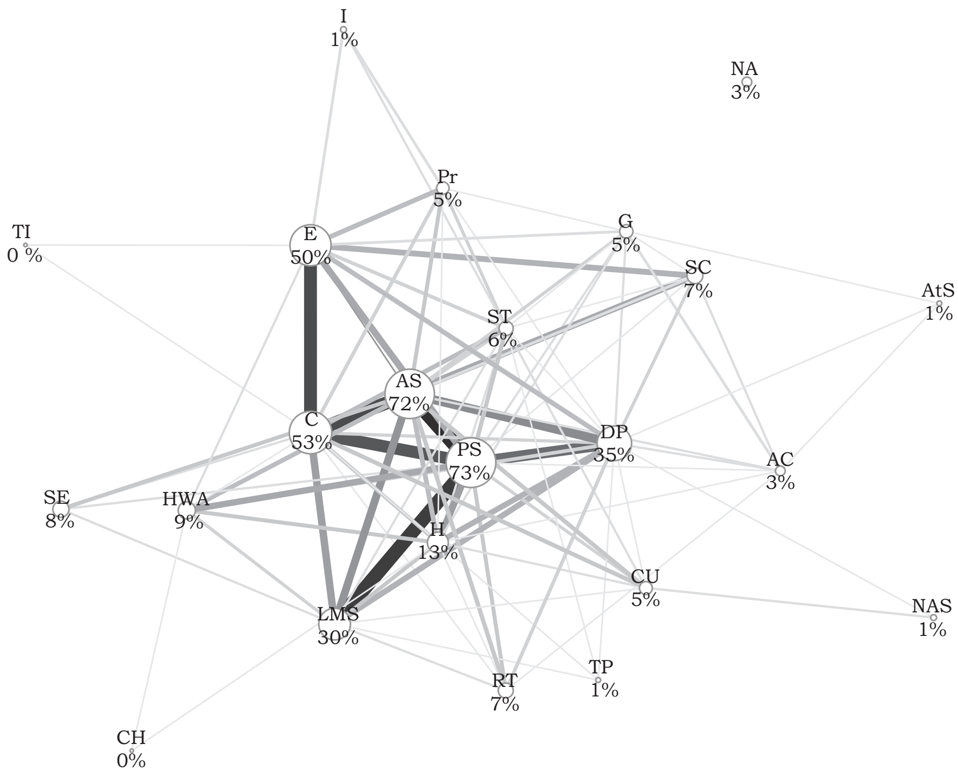


Figure 5: This network of category shows the percentage of times each label was used in the coding procedure of the four surveys. Table (2) has both the key to the abbreviations and examples of code use. The thickness of the link between AS (*At School)* and PS *(Problem Solving)* signify how many times the two categories co-occur in the code to answers. The strong link between AS and PS, as indicated by the thick line, signify that many of the reported interactions involve *Problem Solving At School*.

dents were told that any communication they could remember was important, even if they could not remember what they communicated about. This also meant that out of school and in school communication was important, as was the nature (e.g.. group, individual, or experimental work) of the communication.

The student answers were labeled with different category labels as shown in Table 2 along with examples of how answers were coded. These categories were formed on the basis of a thematic analysis (Braun and Clarke, 2006) of student answers.

To analyze the emergent patterns involving the coded categories, the categories and their interconnections were represented in a network (5). The category network in Figure 5 shows that from the student point of view, problem solving and doing experiments are the primary activities they remember. They often (53% of the time) mention a *Specific Concept* (labeled C) in connection with either *Experiment* or *Problem Solving*. Many times they are also unspecific with regards to the subject as represented by category *Doing Physics* (DP, 35%). They primarily list that they do physics *At School* (72% of the time) and rarely mentions specifically doing physics *Not At School* (NAS, 1%).

Other striking features are the things which are not there or are mentioned only marginally (<10% of the time). *Specific Tasks* (ST) are rarely mentioned, and discussion of physics concepts and language use is almost not present. The question is, whether these categories are marginally represented or absent because these things are not stressed in the classroom or because students simply do not consider them important or memorable. It is not possible at this point in the analysis to answer that question.

Considering the categories used 30% or more of the time would leave us with *Problem Solving* (PS, 73%), *At School* (AS, 72%), *Specific Concepts* (C, 53%), *Experiment* (E, 50%), *Doing Physics* (DP, 35%), and *Learning Management System* (LMS, 30%). The LMS category shows that students remember doing activities (mainly problem solving) in the school's learning management system.

Developing interaction categories

The analysis of student answers resulted in the categories shown as a network in Figure 5 on the facing page. The central categories are also the most used categories. They are intertwined with each other, in the sense that they co-appear with this particular use of the coding of student answers. This section interprets student categories and develops interaction categories using

- physics learning as mastery of representational forms (Dolin et al., 2001; Dolin, 2002) with language as a gateway into this scientific domain (Lemke, 1990; Ogborn et al., 1996).
- a learning matrix (Wenger, 1998, p. 240) consisting of four learning categories: negotiation of meaning, design/emergent, local/global, and identification/negotiability.

The interaction categories form the basis of interaction statements checked by the students during data collection. Figure 6

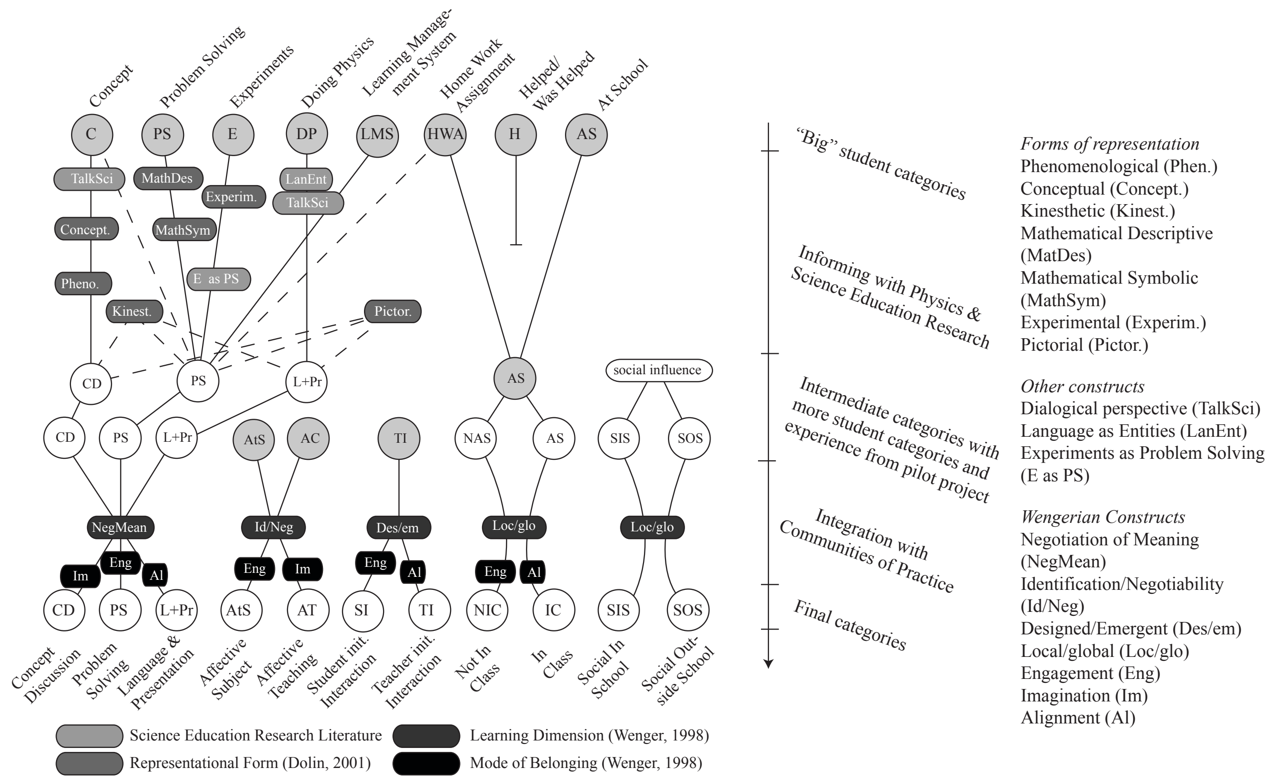


Figure 6: Starting with the most prominent categories from Figure (5), theoretical concepts from physics and science education research inform and change the categories. For example, the *Concept* category is changed to the *Concept Discussion* category is now understood through the lenses of a dialogical Lemke (1990) perspective and from the conceptual and phenomenological forms of representation (Dolin et al., 2001). These changes are made for the categories in Section (). In the further analysis, Wengers learning categories (Wenger, 1998, pp. 231-236) further inform the category. The *Concept Discussion* category, for instance, is now understood in terms of the *Negotiation of Meaning* learning dimension as articulated by the *Imagination* mode of belonging (Wenger, 1998, pp. 175-178).

illustrates the process of development. The following sections elaborate on the process of transforming categories through a description of *moves of change*, defined as *an action where we use theoretical considerations to argue for the change*.

The driver for making a move of change was to integrate student categories with existing research, thus pinning them to a theoretical and research based understanding of learning.

Informing prominent coding categories with physics and science education research

The analysis begins with the most prominent coding categories. These categories appear many times in the coding of student answers, and they appear central in the network shown in Figure 5[8]. Thus taking these as the starting point of the analysis seems natural. From here, the analysis proceeds with moves of change to modify the categories according to theoretical considerations. Physics and science education research motivate each move of change.

The first move of change is to merge the *Experimental* (E) category with the *Problem Solving* (PS) category. Jacobsen (2008) argues that while experiments are usually seen in the literature as a way of achieving cognitive, procedural, and affective skills, they may fruitfully be seen as a way of engaging with problem solving in physics. While students and teachers might not see lab experiments as a way of solving a problem, we can still give examples of problem solving which would also be applicable to experiments. For example, many physics experiments involve reading graphs, and performing some kind of mathematical manipulation of data in addition to knowing how to use equipment and to perform experimental procedures.

Problem solving is usually seen as a central part of doing physics. Here, problem solving is instead seen as one activity among many, which students may engage with to develop models using representational forms. In this view what characterizes problem solving is that students reach some sort of answer to a problem. The representational forms (Roth, 1995; Dolin et al., 2001; Dolin, 2002) students may explicitly use to reach these answers are mathematical and experimental. This means that when students use formulae, produce and read graphs, or measure the change of some quantity by regulating another, they are using the associated representational forms to reach an answer to a problem. The merger of the PS and E category are illustrated with the lines going from the gray PS and E categories on Figure 6 to the white PS category. The theoretical considerations with regards to representational forms are shown as the green shapes which the lines pass through.

Since the *Learning Management System* (LMS) reportedly serves largely as a way for students to do problem solving, it is assimilated into the PS category. The information about whether students did pen and paper or digitally based problem solving is lost through this move, keeping the number of categories down. Problem solving at this state may entail interactions where different representational forms are in use: math-

[8] Calculating the PageRank of each node in network in Figure 2, reveals that SC are ST are more central than HWA. However, these codes are linked to *specific* computer programs and tasks. It would introduce too many categories to introduce all the possible specific tasks and computer programs students will engage with during the course of a semester, so we discard these.

ematical, pictorial, experimental, and kinesthetic. However, it should not contain phenomenological or conceptual forms of representation.

The purpose of creating the *Problem Solving* category without the *Concept* category is to recognize the literature suggesting that sometimes students only learn to pick the right formulae (Halloun and Hestenes, 1985a; Hestenes and Halloun, 1995; Halloun and Hestenes, 1985b). They are not encouraged to discuss physics concepts outside the narrow scope of standard text book problems. Although many studies (Mazur, 1997; Hestenes, 2006) have shown that conceptual discussions have a positive effect on student conceptual understanding, the standard practice in many physics classrooms may not be informed by these studies. Thus, the objective here is to keep conceptual interactions separate from problem solving interactions.

In Figure 5, the *Concept* node (C) is heavily tied to the *Problem Solving* (PS) and *Experiments* (E) nodes, because students recalled a specific concept in connection with problem solving. This is illustrated by the dotted line from the gray C to the white PS in Figure 6. However, recalling that you did a problem related to a parallel or series connection does mean that you have discussed how current behaves in an electrical circuit. But the way students understand current influence how they use the formulae as illustrated by Bruun (2011).

The move of change on the *Concept* category is to add *Discussion*, thus creating the *Concept Discussion* (CD) interaction category. In terms representational forms, the phenomenological and conceptual representations are at play here. Phenomenological refers to “putting words to what is immediately perceptible” (Dolin, 2002, author’s translation), thus creating a first set of concepts. Mastering the conceptual form of representation for Dolin (2002) is associated with mastering the use of the generalizations and relations between physical phenomena developed by physicists.

This line of thinking is akin to the knowledge in pieces view on conceptual understanding proposed by Andrea diSessa (diSessa, 1993, 2002). diSessa identifies phenomenological primitives (p-prims) as atomic knowledge bits which persons use to make explanations of physical phenomena. He argues that an expert understanding of some physics subject can be described through the concept of a coordination class, where p-prims have been honed to serve the purpose of contributing to correct explanations of physics situations. The CD category is meant to capture interactions between students, where they are on the way towards an understanding of physics which lies somewhere between novice and expert. The concept discussion (CD) category entails phenomenological and conceptual representational forms but also kinesthetic (in the form of gestures) and pictorial (conceptual drawings) forms of representation. The part two forms of representation may play in interactions of the kind captured by the categories C*onceptual Discussion* and *Problem Solving* are illustrated in Figure 6 by the dotted lines from the green shapes representing these representational forms to categories.

Clearly, language plays a part of both the newly formed *Conceptual Discussion* and *Problem Solving* categories. But in in principle students can solve problems and discuss physics concepts without using words which have specific meanings in physics. Indeed, when learning, students sometimes refer to concepts by pointing, using words like "that thing" or everyday concepts (Roth and McGinn, 1998). Indeed when bridging between everyday understanding and scientifically correct ways of describing science, teachers may use metaphors and analogies which intertwine every day words with scientific concepts pointing out what the scientific concepts are (Ogborn et al., 1996). Furthermore, students continuously refine their language to approximate the language of science (Lemke, 1990; Bruun, 2011). For students then, *Doing Physics* (DP) is probably a combination of solving problems, understanding concepts, and formulating things in a way consistent with physics vocabulary. Since problem solving and concept discussion are already categories, the move of change on the DP category is to transform it into the Language and presentation (L+Pr) category. Presenting and talking physics involve all the different forms of representation used in physics, since these are arguable the representational forms physicists use to communicate.

The *Help* (H) category is problematic, because it provides a reason for bias. As noted by some researchers (Liljeros et al., 2001; Liben-Nowell, 2005), people tend to report interactions which put them in a favorable light. Helping someone as opposed to being helped seems favorable, and eliminating this category could reduce over or under reporting interactions, thus giving a more true picture of the actual interactions. The cost is that information about the interaction is lost, but even if someone believes that he helped someone, it doesn't mean that he did not learn from the interaction. This is the reason for eliminating the help category.

The category *At School* (AS) is large, and it is not surprising that many physics interactions happen at school. The *Not At School* (NAS)category is only marginally present, which could reflect that most interactions do not happen outside the school context. The home work assignment category is absorbed into the *Problem Solving* and *Learning and Presentation* categories, and thus we lose information on student collaborations on home work assignments.

The three physics categories developed here, *Problem Solving*, *Conceptual Discussion,* and *Learning and Presentation*, have crossovers between the forms of representation used. To further separate them, the next step will be to inform them with Wenger's (1998) learning theoretical framework, specifically learning dimensions and modes of belonging. Wenger's framework serves as a way of incorporating McCormick et. al's (2011) thoughts of links as processes. For example, the negotiation of meaning can be seen as a process consisting of participation and reification.

Before entering the realm of communities of practice, we have a set of intermediate categories, which consist of the developed physics content related categories (*Problem Solving*, *Conceptual Discussion,* and *Learning and Presentation*), the remaining categories from the analysis of student answers (*Affective towards Subject* (AtS), *Affective towards Collaborator* (AC), *Teacher Initiated* (TI), *At* School (AS), and *Not At School* (NAS)), and two social categories representing *Social Interactions at School* (SIS) and *Social interactions Outside of School* (SOS) [9].

[9] The analysis leading to these two categories is not described here, but it is analogous to the analysis of student answers to the initial physics survey. At the same time as answering the physics survey, student answered a

Wengerian categories

Communities of practice (Wenger, 1998) is a very broad theory, and the different concepts in the theory hold many potential meanings. For example, Wenger describes communities of practice in terms of mutual engagement, a joint enterprise and a shared repertoire. However, even if mutual engagement is described in terms of participation and reification, the two "can be seamlessly interwoven". Likewise, a joint enterprise can exist "without being reified, discussed or stated as an enterprise". This makes the theory hard to operationalize, without leaving out some of the potential meanings embedded in his concepts.

In terms of communities of practice, this study is concerned primarily with learning dimensions. Wenger lists four learning dimensions Wenger (1998, chapter 10), and describes them in terms of dualities; as two parts which are intertwined in such a way one cannot exist without the other and each part holds different perspectives. These learning dimensions are negotiation of meaning (engagement/reification), identification/negotiability, designed/emergent, and local/global.

Here, the physics content related categories are seen describing negotiation of meaning. Behind these lie the reifications which students use to participate in the communities they are part of in their studies. When students engage with each other it is through reifications like problem sets, lab exercise manuals, computer programs, text books, notes (both their own and the teacher's) and even ways of speaking about physics. These are the reifications around which students can participate in physics.

Wenger also develops three different modes of belonging - engagement, imagination, and alignment - which can be used to characterize a learning dimension. Negotiating meaning in terms of engagement describes how students use and develop practices when they engage in problem solving with each other. It is not only problem solving practices, but also how concepts and language applies to problem solving.

This is different from concept discussion, where students imagine how different concepts work. In terms of negotiation of meaning, they engage with each other to discuss the nature of concepts and interaction of concepts. For example, Bruun (2011) shows two students arguing from different models of electricity about the ranking of light bulb intensity in a specific electric circuit. They use different explanatory models, and their models influence how they discuss concepts. They use the available materials (computers, pen, worksheets) throughout the discussion, but only to enact their explanations about electricity. In short, they use their imagination to negotiate meaning.

Communications about how to present things right or how to use the language of physics in the correct way amount to aligning communication with the styles and discourses of a larger physics community. Distinguishing in words between *weight* and *mass* can be very important in a discussion about how a pendulum moves on the Moon (Bruun, 2008) or when solving a problem about it.

So far *Problem Solving* has been linked to engagement, *Concept Discussion* to imagination, and *Language and Presentation* to alignment. At the same time, it is clear that prob-

social survey.

lem solving can entail both conceptual discussions and aligning language, and the same goes for the other two categories. However, by listing the categories as explicitly different, students could reflect upon how they perceived their physics interactions.

The two small categories, *Affective towards Subject* (AtS) and *Affective towards Collaborator* (AC) are difficult to interpret, because they are so small. Also, the few AC statements students do provide, do not really characterize the process. They take the form of an assessment of the other person. Student evaluation of peers is beyond the scope of this study because (1) it is a potential source of mischief between students and (2) this study is about processes, and only tangentially about relations. However, since collaborating with peers might be part of a teaching situation, and we are interested in student communications about teaching, the move of change on AC is to convert it to *Affective towards Teaching* (AT). The AT category thus entails student opinions about teaching and teaching activities.

Communications about students' affection towards teaching seems like a way to negotiate identity in the class room. Arguably, the subject relevant categories may capture some of the identity of students, but talking about teaching is another layer. It positions students with respect to teaching, and may also engage students in reflections about their own learning practices. Thus, the category may articulate the identification/negotiability learning dimension with respect to imagination.

Students affection towards teaching is a meta layer compared to the "pure" physics subject interaction categories, and it may reflect parts of their capacity beliefs (Andersen et al., 2004) and motivation. For instance, if they do not feel that group work stimulates their learning, they may communicate concerns about teaching at times where group work is in focus.

As illustrated in Table 2, students may feel strongly about physics as a subject. When communicating about their affection towards physics as a subject, they may refer to situations where they actually did physics. So it is a reflection of how it is to do physics at this moment, how it was to do physics at some point. In this understanding communication entailing affection towards physics, some kind of engagement is implied, while alignment and imagination are downplayed.

Communication about affective issues regarding the subject of physics as seen from an alignment perspective would imply power relations. As seen from an imagination perspective, students might communicate about affective issues regarding their future dispositions. Thus, the understandings of the affective categories are tentative. For now, they belong to the identification/negotiability learning dimension and the engagement (*Affective towards Subject*) and imagination (*Affective towards Teaching*) modes of belonging.

The *Teacher Initiated* (TI) category was only used to describe one student comment out of all student comments in four weeks. Still, student interactions could be broadly viewed as either teacher initiated or student initiated. Clearly, when students work together in a laboratory exercise, the teacher has initiated this even if he may not have designed the composition of the groups. On the other hand, student might be given free time during class to work on problems, or they might help each other with home work on their own initiative. Thus, the

Teacher Initiated category splits into two categories, *Teacher Initiated* and *Student Initiated* (SI).

This partitioning can be described by the designed/emergent dimension, and student initiated communication corresponds to the engagement mode of belonging. It is quite clearly "situated improvisation within a regime of accountability" (Wenger, 1998, p. 240). Likewise, teacher initiated communications is nicely described by the alignment, as it implies coordination, feedback, and renegotiation.

The *At School* (AS) category could intuitively be split into a *Not at School* (NAS) category and the *At School* category. However, since students answer that they do not communicate a lot outside of school, it makes more sense to make the division between *In Class* (IC) and *Not In Class* (NIC). This division reflects that some times communication might take place in other types of context than in the classroom; it is meant to reflect Wenger's dual concept of local/global. For Wenger, the global is made up of many localities, here the global is operationalized by adding *Not In Class*. A communication taking place outside a school context let students expand the area in which they apply their knowledge. At school, students probably align their communication to the standards enforced by the teacher. Communication about physics not at school may signify a deeper engagement in the communities in the physics class.

As seen in Figure 2, the networks describing physics communication/collaboration can change, and as noted in the legend, some of this change might be rooted in purely social circumstances. Therefore, this study includes the categories *Social interaction In School* (SIS) and *Social interaction Outside of School* (SOS).

The interpretation of categories developed here is of course tentative. However, they define a means to develop the statements presented to students in the weekly surveys. The next section describes this development.

Survey statements from categories

The analysis of the previous section and the resulting categories formed the basis for the second survey instrument administered to the students of the class. Table 3 lists a translation of the questions along with the categories they address in the order in which they were presented to students.

As noted by Pustejovsky and Spillane (2009) network surveys may be subject to biases just as normal surveys are. One task is to formulate the categories as presented to students in a way which minimizes biases. Participants might be cognitively primed by the wording of a question to answer in a certain way.

A guiding assumption was that if students were to rank their experiences with others or if they were prompted to select the ones they preferred working with, then the answers would be biased. For example, a bias might be towards students naming their friends rather than the ones they actually remembered communicating with about physics. But since students were to answer the survey week after week, students might make deals with each other (*If you rank me high, then I'll rank you high*).

This is the reason the survey uses the word "communicate" rather than "work with" or "collaborate". A student who feels that (s)he is working alone, will not indicate working with oth-

#	Category	Code	Category description (translated from Danish)
1	Problem solving	PS	We communicated about how to solve a task in physics. (How to perform calculations, what formulas you need, how to read graphs and the like).
2	Conceptual understanding	CD	We communicated about understanding one or more physics concepts. (What current is, what the normal force is, how radioactivity works, and the like).
3	Language, presentation, form	Pr+L	We communicated about how to use the language of physics, or how to present physics. (How you say things right, how you properly write a report, how you structure a report).
4	Affective towards teaching	AT	We exchanged opinions about what we think about the teaching methods. (What you think about the way the teacher teaches, about group work, and the like).
5	Affective towards subject	AtS	We exchanged opinions about what we think about the subject. (Whether or not it is exciting, boring, difficult, or easy).
6	Local physical setting	IC	We worked together during class, at a “homework café”, or similar
7	Global physical setting	NIC	We worked together outside of class. (At one of your homes, after school hours at school, during recess)
8	Designed collaboration	TI	We were put together by the teacher. (Teacher-made groups, work with your neighbor, etcetera.)
9	Emergent collaboration	SI	We chose to collaborate ourselves. (You chose your own groups, chose to collaborate during a class, etcetera)
10	Local social interaction	SIS	We talked in school about something other than subjects in school.
11	Global social interaction	SOS	We talked outside of school about something different than subjects in school.

Table 3: A list of the 11 questions the students answered 9 times during 16 weeks in the fall semester.

ers, even if the student asked someone for help or helped someone. However (s)he might be more likely to list having "communicated" with someone. The same kind of reasoning lies behind "exchange opinion" in stead of "argue" or "discuss". Discussing might have positive connotations excluding other types of opinion exchange, while argue is typically negatively connotated. This means that this survey takes communication as a proxy for interaction.

Pustejovsky and Spillane's analysis of question-order effects also show that listing a category which produces many namings first is preferable to listing it after a category which produces many namings. Since the primary student category was *Problem Solving*, this category was listed first. Thus the survey prompts for names within a category which should produce the most namings. Pustejovsky and Spillane investigated two categories of the same nature, whereas this study utilized multiple categories in different groups. 11 categories of naming will probably be a severe cognitive load for many participants, and placing the IC category midway through the physics related categories may give more names at a time where students are getting fatigued.

Networks from survey describing learning dimensions

This section describes the network structure and development in terms of the theory developed in this paper so far. However, the understanding of the nature of links and nodes developed so far may not sufficiently to explain the phenomena the networks display. The validation session described previously, will help to hypothesize about the phenomena described in this section.

In the networks, each node corresponds to a student's choices within each category. The networks have been constructed so that all students who have named or been named in one category at least at one time during data collection are represented by a node. Links within a given network represent the namings students have made each week. Students have been asked to name other students whom they remember having had interactions with in the preceding seven days.

Nine surveys from weeks 35 to 46 formed the basis of 97 different networks. This is too many networks to be feasibly portrayed in a paper. Instead, they are available online[10]. Figure 7 shows two drawings of one of the networks as it appears online. It is possible to investigate all networks in this study.

Looking at the networks online, *Problem Solving* (PS) seems to have the most links, which would signify that a lot of the activity in class is about solving physics problems. Also, the *In Class* (IC) network is very active in this way. But the social networks, *Social interactions In/Outside School* (SIS/SOS) show far more activity than either of the PS and IC categories. Also, at a glance the Teacher Initiated (TI) networks are hardly networks: They only have a couple active nodes.

The Kamada-Kawai lay-out algorithm (Kamada and Kawai, 1989) used to display the networks can be used as a coarse grained way of identifying structure in the network. Looking

[10] www.jbruun.org/studentNetworks

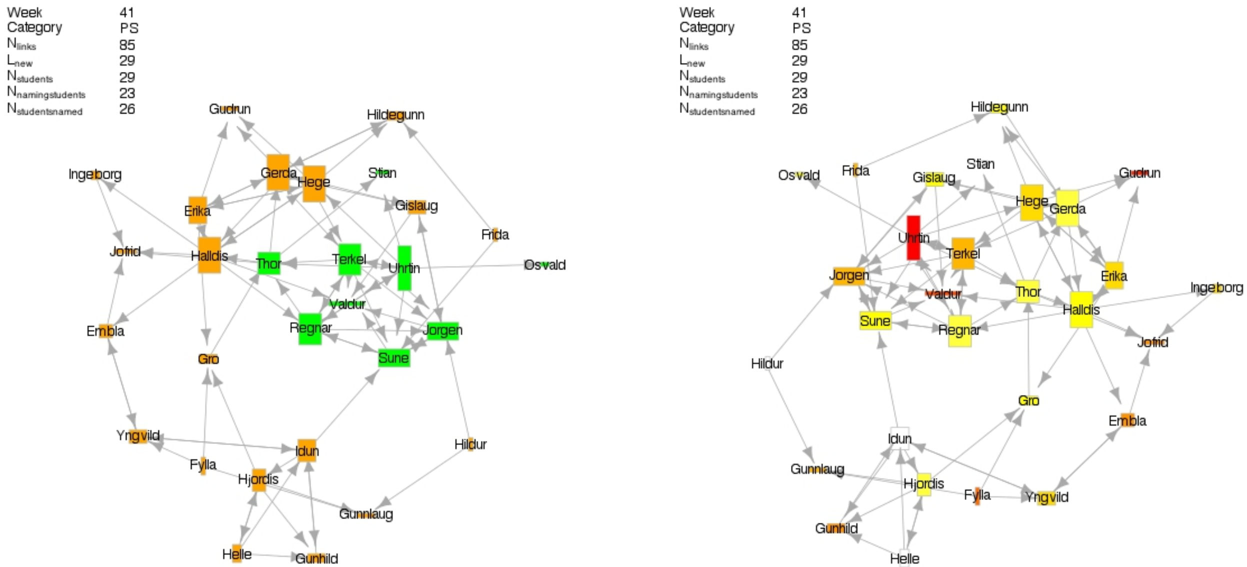


Figure 7: Two drawings of one of the networks available online. The networks can be found here: http://www.jbruun.org/om/studentNetworks/. The width of a node is proportional to the number of times the student has been named (indegree) while the height is proportional to the number of names the students has provided (outdegree). In the left drawing, orange nodes are girls, and green nodes are boys. To the right, nodes have been colored according to the grade from a test in week 44: Red nodes have low grades and more yellow nodes correspond to high achieving students.

at the networks (see Figure 7), you intuitively see that students seem to be segregated with respect to gender but not with respect to grade.

Displaying the networks like this is be informative, but it is difficult to keep track of the many features they exhibit. For this reason the measures developed in Section (Figure 2 and Table 1) are now examined and compared with selected information from the teacher's plan for teaching.

In this plan, the teacher lists the general subject (e.g.. "sound and music"), the specific topic of the week (e.g.. "standing waves on a string"), and the teaching activities (e.g.. "experimenting with resonant frequencies of a string"). The teaching plan also includes the dates for the teaching unit, making it possible to extract how many physics classes the students had in a given week.

Figure 8 shows how the number of links, N_l, vary in the *Problem Solving* (PS), *In Class* (IC), *Concept Discussion* (CD), and *Learning and Presentation* (L+Pr) networks. Below the weeks the number of teaching units in the preceding week is shown. Weeks 40, 43, and 46, where the students had only one or zero classes, show dips in the number of links. During the fall break students were not at school and very few answered the survey. The variation in N_l seems to be roughly consistent with the number of teaching units in the preceding weeks. However, week 44 has less links than week 45, even if week 43 had three teaching units and week 44 had two. Thus, other factors might help explain the number of links, for example, what day of week students answered, or what kinds of teaching activities they engaged with during class.

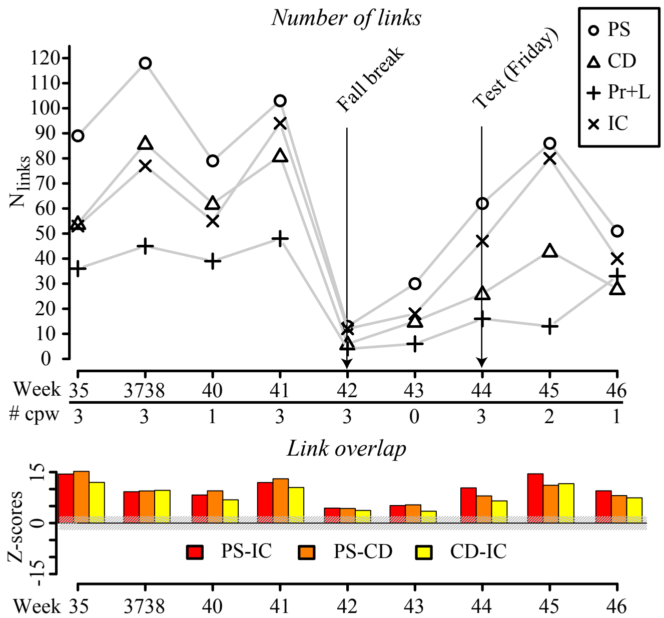


Figure 8: The number of PS, IC, CD, and L+Pr links. Teaching did not allow data collection in week 36 and 39, while week 37 and 38 are inseparable due to a technical error. The number below each week is number of teaching units in the preceding week. Thus, week 39 had one teaching unit. The Z-scores at the bottom shows the overlap between links of three of the categories compared with 1000 random versions of the network. The significant region is marked by the shaded gray area.

The networks for the rest of the physics categories follow the same trends, although they all have a lower amount of links each week than the *Problem Solving* networks. The social networks have a lot more links (often around 2-300 links compared with max 86 for *Problem Solving*) signifying that social interactions are much more prevalent for these students than physics interactions. The *In Class* category mostly has an amount of links comparable with the *Problem Solving* network, while the *Not In Class* network is consistently has fewer links.

Figure 8 also shows Z-scores for overlapping links, for example, $Z^{week35}_{L_{PS\cap CD}} = \frac{L^{w35}_{PS\cap CD} - \langle L^{w35}_{PS\cap CD}\rangle_r}{\sigma^{w35}_r}$. The amount of overlapping links between each of these categories are significantly higher than we would expect randomly[11], meaning that students often name each other in multiple categories. Thus, there an overlap between PS and *In Class* (IC), which we would expect from the initial student answer, but also between *Problem Solving/Concept Discussion* and Concept Discussion/In Class. Although not shown here, this is also true for the overlap between the other categories.

The $L_{g_i \cap g_j}$ measure can also be used to find the number of new, reestablished and predicted links. For example, the intersection $L^{ICS}_{w_{40}\cap w_{41}}$ is the number of links in *In Class Social* networks from both weeks 40 and 41. Subtracting this number from the total number of links in ICS week 41 yields the new links compared with the week before, L_{new}. In a similar manner, we can ask how many links are in the network of week 41 *and* week 35, L_{pred}, taking week 35 as a predictor. Also we can ask how many links in week 41 are in any one of the preceding weeks, that is the number of links which are reestablished, L_{ree}. Figure 9 shows the these three measures normalized to the total number of links in a weeks network for six different categories.

Notice that *Problem Solving*, *In Class* and *Student Initiated* in the first row of Figure 9 communication links follow roughly the same pattern. The *Concept Discussion* and *Learning and Presentation* categories are not shown, because they follow roughly the same pattern from week 41: New links comprise the largest fraction, followed by reestablished and predicted links. From this week, the number of predicted links seem to stabilize around 30% while the other two vary more.

This picture is contrasted by the *In Class Social* (and the *Not In Class Social*, not shown), where 70% of future links can be predicted from week 35. Moreover, the fraction of reestablished links tends towards 1.0, and the variation in new links is around 30-40%. Thus the picture here is the reverse.

Finally, the affective networks are erratic, although the number of predicted links are always low. However, there seem to be a shift in the fraction of new links from week 41 to week 42 and onwards in both of these types of networks. The starting from a low fraction of new links, the networks tend towards having a higher fraction of new links each week. There is a difference between the *Affective towards Subject* (AtS) and

[11] The networks were randomized by repeatedly selecting two pairs of of connected nodes (A,B) and (C,D) with no interconnecting links and switching the links so the new connections become (A,C) and (B,D) (Maslov et al., 2004). In this work, each network was rewired 10 times the number of connections in the network. After rewiring each network, the overlapping connections were counted, the Z-scores are based on 1000 of such iterations.

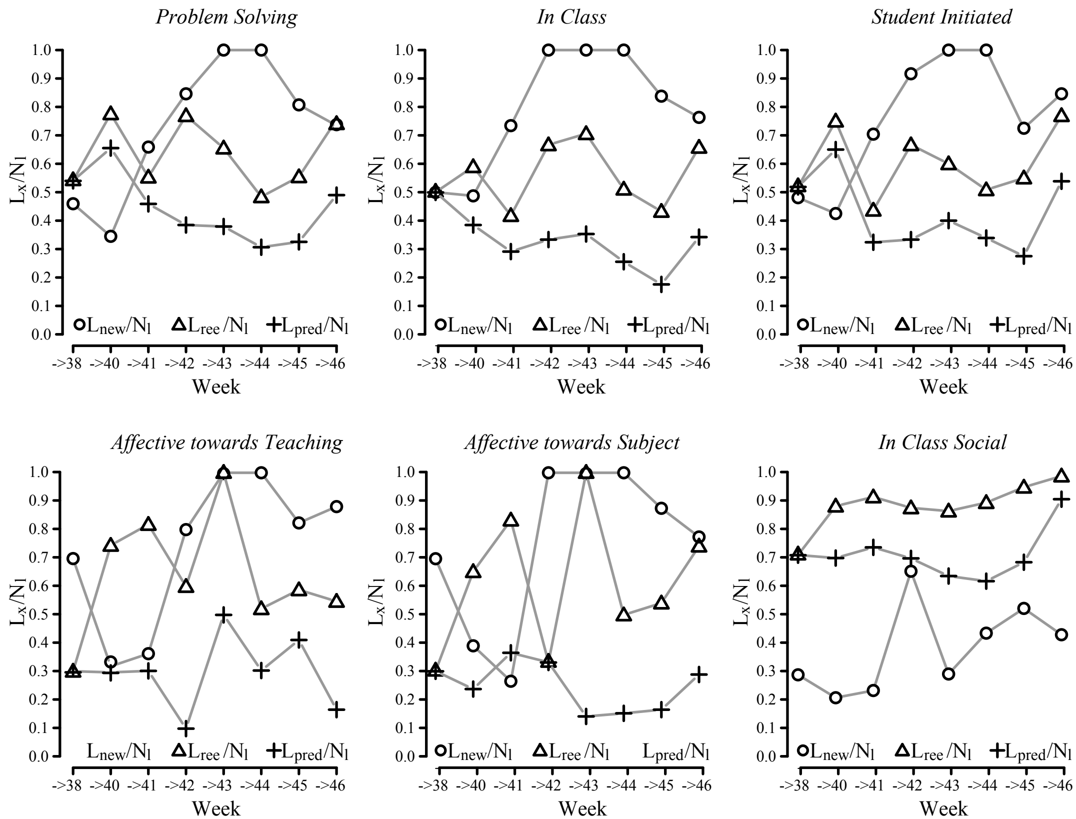


Figure 9: For a week, new links, L_{new}, are in the network for that week, but *not* in the preceding week's network. Reestablished links, L_{ree}, are in the network of a week *and* in at least one of the preceding weeks' network. Predicted links are the links which are present in the network of week 35 (not shown) *and* in the network for a week. The figures show the fraction of new (L_{new}), reestablished (L_{ree}), and predicted (L_{pred}) links relative to the total number of links that week (N_l). In the In Class Social network for week 41, 90% of the links are present in either of weeks 35-40. Around 70% of the links in week 41 are also in week 35, and around 20% of the links are new compared with week 40.

Affective towards Teaching (AT) networks in week 46, where L_{ree} rises from 50% to 70% in AtS network and drops from 60% to 50% in the AT network. However, the erratic nature of the plots makes it difficult to discern patterns for these networks.

The different link patterns show a connection between *Student Initiated*, *Problem Solving*, and *In Class*, which is consistent with the student answers. In the context of the developed framework, it could be taken as an indicator that student engagement with physics emerges from interactions locally in class. It is the students' reaction to the teaching activities for which the teacher designs.

Figure 10 shows how the Z-scores for target entropy, and hide in the *Problem Solving, Concept Discussion* and *Language and Presentation* networks. In the first three weeks of the study, the target entropy in the *Problem Solving* network is significantly lower than randomly expected, meaning which could be interpreted with students having less learning opportunities in these networks (Section). This is most prominent in week 41, but notice that *Language and Presentation* is also significantly lower in week 3738, and *Concept Discussion* is significantly lower in week 41.

Interestingly, the most prominent deviations in target entropy from random happen in connection with experiments for *Problem Solving* and a homework assignment for *Language and Presentation*. Perhaps these two activities trigger students communicating only with a select few other students thus not facilitating many diverse interactions. These highly prominent features are matched with high levels of hide, which in turn signifies that the communication in the network is less effective than randomly expected.

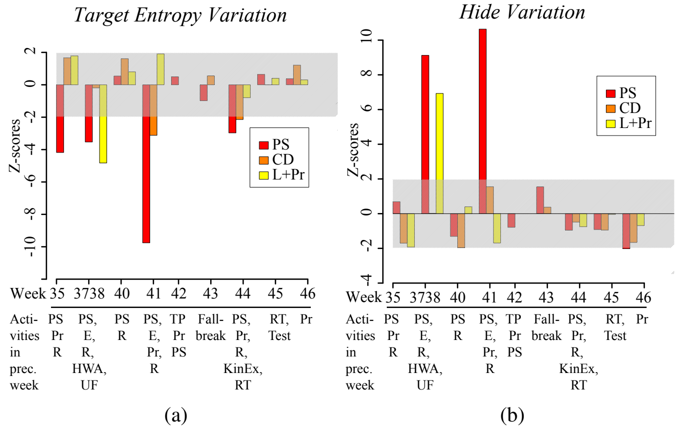


Figure 10: The target entropy (a) and hide (b) Z-score variation from week to week. The codes for activities at the bottom are *Problem Solving* (PS), *Student Presentation* (Pr), *Reading* (R), *Student Experiments* (E), *Home Work Assignment* (HWA), *Teacher Presentation* (TP), *Kinesthetic Exercise* (KinEx), and *Reading for Test* (RT).

Validation of category questions and hypotheses

It is possible to interpret patterns in network measures with respect to teaching and learning. However, a more thorough understanding of how students perceive their remembered communication is needed to validate the meaning of the interaction patterns. This understanding may in turn be used to hypothesize about why these patterns emerge.

To achieve this understanding, students participated in a 1½ hour validation session. Here, students first responded to an individual free-text on-line questionnaire. Later, they worked in groups of 3-5 to discuss their answers. In total, 24 students where present for the validation session. The following two sections analyze these two parts of the validation session, Section combines the theoretical framework, the networks, and the validation session analysis to generate three hypotheses based on the results shown in Figures 8-10.

Individual part

In the individual part of the session, students sat in class each with a laptop computer and answered a questionnaire. The questionnaire consisted of four free-text questions. In each question the students were reminded of a group of the categories (e.g.. #1-3 in Table 3). Then the text prompted them to elaborate on when they had checked such a link.

For #1-3 and #4-5, the survey asked them for examples of situations which made them check a link. For #6-7 and #8-9, they were asked if they had experienced any doubts when choosing whether to check or not check such a link. They were not asked to elaborate on #10-11, the social categories.

The purpose of the individual answers was to elicit first hand student responses. Such responses are not un-biased, since the students are influenced by the context and communities of which they are part. But they are not influenced by group discussion, and thus serve as a representation of what the individual student thinks at that particular moment. In this way, the individual answers are first hand.

Students used the free text format in primarily two different ways. The first way was to separate their answer into the number of categories they considered. For example:

> "If we in a class have solved problems together and talked about which formulae and so on to use. 2. I don't think I have experienced checking this. I guess it is because the teacher explains it very thoroughly in class, so that we know it and do not have to discuss it." - Gunhild

The second way of answering was more narrative:

> "It was a day in November, actually the day before the physics test. Jorgen and I sat in Jorgen's parents' basement. We went through a physics compendium to be ready for the test. As we reached the subject of atoms and photons, we didn't understand much, so we communicated about how to work with the problems, and how to understand them..." - Valdur

The first type of answer was coded as two separate answers. This means that *Problem Solving* (PS) and *Use of Formulae* (UF) would be noted to co-occur, while *Not Relevant* (NR) and *Teacher Presentation* (TP) would also co-occur. However, PS and NR would not co-occur in this answer. In the second, PS, UF, *Specific Concept* (C), and *Problems Understanding* (PU) would be listed to co-occur.

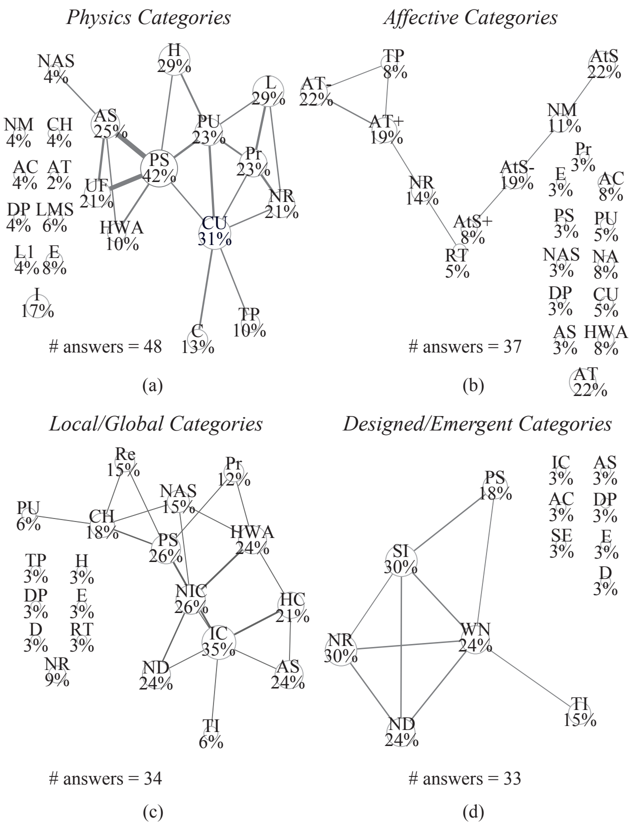


Figure 11: Category networks based on codes of individual student answers in the validation session. The categories are linked if they co-occurred in an answer more than once. Percentages are normalized to the number of answers in the four groups of categories.

Coding students answers for the four groups of categories (#1-3, #4-5,#6-7, and #8-9) made it possible to create four category networks (as in Figure 5). Figure 11 shows four category networks of coded student answers. The initial categories shown in Table 2 and Figure 5 served as a basis for the new categories, but additions were needed for a fuller picture. The *Use of Formulae* (UF) category is one example, as are codes representing negative and positive answers in the affective categories (*Affective towards Collaborator* (AC), *Affective towards Subject* (AtS), *Affective towards Teaching* (AT)). The category networks in Figure 11 are for the three physics link categories (a), the affective link categories (b), the local/global link categories (c), and designed/emergent categories (d).

The Physics Link Categories validation category network (Figure11 (a)) shows that problem solving is linked to *At School* (AS). There is a small link from *At School* to *Not At School* (NAS), reflecting a couple of students adding that they have also indicated situations where the communication was outside school. *Problem Solving* is also heavily linked to *Use Of Formulae* (UF). This is not surprising since it is part of the category description, but it may be surprising that other parts of the description are not mentioned.

Conceptual Understanding (CU), *Language* (L) and *Presentation* (Pr) are also present, but notice that they are all linked to the *Not Relevant* (NR) category. This category depicts whenever a student indicates that they rarely use the category, or when they say that discussing concepts is not relevant because the teacher has already explained them.

In the coding, language and presentation are separate. *Language* codes statements like "I have discussed the difference between words as they are used in math and in physics" *(Jorgen). Presentation* codes statements like "Since we had an answer book, we also tried to figure out how to present the results to satisfy [teacher]" (*Valdur*).

The *Problems Understanding* (PU) category appears central in the network. This means that students relatively often make a connection between communicating with others and having problems understanding something. For example, Gro writes: "Talking with someone else about a problem you do not understand, may imply that you hear the problem formulated in a different way and thus understand it differently. This may help you solve the problem."

In the Local/Global Link Categories network (Figure 11(c)) describing how students view the difference between *In Class* (IC) and *Not In Class* (NIC) categories, the IC and NIC nodes both link to *No Doubt* (ND). In 24% of the 34 answers for this group, students explicitly listed that they had no doubts as to when they checked IC or NIC. Students described more what they meant with IC and NIC than describing any doubts.

Disregarding the weak link between IC and NIC (they co-occur twice), NIC is further connected only to *Home Work Assignment* (HWA) and *Not At School* (NAS). This means that students do not explicitly mention problem solving while not being in class. However, they mention other things as part of communicating outside of the classroom; *Online Chat* (CH), *During Recess* (Re), and *Not At School* (NAS). *In Class* is further connected to *Problem Solving*, *Homework Café* (HC), *Teacher Initiated* (TI), and *At School* (AS). In sum, students do not seem to have any doubts as to what IC and NIC entails.

When viewing each student answer, it seems that they do not have any doubts when distinguishing between *Student Initiated* (SI) and *Teacher Initiated* (TI) communication. However, notice on Figure 11(d) that both SI and TI are connected to *Work With Neighbor* (WN), indicating that students might disagree on a central point. As Sune writes: #[I list TI] when we got the instruction to 'work with your neighbor"'. But Hildur writes: "I often choose to work with the person I am sitting next to." Frida explains, "I sometimes have doubts, when you are working with your neighbor. I do not think you can always say that you have chosen the person yourself." It seems that for some students the separation is clear cut, but others think very carefully when answering this question.

For many students the question of who initiated the interaction is *not relevant* (NR). This is not to say that they do not care, the code merely reflect that students write that the teacher rarely puts them in groups. If they are to work in groups, they are free to choose them themselves, and some of the students might see this as SI even if the teacher gives the instruction to work with others.

The Affective Link Categories network consists of two disconnected components, when we disregard links of weight 1. The two components correspond to the *Affective towards Subject* (AtS) and *Affective towards Teaching* (AT) categories. Students mostly seem to exchange opinions about the subject matter, when they are introduced to *New Material* (NM). Also, some students associate negative connotations (as represented by the node *AtS-* in the network) with these exchanges of opinion: "This comes up mostly by itself (for example, when we are doing homework). For example, [I would say]: 'This subject is mega hard - I do not understand it."'- Frida. Another student remarks that they often talk about their opinions toward the subject when a there is an upcoming test.

The *Affective towards Teaching* (AT) category is disconnected from other categories in the network. In the coding, *Affective towards Teaching* appears along with any other category maximum one time. In a network with only link weights higher than 1, AT becomes disconnected. However, AT coappears once with 10 different categories, reflecting that students have many different associations with *Affective towards Teaching*. When students differentiate between positive affection (AT+) and negative affection (AT-), they sometimes link it to *Teacher Presentation* (TP).

In five out of the 34 answers, students explicitly mention that they rarely or never use this category. Notice that it is connected to the AT+ node, which could indicate that students find affective talk about teaching irrelevant to communicate about, when they are satisfied with the teaching.

Group discussions

As the students finished the individual survey they were put together in groups of 3-5. This gave a total of five group answers They went to another room where they were given the task to discuss, compare and contrast their answers to the individual part and note key likenesses and differences in an online survey. The point of this part of the session was to elicit answers which individual students might not have thought of and to highlight agreement/disagreement between students.

For the *Problem Solving* category, the group answers can elaborate on the individual category networks. Only Group 1 had a discussion about whether problem solving entailed experiments. Frida mentioned that problem solving was also when you did experiments, but the other people in the group had not made this connection. Whether students in the class generally link problem solving as described in the questions with experiments, is an open question.

In Group 2, Gudrun and Valdur argued that the *Concept Discussion* category entails to discuss what the different variables in a formula mean, while Regnar and Yngvild believe that the category entails discussions about the meaning of a physics concept, like current or normal force. The "meaning of variables in a formula" seems more connected to problem solving than "conceptual discussions about concepts", which seems more philosophical. Or as Hildegunn, Gislaug, Stian, and Jorgen (Group 3) writes: "It is more when we have doubts in connection with concepts and more theoretical things, rather than problem solving". For this group, concept discussions are theoretical, where as problem solving is not.

Students rarely seem to have felt a need for checking the Language and Presentation category. They argue that they rarely are to hand in lab reports, and if they are, they talk to the teacher about how to present things. In Valdur's experience, though, students do communicate about about language and presentation before tests.

With regards to the affective categories, one group offers an explanation for why these might not be checked as much as the physics related categories: "We are very bad at remembering having talked about this (*Affective towards Teaching*) or who we talked to about it, because it is often in a context where we just mention it and then move on with the discussion" - Group

3. Interestingly, one group writes that they do not use the *Affective towards Teaching* category often, because it they are satisfied, but that "it can be used when we have group work." If the statement of Group 3 is significant for the whole class, then the affective networks do not reliably capture affective communication.

The *Affective towards Subject* (AtS) category is linked to change and something which can happen outside the classroom for two groups: "This is often used , when we begin a new subject and round off another. Apart from this, we also use it when we discuss the subject outside the physics classes" Thor, Hjoerdis, Gro, Embla, Gerda. Group 3 links the *Affective towards Subject* category to communications when they are reading for a test, because "there is a little more pressure to be able to know the subject in the tests". Again this group points out that they are less likely to remember these communications than physics related communications.

The difference between the *In Class* (IC) and *Not In Class* (NIC) categories seem fairly clear. Sune, Terkel, Hildur, and Gunnlaug define *In Class* "as when you work in a school related context, where there is a teacher present." *Not In Class* is when "you work together in an context where there is not a representative form the school present." The answers from other groups are consistent with this definition, although it seems a bit fuzzy if *communication during recess* is categorized as IC or NIC interactions. Interestingly, three groups explicitly mention that NIC interactions mostly happen via online or mobile tools.

The students seem to agree that the teacher rarely puts them in groups. For the students the distinction between *Student Initiated* (SI) and *Teacher Initiated* (TI) groups is whether the teacher explicitly decide the groups: "We would check this (TI), if the teacher made the groups for an experiment or such, but we have never checked it, because it has never happened - *Hildegunn, Gislaug, Stian, Jorgen* ".

Hypotheses derived from validation, pilot, and learning theory

Figures 8-10 give rise to three hypotheses. Each figure stimulates some kind of question about why they look as they look. The developed framework can be used to generate an explanation and a prediction.

Hypothesis 1 Figure 8 shows more activity in *Problem Solving* than in *Concept Discussion*, which in turn has more activity than *Learning and Presentation*. The analysis has shown that this is because students connect doing physics with problem solving, specifically with using formulae. Physics education research emphasizes that other representational forms and developing language are important for a more full understanding of physics (e.g. Dolin et al., 2001). In the sense of communities of practice as developed for this study, *Concept Discussion* could focus the mode of imagination, letting students explore different modes of representation. Thus a hypothesis to test would be:

If student learning focused on activities emphasizing novel (from the perspective of the student) use of other representational forms than those pertaining to use of formulae, then the

Concept Discussion network would be more than the Problem Solving network.

This could be tested by creating teaching units emphasizing the phenomenological and conceptual forms of representations in activities rather than mathematical symbolic forms. It would give useful information for understanding teaching and learning situations.

Hypothesis 2 In Figure 9 the *In Class Social* network students communication patterns seem much more fixed than in the *Problem Solving*, *In Class*, *Student Initiated* networks. The question is why these social interactions are so different and if they can be used in a positive way in connection with teaching.

A straightforward interpretation of both networks and the validation analysis is that of physics communication for these students is mostly centered around solving problems in class. Even without an analysis of student social relations, it seems fair to assume that this situation is very different from how they interact socially. A further interpretation of the data in Figure 9 could be that these students do see a discrepancy between being social and doing physics. But if they are more willing to communicate if they view the situation as social, then perhaps physics teaching could benefit from integrating social aspects in teaching and learning activities. Thus the hypothesis is:

Integrating social activities in physics teaching and learning activities throughout a school year will create a likeness between the behavior of social interaction networks and physics interaction networks. This would have a positive effect on students' communicating about physics outside a school context and on their motivation to do physics.

This key problem here is to find a way of integrating social activities with physics teaching and learning activities which is beneficial to student learning outcomes. Notice that the time aspect includes a whole school year. This is to indicate that changes in social networks might only be seen in longitudinal studies. The implications would relate to scientific literacy; if physics communication becomes part of students' everyday lives they could be expected to increase their scientific literacy.

Hypothesis 3 In Figure 10, the target entropy is low and hide Search Information is high, when there are experiments in the preceding week. The activities in the form of the number of links, N_l, are also high for these weeks, but later weeks show the same amount of activity without the pattern in target entropy and hide Search Information.

The analysis shows, that students form groups themselves when doing experiments, and if they do no not communicate much outside the group (note that this hypothesis can be tested with the current data) this could lead to networks with high level of predictability (low target entropy) and low communication efficiency (high hide Search Information). However, if students did not isolate themselves in their groups, more connections would be made outside each group, signifying more learning opportunities and higher communication efficiency. Thus the hypothesis is:

Incorporating accountability between groups doing an experiment, will increase communication efficiency and learning opportunities as measured by Search Information and target entropy respectively. This will result in a better and more lasting understanding of the subject matter.

This could be tested by comparing communication patterns for a class when throughout a year when they did experiments. Varying between an accountability format and a non accountability format would then give rise to a number of networks, representing communication patterns for the weeks were students did experiments. For the hypothesis to hold researchers should see a higher target entropy and lower Search Information in the networks describing the accountability exercises. The last sentence is included to show how this way of using networks could be incorporated with other ways of describing learning situations. Of course, what a better and more lasting understanding entails would have to be defined.

Suggestions for use and further development of survey and study

The purpose of the study was to develop a method which can be used to integrate network theory with educational theory. The study shows that it is possible to design such a framework, which can be used to create hypotheses to explain why we see patterns in networks describing learning situations.

One way of using this framework in the context of physics education would be to try out different teaching formats and see how students respond to these. For example, a task could be to design a teaching exercise which resulted in more and more efficient conceptual discussion. Coupling the framework developed here with other data (for example, observations of class room interactions) might then be used to connect learning outcomes with network data. Comparing different classes doing the same exercise may then show us quantitative differences which say more about the learning outcome than conventional pre-test post-test research designs.

In another approach, these kinds of network data - based on student self-reports of communication events - may be related to other kinds of network data, for instance, networks based on digitally measured proximity or self-reports on who students *prefer* working with. If self-reports of communication events tell us about who students remember having communicated with, proximity measures may indicate who they could *possibly* have communicated with. Coupling this paper's communication links with self reports on preferences would then tell us how much collaboration with preferred partners, the teaching situation allows for.

In future developments of the method and survey, it seems that some changes in the amount and meaning of categories are in order. For example, the affective categories were rarely used by the students in this study, and if the explanation for this is that they simply forget having communicated about affective issues, then there may not be much of a point to ask them about this. On the other hand, leaving out categories reduces the amount of information we *can* gain from network data.

Acknowledgements

I would like to thank Morten Brydensholt and the staff at Virum Gymnasium for their help with collecting data for this paper. Also, thanks to 2b Mid-Level Physics Class for answering all the surveys. Finally, thanks to Professors Jens Dolin and Robert Evans for suggestions and discussions.

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Appendix 2 - Time Development of Early Social Networks: Link analysis and group dynamics

Time Development of Early Social Networks: Link analysis and group dynamics

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Empirical data of early network history are rare. Students beginning their studies at a university with no or few prior connections to each other offer a unique opportunity to investigate the formation and early development of social networks. During a nine week introductory physics course, first year physics students were asked to name who they had communicated with about problem solving in physics during the preceding week. The student namings were used to produce networks of student interaction for several weeks, which are shown along with these students' grades and gender. Changes in the weekly number of links are investigated to show that while roughly half of all links change from week to week, students also reestablish a growing number of links as they progress through their first weeks of study. To investigate how students group, Infomap is used to establish groups. Further, student group flow is examined using alluvial diagrams, showing that many students jump between group each week., Finally, a segregation measure is developed which shows that students structure themselves according to gender and laboratory exercise groups and not according to end-of-course grade. The results show the behavior of an early social-educational network, and may have implications for theoretical network models and physics education.

I. INTRODUCTION

The formation and evolution of (social) networks has been modeled by many researchers, who have have investigated theoretical models of mechanisms for producing networks resembling empirical networks [1–5]. However, longitudinal network data is rare [6], and for the most part it is difficult getting access to network data from a time t_0, where the network does not exist.

Students beginning at a university with few or no prior connections to each other, are in a new situation, and will probably make new connections with other students as part of their studies. Many of them will also become socially involved, which also involves making new connections to other students. As the students become both academically and socially involved they may change the way they are connected, and this might happen on a short time scale, perhaps daily or weekly. Thus, investigating high resolution network data from such students may offer insights as to how a network forms from scratch.

If students beginning their studies do not know the other students, we could expect them to try out many different possibilities for interaction. Some of these interactions are deemed worth while by the student, and might continue each week. Another prediction would be that they do not interact much at all, but work alone, and yet another prediction would be that the high performing students would become central, because of word of mouth (i.e. as a student you would hear about student X who is a high performing student, and then you would try to communicate with him to understand the subject better). These early patterns of interactions in social networks are largely unknown from an empirical point of view.

This study investigates these early patterns of interactions between approximately 170 students enrolled in an introductory mechanics course at the University of Copenhagen. Students report which interactions they remember in different categories (see Figure 1) related to physics learning and social communication. Naming another student naturally involves a direction, so the networks are directed. Self-reported measures are notorious for being biased [2, 7], but unlike an objective measure, they can tell us what students are interacting about. Also, using e-mails, phone calls, or digital proximity as proxies for social ties, may be misleading in its own right. For example, it has been found [8] that people remember their friends rather than remembering everyone they are near to as measured by digital means. In contrast, asking students who they remember having communicated with about some subject (in this work physics and social interactions), does indicate what the interaction was about. In this work we try to minimize bias [7, 9, 10] by only asking about remembered interactions and not asking students to rank these relations in any way.

To gain understanding of the processes underlying the formation and evolution of social networks, researchers have related network measures to non network node properties. For example, for university students the the probability of making new social connections has been tied to the number of classes taken together [6]. Also in a twenty year long study, people with increasing body mass index (BMI) tend to cluster together [11]. Thus relating the calculations we can perform on networks to the socially rele-

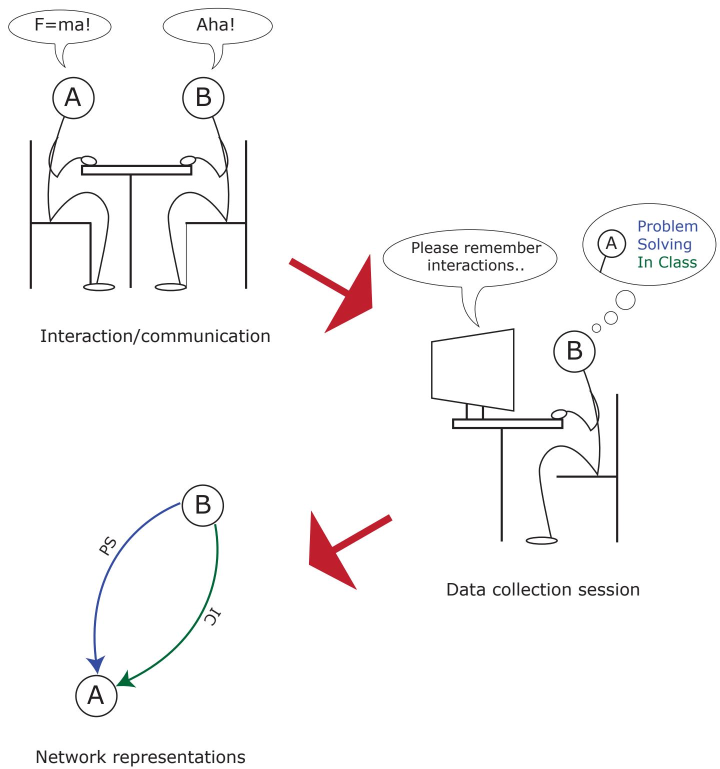


Figure 1: Students interacted when studying and subsequently reported the interactions they remembered via an online survey. Each remembered interaction becomes link in a network.

vant variables leads to knowledge of the social processes relevant to network formation.

One process occurring in social networks is the tendency for similar nodes to connect or be connected, referred to as homophily [6]. However, in many networks, nodes group together in clusters (or modules) where all nodes are not necessarily directly connected. In these groups information presumable flows more easily, and even if the probability for a single node to associate with similar nodes is high, mere chance could mean some connections would be to non-similar people, thus diminishing the similarity within the group.

Recently, the Infomap algorithm [12] has been used to find clusters and the structure between clusters in networks, employing a information flow based perspective on grouping. Since the links in this study represent student communication, this perspective seems appropriate. Also, Infomap has been shown to perform well on directed networks compared to other grouping algorithms [13], and changes of Infomap groupings of different networks with overlapping nodes can easily be visualized with alluvial diagrams[14]. Thus, Infomap might help find relevant groups of students interacting with each other.

Such groupings allow researchers to ask how students structure themselves during the first time at university. To quantify how students structure themselves in groups this paper develops a measure of segregation based on Kullback-Leibler divergence [15]. This measure is applied to each group to see how far these groups were from the whole cohort's distribution of grade, gender, and class number. Further, by giving each group's segregation a weight proportional with the number of students in it, the segregation for the whole network can be calculated. Thus, the segregation measures of how each group and the whole network is structured according to an attribute, compared with the cohort's distribution.

After explaining the background for data collection (Section II), this work shows empirical networks of self-reported interaction networks from an introductory physics course at the University of Copenhagen (Section III). Here it is shown that link patterns change from week to week but that many links are reestablished later on. Further, in Section IV an alluvial diagram shows how students jump between groups but that groups seem to stabilize at the end of the measurement. Finally, the segregation measure is developed in Section V, and in Section VI used with groups found with Infomap to show that students do not structure themselves according to grade but primarily according to their laboratory exercise groups, and somewhat according to gender. This is followed by a conclusion in Section VII.

II. BACKGROUND

A. Cohort and context

Students were allotted time during the obligatory weekly laboratory exercises to fill out online self-report surveys. Typically, students would fill out the survey at some time during the lab exercise, while some chose to fill it out at home. They were encouraged to fill out the survey at the beginning of a lab class, but some fitted in the survey when a natural break came in their lab activities. Students where told that their answers would be confidential and could not be used by their instructors/lecturers to identify them as individuals. Students not wishing to participate where allowed not to. The university uses a single laboratory for the introductory mechanics course, and students are divided into seven lab classes each of which have a weekly 3-hour time slot to do the lab exercises and also work together in *problem solving sessions* with a tutor to guide them. These are spread out throughout the week, so student responses are collected throughout the entire week.

Mostly, we can assume that a student answered the survey at the same of week from one week to the other. That is, if student A answers the survey on a Tuesday afternoon one week, chances are that student A answers the survey on the following Tuesday again. However, students were allowed to switch lab exercise hours, if for some reason they were not able to make it to the scheduled one. Thus, there is some fuzziness with regards to when student answers are recorded.

The measurements where done during a course in introductory mechanics and special relativity in a Danish University. Students are primarily ethnic Danes. The majority of students are physics majors who have just started their studies. Some major in other disciplines (for example mathematics), but are allowed to choose this physics course as part of their study plans.

B. Description of survey and data collection

The online survey was divided into two each week; an academic part and a social part. The academic part consisted of 9 interaction categories, while the social part consisted of 3 interaction categories. The categories where developed through a mixed methods pilot project prior to data collection [16], and in this study we only examine the category pertaining to *communication concerning problem solving*. A weekly format was chosen based on [8] who found the greatest correspondence between self reported networks and digitally measured proximity networks if the interactions were reported within a week.

Students where given a login to a learning management system, where they could locate the survey each week. For each interaction category, students marked each of the students they remembered having had interactions with. Names of possible students (all students enrolled in the course) where given in a roster[9]. The researcher was present throughout most data collection sessions, and students where invited to ask if they had doubts on how to answer the survey. The researcher emphasized repeatedly that they should mention only the people they remembered, that their answers where anonymous, and that there was no implicit ranking of their friends.

III. RESULTING NETWORKS

The introductory course has a duration of nine weeks, but there are only seven networks, four of which are displayed in Figure 2. Due to initial confusion about how to respond, many students did not answer the first two surveys, so they were pooled together, representing course week 1 and 2. Further, due to a technical error, course week 6 data was not recorded. The networks do not seem to indicate a lot of segregation according to grades. The non-passing students (red and orange nodes) seem to move from being well integrated in the network to be placed in the periphery in week 5 and most red nodes are gone in week 9.

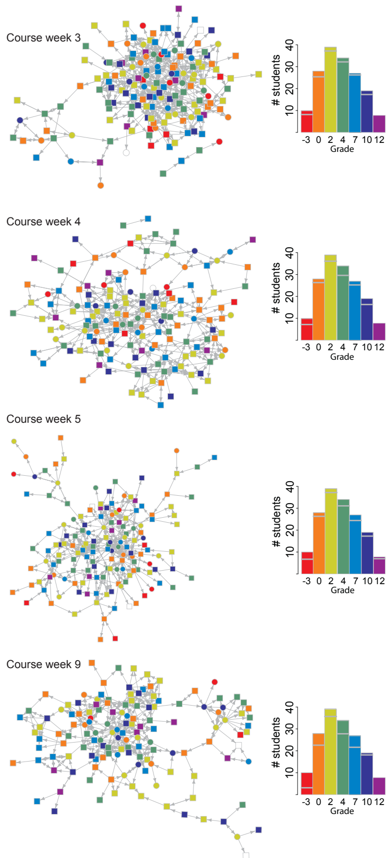


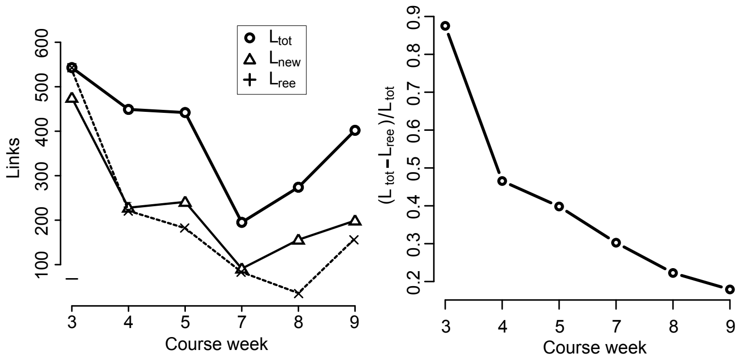
Figure 2: Four networks from different weeks. The density $\rho \approx 0.02$ for each network. Color codes represent end-of-course grades: -3 and 0 are failed. The histograms show how many students with a particular grade are present in the network compared with the total number of students getting a grade. Girls are represented as circles, boys as squares. The total number of of nodes in the networks are 161, 152, 154, and 139 respectively.

This may be explained by the way structure of the course, where students are continuously solving problems and getting grades. Thus, students who do poorly might simply have left the course. On the other hand, they might be pursuing the course but are neither participating in the survey nor being named by others.

Turning to the structure of the networks, they seem evolve from a compact to a more stringy nature. This may signify students tendency to form groups which are connected by bridges [17, 18] as proposed by social network analysis. However, there may also be effects from the survey participation; the number of students naming at least one other student is respectively 124, 115, 98, and 83 for the four networks. This might be due to student fatigue with respect to the survey [10], to an increased workload on the students towards the end of the course, to student drop out or to a combination of the three.

Figure 3(a) show how the total number of links, L_{tot} change from week to week. The dip in course week 7 corresponds to the traditional Fall Break in most Danish educational institutions. However, it is peculiar, since this is an intensive course with no scheduled fall break. However, this would explain both the dip and the slow recovery in course week 8: In week 7 a larger number of students would be absent thus not answering the survey. In week 8 few people would list having had physics interactions with these students.

There are a considerable amount of new links, L_{new}, each week compared with the preceding week. Roughly half of the links each week are new compared to the preceding week. However, the number of re-established links, L_{ree}, comprise a larger and larger fraction of the total. For a week, the number of re-established links are the number of links in the network which are present in one of the preceding networks.



(a) (b)

Figure 3: (a) Number of total, new, and reestablished links. New links, L_{new}, calculated based on the preceding week. L_{ree} is the number of links in a given week which are also present in at least one of the preceding weeks. L_{tot} = 160 for week 2 (not shown). (b) The fraction of completely new links relative to the total number of links, $\frac{L_{tot}-L_{ree}}{L_{tot}}$, seems to decrease exponentially. The number of unique links for all weeks is 1214, which is about 4-6% of the total number of possible links in a directed network with 140-160. This means the the decrease in completely new links is not due to a saturation of the network links.

Together, the variations in L_{new} and L_{ree} may be used to form a hypothesis to explain how bonds are created during the early stages of this particular student network's history: Students try working with a lot of different collaborators. As they do this, they find out who they want to work with and return to them again. This is supported by Figure 3(b), where the fraction of completely new links, $\frac{L_{tot}-L_{ree}}{L_{tot}}$, is shown to decrease over time.

IV. GROUP FLOW

Figure 4 shows the alluvial diagram [14] for student groups in the four networks displayed in Figure 2. The height of each is box proportional to the accumulated PageRank of the group. The color of the boxes mark what range the group mean grade falls in. Though there does not seem to be a connection between group mean grade and accumulated PageRank, the group mean grades seem to become more homogeneously distributed among the groups in the diagram in course week 9 compared with the preceding weeks.

The stream lines between each column indicates shifts in PageRank from one week to the other. Some groups seem stable throughout the course, but many changes happen between weeks. However, there are fewer stream lines between week 5 and 9 (38) than between the other weeks (46 and 51 respectively), which indicates that groups seem to stabilize somewhat over time. However, on a week to week basis groups seem to change a lot, especially in the beginning of the course.

Most boxes have a light and dark shade. The light shade indicates the accumulated PageRank of the students which are not significantly (90% confidence) attached to the group in question as found by a bootstrapping procedure [14]. This indicates that the students which these light shaded boxes represent could not be reliable assigned to the group in question in a bootstrap world of the network.

Network maps are also shown in Figure 4. These map show

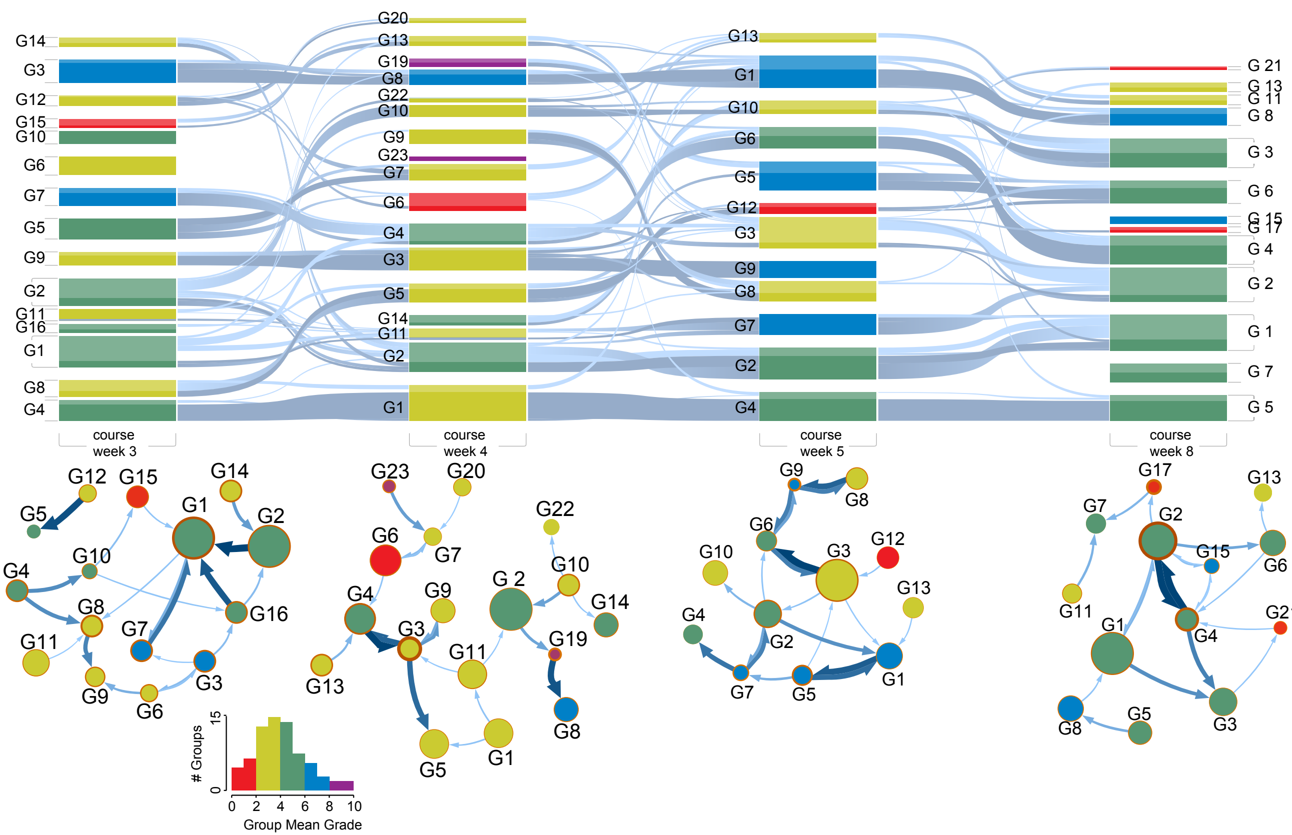


Figure 4: Top: Alluvial diagram for groupings of the four networks displayed in Figure 2. Each column in the diagram account for approximately 80% of the total PageRank corresponding to around 70% of the students in the network for the corresponding course week. The height of a block representing a group is proportional to the accumulated PageRank of the group, and the lighter colors in each group indicate how much PageRank is insignificantly clustered. Finally, the thickness of the gray streamlines between groups in different weeks indicate the initial and final PageRank the nodes making the transition from one group to another. Bottom: Flow maps of the same groups shown in the alluvial diagram. Node sizes are proportional to the number of students in the group. Arrows are proportional to the information flow between groups as calculated by Infomap. The total number of groups each week is 28,28,22,22, respectively. Groups are color coded according to their mean grade. The histogram shows the distribution of grades for all groups.

groups of students as nodes with sizes proportional to the number of students in each group. They are again color coded according to the mean grade of the group, showing that there does not seem to be a clear cut relation between the number of people in the group and the group mean grade.

The arrows indicate probability flow which we can relate to how much students in one group name students in another group. Since these namings indicate communication about problem solving, these arrows might indicate which groups are important for how problem solving behavior is spread in the network. As such they might be used to identify groups to help or influence when teaching introductory physics.

PageRank is an interesting measure for many networks, but here we can note that it would probably be more interesting if the stream lines and height of boxes represented actual students. Then we would be able to see more clearly which students move, what their attributes are, and how this affects group composition. We would also be able to see clearly, which students are difficult to assign into groups. Thus, in future work, it might be beneficial to change the interpretation of sizes in the alluvial diagram.

V. STUDENT SEGREGATION

To quantify, how students with different attributes segregate, assume an attribute with n possible values. For example gender would have $n = 2$. A grouping algorithm now creates a grouping M with N_g different groups. Choosing a node at random in the network, the probability of choosing a girl is $q_{girl} = \frac{N_{girl}}{N}$. However, groups potentially have other distributions of boys and girls, so in the $i'th$ group, $p^i_{girl} = \frac{N^i_{girl}}{m^i_{students}}$. In an information theoretical approach [15], the *surprisal*[19] is $\delta^i_{girl} = log_2(\frac{p^i_{girl}}{q_{girl}})$. For the whole group, our expectation of surprisal for the $i'th$ group is $D^i = p^i_{girl} log_2(\frac{p^i_{girl}}{q_{girl}}) + p^i_{boy} log_2(\frac{p^i_{boy}}{q_{boy}})$. This is an instance of Kullback-Leibler divergence [15]. Treating the groups as independent sub-systems, the total weighted segregation for this particular grouping M is $D(M) = \sum_i \frac{m^i_{students}}{N} D^i$.

In general, an attribute may have more possible values. If a student may only take one of these values, probability distributions $\{p_{ij}\}$ and $\{q_j\}$ can be defined so that:

$$D_{seg}(M) = \frac{1}{N_s} \sum_{i \in M} m_i (\sum_{j=1}^{n} p_{ij} \log_2(\frac{p_{ij}}{q_j})) \quad (1)$$

where $N_s = \sum m_i$ is the total number of students in the grouping. The range of $D_{mix}(M)$ can be estimated as follows: If $p_{ij} = q_j$for all j in all groups m_i, $D^{min}_{seg}(M) = 0$. For perfect segregation, where for each group $p_j = \delta_{lj}$ for the $l'th$ of the n categories the segregation is $D^{max}_{seg} = -\sum_{j=1}^{n} q_j log_2(q_j)$[20].

Here, the segregations according to different attributes are calculated: Gender ($n = 2$), grade ($n = 7$), and lab class ($n = 7$). To see how different the segregations are from a random distribution of gender, grade, or lab class, the attributes are resampled, while keeping the module structure M. It corresponds to changing the distribution, $\{p_{ij}\}$, in each group at random, while keeping the prior distribution, $\{q_i\}$. This resampling is done a number of times (here, 10^4) and each time $D^r_{seg}(M)$, is calculated. Finally,

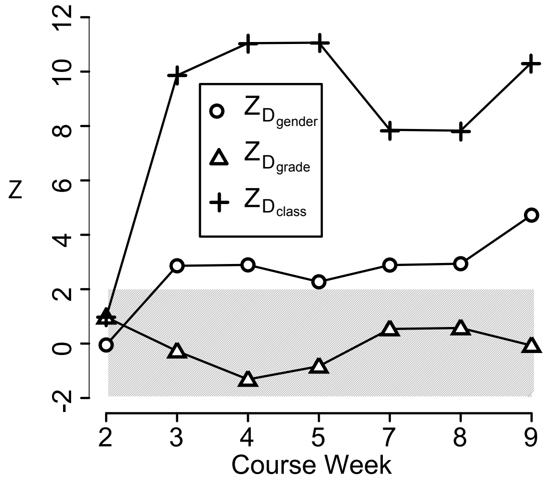


Figure 5: Segregation Z-scores for gender, grade, and lab class for each week. The shaded area indicates the non significant region.

the Z-score, $Z = \frac{D_{seg}(M) - \langle D^r_{seg}(M) \rangle}{\sigma_r}$, is calculated. Thus, the results will show the deviation from random variations.

VI. SEGREGATION RESULTS

The results of the calculations for the whole network segregation from week to week during the course are shown in Figure5 (a). The expected distribution $\{q_i\}$ is calculated from the student present in the network. The first week shows no significant segregation or non-segregation. During the following weeks, students segregate significantly according to lab classes and to a lesser degree according to gender.

While there is significant segregation according to gender and lab class , students do not segregate or mix near perfectly. If students segregated perfectly, calculations show that the Z-scores would be around 20 for gender and around 40 for grade and lab class. If they did not segregate at all, that is if $D_{seg} = 0$ corresponding to perfect mixing, the Z-scores would be around -2 for gender and -4 for grade and lab class. Thus, groups do not consist for example of students from only one lab class but of clusters of students from different classes.

VII. CONCLUSIONS

This study examined the early stages of network formation based on student reports of who they remember having communicated with about problem solving in physics. Seven networks made from weekly reports these types of communication in an introductory physics course were analyzed. In these networks less and less low performing students are represented, but the remaining students do not to segregate according to end-of-course grade.

The link analysis showed that roughly half of the links in a week were new compared with the preceding week. However, as the weeks go by, students communicate with former communication partners, which is indicated by the relative decrease in total links. Group flow patterns were were examined. Stream lines in alluvial diagrams show that while some groups seem stable throughout the different weeks, students seem to flow extensively between groups. Also, the alluvial diagram revealed that many students could not be significantly clustered together in groups. Finally, student segregation was analyzed using a divergence measure, called the segregation, which was applied to groups found with the Infomap grouping algorithm. This analysis showed that students segregate significantly compared to according to lab class number and to lesser extent gender, but not according to grade. It was also shown how individual groups found by Infomap could be analyzed.

The overall picture painted by these analyzes is one where students try out many different possibilities for collaboration the first weeks but gradually settles to communicate with the same people. Some find study partners based on lab/problem solving classes and to some extend gender, with which they continuously collaborate or reconnect with during the course. However, it is generally difficult for infomap to find assign these students to only one group. Further research could use the results from the link analysis to constrain models of network development. Another direction would be to further investigate Infomap groupings. Since Infomap yields information about which alternative groups it could find using bootstrap worlds, these groups could also be investigated with the segregation measure. As a final note, these results also have value for physics education research.

ACKNOWLEDGMENTS

I would like to thank Professor Ian Bearden from the Niels Bohr Institute for his tireless work during the period of data collection. Without this work, there would be no network data. Also, thanks to Professor Kim Sneppen for useful suggestions and discussions.

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[20] $D_{nomix}(M) = \frac{1}{N_s}\sum_{i\in M}\sum_{l=1}^{n} m_i^l \log(q_l^{-1})$. Adding groups with same l yields $D_{nomix}(M) = \frac{1}{N_s}\sum_{l=1}^{n} m_l \log(q_l^{-1})$. Finally use $q_l = \frac{m_l}{N_s}$.

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Appendix 3 - Talking and learning physics: Predicting future grades from network measures and FCI pre-test scores

Talking and learning physics: Predicting future grades from network measures and FCI pre-test scores

Jesper Bruun & Eric Brewe

The role of student interactions in learning situations is a foundation of socio-cultural learning theory, and Social Network Analysis can be used to quantify student relations. We discuss how self-reported student interactions can be viewed as processes of meaning making. Subsequently, we interpret three network centrality measures *Hide*, *Target Entropy*, and *PageRank* in terms of interactions among enrolled students in an introductory mechanics course at the University of Copenhagen. We calculate network centrality measures on self-reported student interaction networks in three categories *Problem Solving* (PS), *Concept Discussion* (CD) and *In Class Social communication* (ICS) in the introductory mechanics course. Correlating these measures with the sum of grades of two subsequent courses, we find that significant correlation patterns of the PS and CD networks can be matched with significant correlation patterns in the ICS network. Using hierarchical linear regression, we find that a linear model which adds the network measures *hide* and *target entropy* calculated on the ICS network significantly improves the base model using only the FCI pre-test scores from the beginning of the semester. Though, we cannot infer causality from these data, our results show that social interactions in class are intertwined with academic interactions, which we interpret as a natural part of learning physics.

I. INTRODUCTION

The role of social interactions in learning is well established and has been a foundation of learning theory [1–3]. Within Physics Education Research (PER), studies using Network Analysis of student interactions in the learning of physics are emerging [4, 5]. We continue this vein of work, and extend it by using self-reported student interactions as one variable in a model to predict students' grades at the University of Copenhagen in a subsequent block of their physics and mathematics course. In order to model students' grades in the block of physics and mathematics courses we surveyed students at the University of Copenhagen weekly, asking about their interactions in three areas, *Problem Solving*, *Conceptual Discussion*, and *In Class Social interactions*. In order to analyze these networked data, we need to conceptualize the underlying assumptions about networks and how these data can be interpreted. Our analysis then utilizes centrality measures, which describe the position a student occupies within a network, to predict students' grades in university physics courses. We argue that this networked perspective has unique potential for quantitative analysis of learning as a social phenomena.

This study of a Danish introductory physics course at the University of Copenhagen was undertaken to investigate the roles social interactions play in learning physics. This study employs Network Analysis, which has been used in PER to show the growth of student-student interactions in Modeling Instruction [6], and to identify the roles that gender and ethnicity play in an informal network of students in a Physics Learning Center [5]. Bodin recently used Network Analysis to document changes in student epistemic networks after engaging in the numerical modeling of a physics problem [4]. This study differs from the others in that it uses Network Analysis to predict students grades within a subsequent course.

Network Analysis is a collection of analytic techniques which can be used to visualize, quantify and test hypotheses based on relationships between entities within a network. Networks are composed of nodes, links and the attributes of the nodes. In our study, the students in the introductory physics classes are the social entities we are studying, thus they are the nodes within the network. These students have attributes, or individual characteristics which we can use to help describe the entities. The students have been been given a weekly eleven question survey that asked them to select the names of students with whom they interacted in a variety of contexts. The responses to these questions indicate an interaction between students, which in this study is the linkage between the nodes.We collected attribute data including Force Concept Inventory Pre scores and grades in the first block of the introductory physics course. The collection of nodes, links and the attributes of the nodes are the constituent parts of any network, thus in our study the students, along with the interactions they reported and the attribute data we collected constitute the networks.

Network perspectives on data are unique in that data is primarily relational, thus we conceptualize students as situated within the learning context which includes physics classes but also other interactional and social settings. As network analysis is relatively new in physics education research, it is worthwhile to consider how data are collected, and how we interpret the meaning of these data. Further, the interpretation of the meaning of a link is an ongoing debate among Social Network Analysis researchers [7]. In the following sections we describe two theoretical perspectives which are employed during the interpretation of the data we have collected.

II. THEORETICAL FRAMEWORK

We are undertaking an analysis of learning that is dependent on the interactions of individuals within a network of learners. Underlying this analysis is a situated or participationist framework of learning [8, 9]. This theoretical framework avers that learning is an ongoing process of transforming participation [10–12] , and has roots in Vygotskian theory [1]. Vygotskian theory holds that learning and development, especially development of language, are so interrelated that they should not be considered independently. Thus, studying learning should be a study of the interactions a student is involved in. While Vygotsky primarily used clinical interviews or student-teacher interactions, others extended the analysis to interactions in naturalistic settings to help account for the contextual or situated nature of interactions [13, 14]. Rogoff, Matusov and White [12] describe the central premise of the participationist perspective on learning as, "the idea that learning and development occur as people participate in the sociocultural activities of their community, transforming their understanding, roles and responsibilities as they participate." The participationist perspective on learning is particularly salient to this study as we are investigating how the interactions that students engage in within and around the physics class are predictors of their success in a subsequent pair of classes. Although we do not know the exact content or nature of the interactions, we presume that the conversations are not strictly about physics, but enrollment in an introductory physics course is the feature that is common across these students, so we also assume that physics is the subject of some of the interactions.

III. THEORY: NETWORK MEANING AND STRUCTURE

The analysis of social networks developed from quantitative sociology beginning in the 1920s [15] as a way of investigating the structure within social groups. Two primary approaches to analyzing social networks have been used. The first is to look at the overall structure of a social network, for example investigating what groups and communities the network reveal [7]. We have taken a second approach which is to look at the structure of interactions within a network, focusing not on the network but on the position of entities within the network. This approach, focusing on the entities and their positions, has been used by Brewe et al. [5], to investigate participation in a network of physics learners, by Dawson [16] to investigate sense of community. One primary approach to investigating the position of entities within a network is to investigate the positional advantage or constraint that an entity experiences due to that entity's *centrality* within the network; this is done by quantifying the adjacency of that entity within the network. Common adjacency-based centrality measures include degree and closeness which we utilize in our analysis and describe in detail in section III A. Other centrality measures are probabilistic, but still describe the relative importance of an actor within a network. We utilize three such centrality measures, PageRank, Target Entropy and Hide, which we describe in section III B.

Network Analysis is built on a basic assumption that interconnected entities influence each other. The interpretation of any network measure or property rests on how we conceptualize the meaning of links and nodes. This basic understanding of a network shapes how we interpret centrality measures, node attributes, and correlations between them. In Social Network Analysis a flow metaphor and a girder metaphor describe two ways of viewing networks [7]. In the flow metaphor, links are viewed as pipes through which resources can flow. In the girder metaphor, links are relations that tie nodes together, "creating a structure ... around which the rest of the social system is draped" [7, p. 45]. These two metaphors are also recognized in network physics as links representing movement patterns and links as pairwise relationships Rosvall and Bergstrom [17].

As an alternative to the flow and the girder (relational) metaphors, McCormick et. al introduce the notion of a link representing a process of meaning making[18] [3, p. 115], and this interpretation best applies to our study. We asked students to recall interactions, so links do not necessarily represent any flow of resources or information; instead, the links represent that the student has identified a relationship with another student. Relations tell us about the choices students make and thus give us insight into the interactions students value and remember [19]. In this process of meaning making, advice and ideas flow back and forth between students. If a student *explains* physics to another student, from a constructivist perspective that information is not simply transferred. Instead, a learner of physics will always construct, refine, or retain his/her own knowledge based on the interaction. Thus, we consider links to be interactions, during which students exchange and modify their ideas, and so we adopt McCormick et al's view of a network as a vehicle for meaning making.

A. Combinatoric binary centrality measures

Network theory offers many different centrality measures, all of which will have different interpretations in our framework. In this section we briefly explain the combinatoric binary centrality measures we use in this study. The measures are combinatoric, because they rely on counting the number of ways nodes are connected, and they are binary, because a link either exists or it does not, and therefore link weights are disregarded. Throughout this section we refer to Figure 1to illustrate the different centrality measures. In the next section, we will describe and interpret probabilistic measures, which do take weights into account.

1. Degree

The degree of a node is defined as the number of other nodes that are connected to it. For example, the degree of node C in Figure 1 is 3. Often degree is split into indegree and outdegree. Indegree is the number of links coming into a node and outdegree is the number of links going out of a node. Node C has an indegree of 3 and an outdegree of 2.

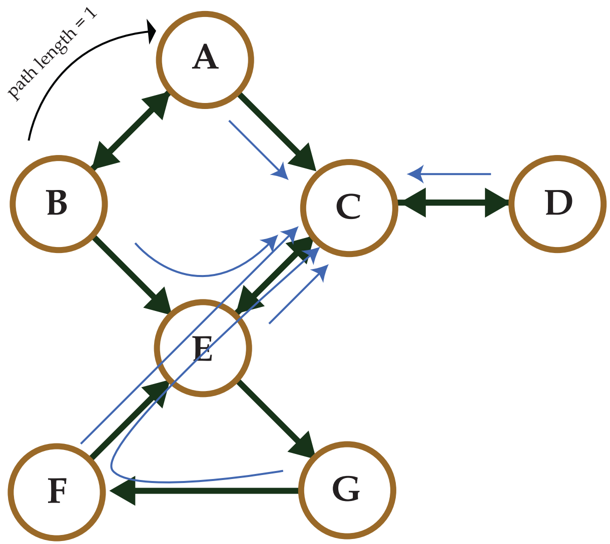

Figure 1. Example binary network for use in Section III A. All links are directional, and thinner,blue arrows show the geodesic distance from all other nodes to C.

2. *Geodesic Distance*

The *geodesic distance* is not used in this study directly, but is used in the calculation of closeness. Geodesic distance is the shortest path between any two nodes. In Figure 1, the geodesic distance between nodes B and C is 2, because no matter whether the path is B-A-C or B-E-C there are two links between B and C. If no shortest path exists, the geodesic distance is the size of the network, $d_{li} = N$ [20].

3. *Closeness*

The closeness, or closeness centrality, of node i is defined as the inverse of the average geodesic distance from i to every other node in the network [19, 21]. In directed networks, in-closeness considers geodesics which end at i, whereas out-closeness considers geodesics starting at i. The formal definition of in-closeness, shown in Eq. 1 is:

$$C^i_{in} = \frac{N-1}{\sum_{l\neq i} d_{li}} \tag{1}$$

Thus, a high incloseness centrality means that the node is reachable in few steps. This implies that if an idea exists somewhere in the network, some version of it might reach a node with high incloseness after a few interactions. So students with high incloseness may have greater access to ideas, while students with high outcloseness may have greater influence. As an example, we calculate the closeness of node C in Figure 1. Three paths have $d = 1$, two paths have $d = 2$, and one path has $d = 3$. The average geodesic distance from any given node in the network to C is $\langle d\rangle = \frac{5}{3}$, which means that the closeness of C is $\frac{3}{5}$, $C = \frac{6}{1+1+1+2+2+3} = \frac{3}{5}$. While closeness gives a good estimate of how easily a student in our network can be reached, it does not address the weights of any link. In order to handle weighted links, we turn to measures of centrality based on probabilities.

B. Probabilistic centrality measures

When we sum up the number of interactions students reported during different weeks (see section IV A 2 on page 11), it becomes possible that student i mentioned student j more than one time. As a result, the link between student i and student j becomes weighted. The weighting of links between students leads to probabilistic interpretations of centrality and thus to more sophisticated calculations of centrality. To interpret probabilistic network measures, we have introduced a hypothetical *student asker*. [22] We imagine the student asker starting at a randomly selected student in our network. He asks this student to name one other student. The basic mechanism is now that the student asker goes to the named student and repeats the process. Thus, the frequency of the student asker showing up at any node in the network is based on the responses students give during data collection. The results may not resemble what the students actually would say in a real situation, but the notion of a student asker is useful as a tool for explaining and giving meaning to different centrality measures. As we explain different centrality measures, we will add additional rules for the student asker.

1. *Strength*

In networks such as ours where student A can mention student C in multiple weeks, the links between nodes can be weighted (see section IV A 2 on page 11). Strength is a measure of the weight of the link and is similar to the degree of a node. Often two types of strength are considered, instrength and outstrength. Instrength is the sum of the weighted links coming into a node and outstrength is the sum of the weighted links going out from a node. In Fig. 2, the instrength of C is 4, while the outstrength is 4. From the perspective of the student asker, the ratio of the instrength of a link to the outstrength gives the probability for i naming j when asked for a name by the hypothetical student asker. That is, $p(j|i) = \frac{w_{ij}}{s^i_{out}}$. In Fig.2, $p(C \mid A) = \frac{2}{3}$. This is the probability of getting the name C when we are asking A.

2. *PageRank*

PageRank is an algorithm conveying the general idea that a node gains importance with the links *to* the node and distributes importance with the links *from* the node. Thus PageRank incorporates both local connections and global position; it pays to have heavy weight links to important people.

To calculate the PageRank, we let the student asker move about the network based on the probabilities associated with each link. Thus, in Figure 2, if the student asker is at C,

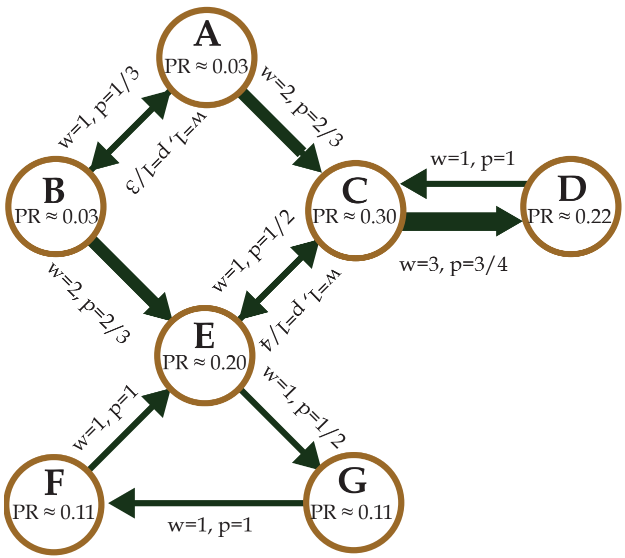

Figure 2. Example weighted network illustrating PageRank and Strength. In this diagram each link shows the weight and the probability of following the link. Inside each node, we show the PageRank, PR, based on a jumping probability of 15%.

he will to D with a probability $p = \frac{3}{4}$ and to E with $p = \frac{1}{4}$. In general, when he asks student i for a name he gets the name of student j with a probability of $p(i \mid j) = \frac{w_{ji}}{s_j^{out}}$.

The asker keeps asking and going around the network many times. The percentage of time he spends at node i is the PageRank of node i, PR_i. In order to avoid the student asker getting 'stuck' in a loop, Brin and Page [23, 24] introduced a jumping probability, $1 - \alpha$. Before asking the next student, the asker first determines whether to jump or to follow links. If he jumps, he chooses a student at random to ask. Otherwise he asks the next student, gets a name of a neighboring student, and follows the link to that student. With the introduction of the jumping probability, the student asker does not get stuck, and we can calculate the PageRank of each node in a fair manner. The PageRank rises with the number and weight of incoming links, and each student adds to neighbors' PageRank by naming them; a node gains importance with the links *to* the node and distributes importance with the links *from* the node.

The mathematical definition is:

$$PR_i = \frac{1-\alpha}{N} + \alpha \sum_{j \in k_{in}} p(i \mid j) PR_j \tag{2}$$

Here, α is the probability that the asker uses a link to navigate the network, PR_j is the PageRank of node j, and $p(i \mid j)$ is the probability of j naming i if asked. In the original paper by Brin and Page [23] , $p(i \mid j) = 1/L(j)$, where $L(j)$ is the number of links on web-page j, here, $p(i \mid j) = \frac{w_{ji}}{s_j^{out}}$.

To exemplify the PageRank we assume a jump probability of 15% in Figure 2.Students A and B have the lowest PageRank (~ 0.03), while C has the highest (~ 0.3). Node E has a higher degree than node D, but, because of the strong link $C \rightarrow D$, D (~ 0.22) inherits enough of C's PageRank to slightly outrank E (~ 0.20). Thus PageRank includes not only the centrality of a single node, but also the centrality of the nodes that are directly adjacent to the node.

3. *Target Entropy*

Target entropy is a centrality measure based on the flow of information through a network. Target entropy gauges the predictability of the traffic around a node in a network [25] based on the assumption that nodes produce and disperse messages, which is consistent with a flow picture of the network. Thus, the target entropy is a centrality measure that presumes that when messages pass through a node that node is more 'important'. The formal definition is [21, 25, 26]

$$T_i = -\sum_{j \in k_{in}} c_{ij} log(c_{ij}). \tag{3}$$

In equation3, the term c_{ij} is the number of 'messages' targeted at i though node j divided by the total number of 'messages' which can reach node . Imagine that each node in the network sends one message. Then for node i, the term c_{ij} is calculated by summing the number of messages incoming to a node i (through node j) and dividing by the number of nodes that are connected to i via geodesics, which

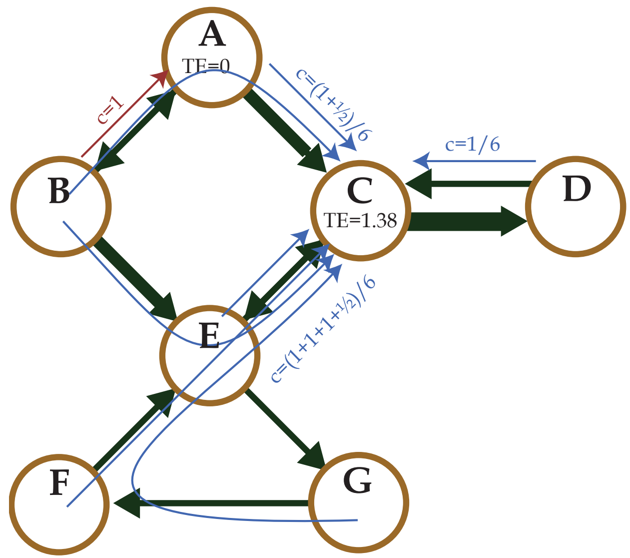

Figure 3. Example weighted network illustrating Target Entropy. The blue lines indicate the shortest path between all nodes and node C.

is denoted M_i. If, as is often the case, a message can arrive at node i from multiple shortest paths (g_p), it is said to have a degeneracy, and then the weight of each path is $\frac{1}{g_p}$, thus the term c_{ij} is:

$$c_{ij} = \frac{\sum_p \frac{1}{g_p}}{M_i},$$

where p is summed over all the paths which reach i though j. In Figure 3, the target entropy of node A is $T_A = 0$, because the number of messages that end at node A is 1, and the number of 'messengers' connected to node A is also 1. For node C, messages arrive from adjacent students A, D, and E. However, the messages could have originated from any of the 6 other students, thus the term c_{CX} varies. This means that $c_{CE} = \frac{1+1+1+\frac{1}{2}}{6} = \frac{7}{12}$, $c_{CD} = \frac{1}{6}$, and $c_{CA} = \frac{1+\frac{1}{2}}{6} = \frac{1}{4}$ and the total target entropy for node C is, $T_C = 1.38$.

Since we employ an interaction picture rather than a flow of information picture, we need to reinterpret the c_{ij}'s. Interacting with many different people increases target entropy. The target entropy will be further increased if they in turn interact with many people who remember them. Thus, if a student is part of a network neighborhood where people are involved in many interactions and remember their interactions, that student will end up with a high target entropy.

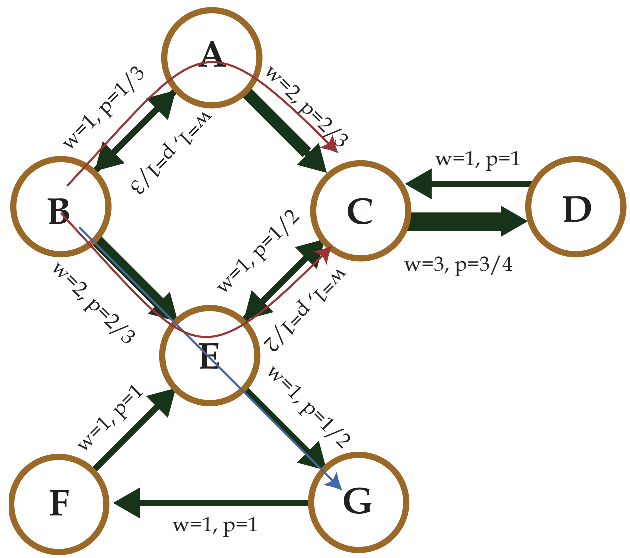

Figure 4. Example weighted network illustrating Search Information. Following the blue arrow, the search information from B to G is $-log(\frac{2}{3}\frac{1}{2}) = \log(3)$. Following the two red arrows, we get $S_{B\to C} = -\log_2(\frac{1}{3}\frac{2}{3} + \frac{2}{3}\frac{1}{2}) = \log(\frac{9}{5})$.

4. *Search Information*

While the PageRank views centrality from the whole network perspective, and target entropy views centrality from a node perspective. Search Information and Hide, a variant on Search Information, describe how easily a node can be found within a network. In our study, we do not utilize search information directly, but instead use a centrality measure, *Hide*, that depends on the calculation of search information. In general, the search information from a node i to a node j is (adapted from [25]):

$$S_{i\to j} = -log_2 \left(\sum_{sp} \left[\prod \frac{w_{lk}}{s_l^{out}} \right] \right).$$

The calculation and interpretation of search information is best accomplished based on a student asker. In Figure 4, imagine the student asker asking his way from student B to student G, separated by student E. Disregarding the weights for a moment, B and E each have two outgoing connections. Only one of B's connections end up at E and only one of E's connections end up at G. If both A and C mention a name at random, then the probability for the student asker ending at B from A is $p = \frac{1}{2^2}$. Instead of choosing randomly, the student asker now wishes to ask if he is on the right path if he chooses a given node. If we assume that B and E both know the route to G from their respective positions, then the student asker can be sure to reach G with $-log_2(p) = 2$ questions. This is the information cost for reaching G from B seen from the student asker, if we disregard the weights. Including weights, the probability of taking the correct path from B is changed to $\frac{2}{3}$, yielding a probability of $p = \frac{1}{3}$, and a corresponding information cost of $\log_2(3)$. Of course, if more than one path exists from one node to another, each contributes to the probability, making the information cost lower.

5. *Hide*

Hide describes, on average, how many steps the student asker must take to find student i from all other nodes. It is defined by the equation:

$$H_i = \frac{1}{N-1} \sum_{j \in G} S_{j\to i}. \tag{4}$$

In the interaction picture the interpretation of hide is straight forward. With a high hide, a student has not participated in many interactions which other students remember. The interactions the first student has participated in may only be one out of many interactions, while students with a low hide have participated in so many meaningful interactions that they are easy to find.

IV. METHODS

This study took place at University of Copenhagen with approximately 170 students taking their first physics courses. The semester at this university is split in two blocks. In the first block students take two courses, an introductory mechanics course and an introductory mathematics course. In the second block, students take another two courses, an advanced mechanics course named Rotations and Oscillations and a linear algebra course. The physics course in each block includes two weekly lectures with all 170 students, two weekly problem solving sessions and one weekly lab session. Students were broken into seven cohorts of 30 students and took the problem solving sessions and lab sessions with the same cohort. Six of the seven cohorts of students are physics majors, and the seventh is a cohort of math majors. The majority of students complete both blocks of the introductory physics course. FCI and network data were collected during the first block. These data were used to predict the sum of the math and physics grades in the second block.

A. Data Collection

The data collected for this study include student surveys as well as attribute data for individual students. Figure 5summarizes the timeline of data collection across the two blocks of the semester.

1. *Survey data*

Each week during the first block, students were asked to answer two online network surveys. Both surveys were administered during the lab section of the course. In each survey, students were allowed to select names from a dropdown menu which included the names of all students enrolled in the physics course. There was no limit on the number of names that could be selected. All surveys were administered in Danish, so questions have been translated for this paper. Three questions were selected for this study and were part of a larger survey. In Table I, questions 1.1 and 1.2 probe for problem solving and conceptual discussion interactions respectively, while question 2.1 probes for social interactions. We focus on student interactions with regards to *Problem Solving*, discussion of physics concepts, social interactions during classes, and social interactions outside of classes. The two first interaction networks, *Problem Solving* and *Concept Discussion*, reflect academic student interactions which are related to success in physics courses, while the third reflects social engagement in physics classes **?**]. The student interactions in each network form the basis for calculating different centrality measures. It should be noted that because these are self-reported interactions, it is possible that students were biased toward listing their friendsLiben-Nowell [27], Liljeros et al. [28], Marsden [29]. Three efforts to reduce this bias were used: first, the surveys were anonymous for everybody but the researchers, second, the questions do not imply that interactions should be ranked, and third, the formulation of the questions do not imply one student seeking advice with another. See Table I.

2. *Transforming survey data to networks*

The survey data were used to construct three networks each week, one based on the responses to each question 1.1, 1.2 2.1. For each week we construct a *Problem Solving* net-

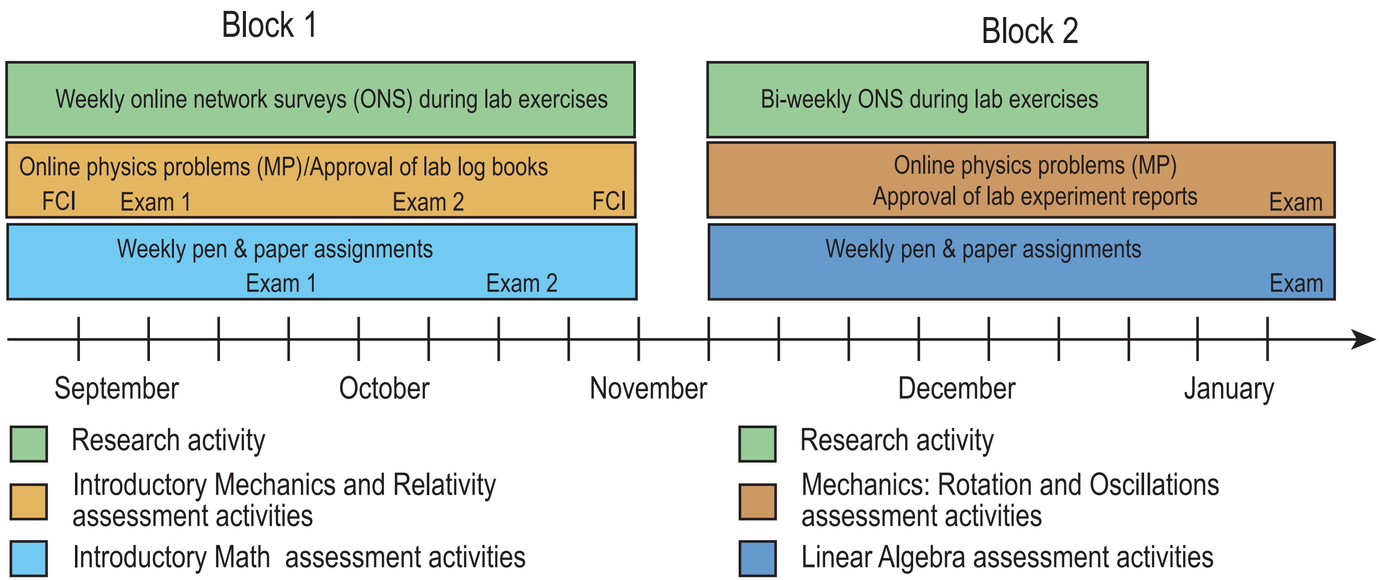

Figure 5. Timeline of assessment activities during the first two blocks along with the research activity relevant to this study. The FCI assessments and the approvals of lab log books/reports are not part of the grade.

work, a *Concept Discussion* network, an in-class discussion network. The network for a single week is directed and unweighted. The direction of a link, l_{ij}, is from student i to student j, if i names j. It is unweighted, meaning that all existing links have weight $l_{ij} = 1$. In order to make data analysis manageable, the weekly networks were summed to create three networks that include the responses from all weeks. This means that the links between students are now

Survey	Category	Question (translated from Danish)
1.1	***Problem Solving***	We communicated about how to solve a task in physics. (How to perform calculations, what formulas are needed, how to read graphs and the like).
1.2	***Concept Discussion***	We communicated about understanding one or more physics concepts. (What current is, what the normal force is, how radioactivity works, and the like).
2.1	***In Class Social***	We communicated socially in connection with a lecture, problem solving session, or lab exercise.

Table I. Survey categories for naming. Each week, students chose who they have had interactions with in each category. Categories in bold are the ones we work with in this study.

weighted links with the maximum weight between node i and node j in any network weight $l_{ij} = 9$. Because some students did not attend every lab meeting, they did not complete the survey each week and thus not all students are included in each week. By summing the weekly networks we have created what we feel is a more stable and representative network.

B. Attribute data

Though networked perspectives on data assume that the data is relational and interdependent, each actor within a network has characteristics, or attributes, which are independent of the other actors. These can be used in conjunction with networked measures to predict outcomes. In this study we used two attributes, pre-instruction Force Concept Inventory scores (FCI_{Pre}) and physics grades in the first course block (GR_{Mech1}). These attributes have been shown to be predictive of student grades in physics [30]. These data were provided by the physics department.

1. *Force Concept Inventory Pre Score*

The Force Concept Inventory [31] was administered as a pre/post test in the first block of the course. We used the FCI pre score attained during the first week of the first block of the course as a predictive measure in the sum of grades in the second block of the course.

2. Grades

In the Danish system, grades are numeric values which range from -3 to 12. We use the numeric grades in two ways. First, we use the mechanics grade in the first block as an attribute which is used as a baseline predictor in the linear modeling component of the study; the possible range is [-3,12]. Second, we use the sum of the physics and math grades (SoG) in the second block as the dependent variable in the linear modeling part of the study. Because this is a sum of two grades, the possible range for this grade is $[-6, 24]$. The Sum of Grades variable is only available for students enrolled in both the Rotations and Oscillations course and the Linear Algebra Course.

C. Summary of variables

This study made use of a large number of variables, which include centrality measures, attributes, as well as the dependent variable, grades in the second block. Table IIsummarizes the variables used in this study.

D. Correlations

The initial analysis of each network was to create a *Pearson's r* correlation matrix including each of the centrality measures, the attributes, and the outcome variable, sum of grades in the second block. Initially, these correlations were carried out on all 12 networks, but the three networks for *Problem Solving* (PS), *Concept Discussion* (CD), and *In Class Social* (ICS) communication showed the highest correlations with network measures. Because not all students participated in each network, each of the three networks include differing numbers of students. Further, students who were named in a category at some point during the two blocks of data collection, but did not name any other student during the first block were removed from the analysis. The number of removed students where $N_{PS} = 25$, $N_{CD} = 43$, and $N_{ICS} = 40$. All analyses were carried out in the R environment primarily with the iGraph package [20] for analyzing network data. As with all network statistics, the independence of measures assumption was violated and thus all statistics were calculated using bootstrap methods, which are incorporated in the boot package of R [32]. These correlation matrices were used to help identify centrality measures which held the greatest promise for predicting SoG, so all correlations with $p \geq 0.001$ were eliminated. This left five centrality measures as candidates to construct linear models for predicting SoG in the second block.

E. Hierarchical multiple regression on sum of grades.

We used hierarchical multiple regression to create models with the Sum of Grades SoG in the second block as the

Centrality Measures	Symbol	Attributes	Symbol	Grade Variable	Symbol
Instrength	s_{in}	FCI Pre	FCI_{Pre}	Sum of Grades in Block 2	SoG
Indegree	k_{in}	Mechanics Grade in Block 1	GR_{Mech1}		
PageRank	PR				
Hide	$\mathcal{H}$				
Target Entropy	T				

Table II. Summary of centrality measures, attributes and grade variables used in this study

dependent variable. The centrality measures and the attributes serve as independent variables within each of the three networks considered. Hierarchical multiple regression is a regression technique used to generate and compare the predictive models for a continuous dependent variable using different sets of independent variables. Typically, independent variables are entered in a specific order according to logical or theoretical considerations. In this study we entered variables in order of decreasing correlation with the SoG variable. A new predictive model is created with each new independent variable (or pair of variables) entered. These models are then compared using an F-test for the differences between correlations as described by Tabachnick and Fidell [33]. Again, the analyses were carried out in the R statistical programming language. In addition, all models were compared with the a regression model using the mechanics grade in block 1. This comparison provided us a baseline which is interpreted as , “Does the regression model using centrality measures do as good of a job predicting the SoG as the grades in a previous block of the course?”

V. RESULTS: CENTRALITIES AND FUTURE GRADES

The results can be grouped in three sections. Global network features give a first impression of the data, and show some of the differences between the three networks . The second section shows correlations between attributes, centrality measures and the grades from the first block. In the third section, we evaluate the the predictive power of different linear combinations of attributes and centrality measures.

A. Global network features

Figure 6 shows sociograms of the three different networks. The layout has been determined by applying a force based plotting algorithm [34] to the *Problem Solving* (PS) network. This network contains the largest number of students in our study ($N_{PS} = 152$). In the subsequent networks, the positions of nodes remain constant. The *Concept Discussion* (CD) and *In Class Social* (ICS) networks have fewer nodes than the PS network ($N_{CD} = 134, N_{ICS} = 136$), since not all students answered the questions pertaining to these two networks. The sizes of nodes are proportional to the centrality measure showing the highest correlation with the Sum of Grades (SoG). We represent the SoG with colors; the low achievers are red, high achievers are dark green. White nodes represent students for whom we do not have SoG data. The link weights are represented with a gray scaling, where weaker links are lighter and thinner than stronger links. The maximum weight is 9, corresponding to a student naming another student for the total 9 weeks. The insets are link weight distributions, on a semi logarithmic scale.

Looking at the PS network which is the basis for our three plots, we notice that high achieving students (dark green and light green nodes) tend to be nested within inner clusters, while low achievers (orange and red) tend to be in the periphery. While it is difficult to see differences in PageRank in the PS network, we notice that in the CD and ICS networks, the green nodes tend to be larger than the red and orange nodes. This mean that there is a tendency for nodes with high Target Entropy to be high achievers. In each of the three sociograms an almost separate cluster of white nodes exists at the top left. This cluster of nodes represents students in the cohort of math majors, who did not enroll in the second block of physics, so a SoG is not available. The cluster of math students are separate, with few ties linking them to the main cluster, indicating that they mostly did not interact with the physics students.

B. Centrality and attribute correlations

With two different attribute variables, five centralities, and one outcome variable for each network, we have 84 possible correlations. In order to show the correlations graphically, identify significant correlations, and report differing degrees of freedom with each variable, we use a correlation network [34]. Correlation networks are diagrams which show the correlations between various measures. In a correlation network two measures are connected if they are correlated and the strength of the link is proportional to the strength of the correlation. Figure 7 shows the correlation network for the measures in our study, picking out only significant correlations. The measures for each network are color coded, and the value in each node represents the number of degrees of freedom in the correlation. The value of the correlation coefficients are represented on the links. We have adjusted the lay out manually, so for each variable the distance from the measures to the SoG is ranked. This means that since Gr_{Mech1} shows the highest correlation in each of the three distinct networks, it is closest to the SoG in the drawing.

The Gr_{Mech1} score correlates with the SoG with $r \approx 0.6$, reflecting that performance in the first block is a strong predictor of performance in the classes in the subsequent block. The FCI pre-test score, (FCI_{Pre}), correlates with the SoG with a coefficient of $r \approx 0.42$. In each network,FCI_{Pre} also correlates with the Gr_{Mech1} score, which is no surprise.

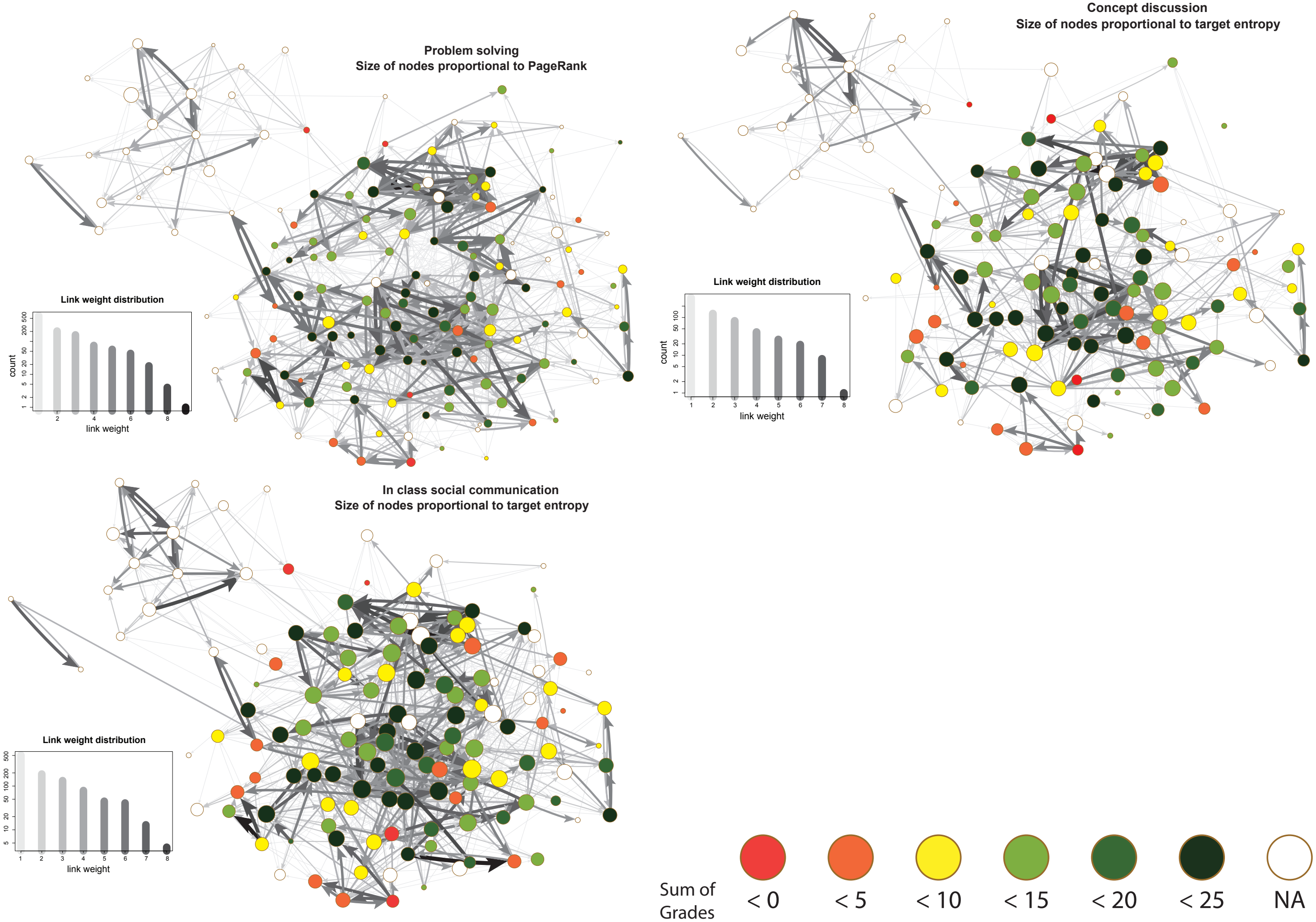

Figure 6. Sociograms of the three networks with insets of link weight distributions. Links are color coded according to strength, with light grey representing weak links and darker grey representing increasingly stronger links. Nodes are color coded according to the Sum of Grades as indicated at the bottom. The size of nodes are proportional to the centrality measure with highest correlation with the sum of grades.

1. Problem Solving Network Correlations

In the PS network, only two network measures, PageRank and Hide, correlated significantly with SoG. The Hide, ($\mathcal{H}$), correlates negatively with the SoG, with a coefficient $r = -0.32$ indicating that the more difficult a student is to find within the *Problem Solving* network the lower the grades in the subsequent block of courses. The PageRank, (PR), correlates with a high-level of significance with both the SoG and with the Gr_{Mech1}, indicating that students who are more central in the *Problem Solving* network also tend to have higher grades in the second block of courses.

2. Concept Discussion Network Correlations

In the CD network, three network measures, Indegree, Instrength and Target Entropy, correlate with SoG. One feature we find interesting is that in the Concept Discussion network, two centrality measures, Instrength and Indegree, are both about how others view the student. This indicates that a student that is memorable to other students tends to have a higher grade in subsequent courses. Finally, we note that the target entropy has a slightly higher correlation with the SoG ($r = 0.45$) than does FCI_{Pre} ($r = 0.42$).

3. In Class Social Network

In the ICS Network, PageRank, Instrength, Indegree, and Target Entropy correlate with the SoG with relatively similar strength. ($r = 0.35 - 0.38$). The ICS network has similar centrality features as the other networks: $\mathcal{H}$ has a negative correlation with SoG, and again the Instrength and Indegree are positively correlated with SoG. One interpretation of this is that students who are identified by other students as participating in social interactions in the classroom setting tend to earn higher grades and students who are not easily found engaging in social interactions within the *In Class Social* network tend to earn lower grades.

C. Hierarchical Multiple Regression Models

In each of the three networks, the GR_{Mech1} score shows the highest correlation with SoG. This result is unsurprising, as grade in previous physics course is an excellent predictor of grade in a subsequent physics course. Thus we created models using GR_{Mech1} as a benchmark, and we search for models using networked variables with equivalent predictive power. Since most of the measures correlate significantly with the GR_{Mech1} attribute, we do not expect that adding centrality measures or FCI test-scores to a GR_{Mech1} model will increase it's predictive power significantly. Instead, we use hierarchical multiple regression [35]. The results, including R^2, F-tests, and Δ-values are listed in Tables

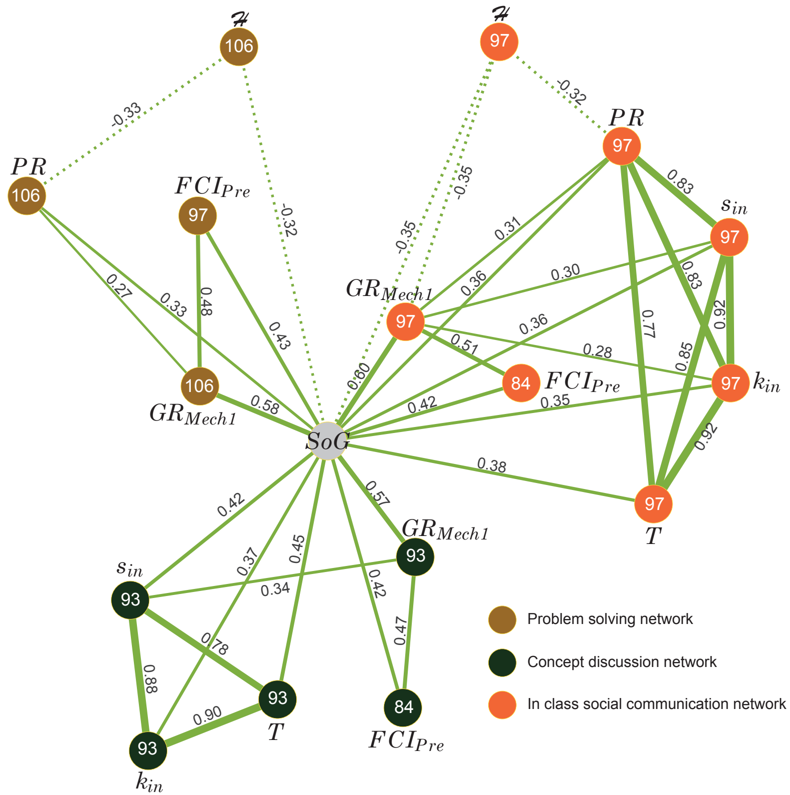

Figure 7. Correlation network of measures that correlate significantly with SoG ($p < 0.001$). Correlation coefficients are included on the links, the number of degrees of freedom are shown in the nodes, and the layout is determined by strength of correlation with SoG.

III, IV, and V.

1. *Problem Solving Network Modeling*

The attribute, FCI_{Pre} correlated most highly with the SoG in the *Problem Solving* network, so FCI_{Pre} was used to create the base model. As seen in Table III this base model was significantly different than a constant only model. When PageRank was added and a new model was created, an F-test indicated that this model was significantly different than the FCI_{Pre} only model. However, the delta in Table III, indicates that the model with FCI_{Pre} and PageRank only improves prediction of SoG by 1.52 standard deviations, thus we consider the model with FCI_{Pre} only as the best predictive model. When compared with a model that includes the grades in the first block of the course, the model with FCI_{Pre} is 1.95 standard deviations worse than the grade-based model, which suggests that a model with FCI_{Pre} only is close to being significantly worse than a model with the physics grade in the first block. Finally, adding all three attributes which significantly correlated with SoG, FCI_{Pre}, PageRank and $\mathcal{H}$ did not further improve the prediction of SoG.

2. *Concept Discussion Network Modeling*

In the *Concept Discussion* network, the target entropy, (T), was the attribute most highly correlated with the SoG. Thus T was used to create the base model, as seen in Table IV. This base model improved the prediction of SoG in the second block. In this network, FCI_{Pre} was added and a second model was created. An F-test indicated that this model was not significantly different than the model with T only. The difference in prediction of SoG indicates that the model with T and FCI_{Pre} improves prediction of SoG by 1.49 standard deviations. Thus the best predictive model is the model with T only. When compared with a model that includes the grades in the first block of the course, the model with T only is 1.33 standard deviations worse than the grade-based model, which suggests that a model with T only is not significantly different than a model with the physics grade in the first block.

3. *In Class Social Network Modeling*

In the *In Class Social* network, FCI_{Pre} was the attribute most highly correlated with the SoG, so it was used to create the base model, as seen in Table V. This base model improved the prediction of SoG in the second block. In this network, T was added and a second model was created. An F-test indicated that this model was significantly different than the model with FCI_{Pre} only. The difference

Model	Variables	$R^2[95\%CI]$	F-tests	Δ
PS_F	$\mathbf{FCI_{Pre}}$	$0.18[0.04-0.31]$	$F(1,97)=21.6^{***}$	NA
PS_{FP}	FCI_{Pre}, PR	$0.29[0.12-0.43]$	$F(2,96)=18.4^{***}$	1.52
			$F_{F/FP}(132,132)=0.66^{*}$	
PS_{FPH}	$FCI_{Pre}, PR, \mathcal{H}$	$0.28[0.12-0.41]$	$F(3,95)=12,3^{***}$	1.59
			$F_{FG/FPH}(132,132)=0.91$	
PS_G	GR_{Mech1}	$0.34[0.18-0.48]$	$F(1,106)=53.7^{***}$	$-1.95^{(*)}$
			$F_{F/G}(132,151)=0.35^{***}$	

Table III. Comparison of linear models in the *Problem Solving* network. $^{*}=p<0.05$, $^{***}=p<0.001$

Model	Variables	$R^2[95\%CI]$	F-tests	Δ
$\mathbf{CD}_T$	$\mathbf{T}$	$\mathbf{0.20[0.07-0.33]}$	$\mathbf{F(1,93)=23.7}^{***}$	NA
CD_{TF}	T, FCI_{Pre}	$0.30[0.13-0.44]$	$F(2,83)=17.7^{***}$	1.49
			$F_{T/TF}(133,114)=0.75$	
CD_{TFP}	T, FCI_{Pre}, PR	$0.30[0.12-0.43]$	$F(3,82)=11.7^{***}$	1.49
			$F_{T/TFP}(133,114)=$ 0.76	
CD_G	GR_{Mech1}	$0.32[0.16-0.47]$	$F(1,93)=43.8^{***}$	-1.33
			$F_{T/G}(133,132)=$ 0.52^{***}	

Table IV. Comparison of linear models in the *Concept Discussion* network. $^{***}=p<0.001$

in prediction of SoG indicates that the model with T and FCI_{Pre} improves prediction of SoG by 1.69 standard deviations. However when both T and $\mathcal{H}$ are added the third model improves prediction of SoG by 2.33 standard deviations over the base model. In this network, the best model includes not only FCI_{Pre} but also the networked attributes T and $\mathcal{H}$. The results of the linear modeling in this network are interesting in that when compared with a model that includes the grades in the first block of the course, the best model with ICS_{FTH} is 0.10 standard deviations better than the grade-based model, which suggests that a model with FCI_{Pre}, T, and $\mathcal{H}$ predicts the SoG as well as a model with the physics grade in the first block.

VI. DISCUSSION: UNDERLYING STRUCTURES

A. Understanding relations among networks through correlations

We argue that the three networks are distinct; the *Problem Solving* (PS) and *Concept Discussion* (CD) networks measure different aspects of student interactions, while the ICS network seems to measure the combination of PS and CD. The structure of the correlation network in Figure 7 indicates that in the PS network PageRank (PR) and Hide ($\mathcal{H}$) correlate with equal strength with the Sum of Grades (SoG) and significantly with each other. However, from the CD network In-Strength (s_{in}), In-Degree (k_{in}), and Target Entropy (T) correlate with roughly equal strength with SoG, and to a high degree with each other. In the ICS network, all measures are present and show the same structure as in each of the other two networks: PR and $\mathcal{H}$ are related and s_{in},k_{in}, and T are related. Furthermore PR is related to the s_{in},k_{in}, T collection of variables. Thus, the three networks give information about different aspects of student interaction.

1. *Problem Solving Network Correlations*

Being easy to locate (low $\mathcal{H}$) in the PS network tends to be associated with academic success. We infer that students who are easy to find, are students who are proficient with physics and share their knowledge with others in a memorable way. High $\mathcal{H}$ students may not be proficient with physics or their contributions to *Problem Solving* interactions are not memorable to other students. The way this study is constructed, a student has to be recognized by another student as memorable to get a link, and this reflects the term mutual engagement proposed by WengerWenger [2]. The named student engaged with the other students in problem solving practices; in a process of meaning making McCormick et al. [3]. He or she might have been mentioned for good or bad, although if he or she never added to the meaning making process of others, it is difficult to understand why other students would continue naming him or her.

Adding to this view, we can also imagine high H students as the ones who do not engage in the problem solving practices of the physics student community, we can imagine that he/she will simply not have access to other students' problem solving strategies or to worked out solutions to problems. On the other hand, the positive correlation between PR and SoG indicates that if others in general recognize you as a collaborator, (i.e. someone who participates in the practices of the class), then you are more likely to gain experience with others' problem solving strategies. From a Vygotskian [1] point of view, this may indicate that students can help

Model	Variables	$R^2[95\%CI]$	F-tests	Δ
ICS_F	FCI_{Pre}	$0.18[0.02-0.31]$	$F(1,89)=19.1^{***}$	NA
ICS_{FT}	FCI_{Pre},T	$0.29[0.13-0.42]$	$F(2,88)=17.9^{***}$ $F_{F/FT}(117,117)=0.54^{***}$	1.69
$\mathbf{ICS_{FTH}}$	$\mathbf{FCI_{Pre},T,\mathcal{H}}$	$\mathbf{0.35[0.18-0.49]}$	$\mathbf{F(3,87)=15.3}^{***}$ $\mathbf{F_{F/FTH}(117,117)=0.39}^{***}$	$\mathbf{2.33}^{*}$
ICS_G	GR_{Mech1}	$0.35[0.19-0.51]$	$F(1,97)=53.2^{***}$ $F_{FTH/G}(117,134)=0.79$	0.10

Table V. Comparison of linear models in the *In Class Social* Communication network. $^{*}=p<0.05$, $^{***}=p<0.001$

each other succeed through interactions within a zone of proximal development. Using the vocabulary of Wenger[2], students may develop a shared repertoire, to which the peripheral students have limited access, because they do not engage with others. Of course, some students will not need the shared repertoire of other students, if they are able to decode the language of teachers and text books by themselves. But at least some students seem to develop their problem solving skills in math and physics (as measured by the exams) through problem solving interactions with other students.

2. *Concept Discussion Network Correlations*

The high correlation between s_{in},k_{in}, and T in the CD network indicate that, to some extent, they measure the same thing. But they do show different correlation strengths and patterns. For example s_{in} correlates with GR_{Mech1} and less with T than k_{in}, and k_{in} correlates less with SoG than T.

It is not surprising that T and k_{in} are related since T relies on the number and distribution of shortest paths to a node. A high k_{in} will in itself contribute to a large T. The fact that T correlates better with SoG than k_{in}, shows that taking the whole network into account does seem to make a difference. From a participationist view point, being in a part of the network where ideas and concepts are discussed (and where these discussions are remembered) by many students has a positive impact on later grades.

While strong connections as represented by the Instrength also seem to have an impact, using the whole network perspective with T has slightly more predictive power when it comes to Sum of Grades.[36] In our interpretation, we can associate being in a part of the network, where ideas are frequently discussed by many students with academic achievement. In that part of the network students have the chance to put forth ideas to others, use physics vocabulary actively, and critically examine others' arguments. Apparently, these are qualities we can associate with good grades. Again, we see the links as proxies for students engaging mutually with each other in meaning making processes.

3. In Class Social Network Correlations

Perhaps the most surprising result is that engaging in non-content related social interactions at a lecture, a problem solving session, or laboratory exercise is connected to students' physics understanding or ability to solve problems. It is not clear what mechanism relates a socially memorable (central) student in class to good grades. This is seen in the *In Class Social* network. Examining the results from the *In Class Social* network from a participationist perspective, we might conclude that the practice of physics is not strictly about discussing concepts and solving problems, but also about engaging with others in social realms. It seems reasonable to propose that the 'off-topic', social interactions that students engage in during class time is an important component in the overall engagement in physics and a component of building a community around the practice of physics. These results suggest two important questions : 1. To what extent is the social engagement necessary? and 2. Does the social engagement lead to on-topic engagement or does the on-topic engagement lead to social engagement?

B. Linear modeling

It is interesting that the network with the most predictive power is the In Class Social network. The linear model which combines FCI_{Pre} with T and $\mathcal{H}$ predicts SoG significantly better than FCI_{Pre} or any of the proposed network measures alone. This was not the case with the PS and CD networks, where the best models were the models with one measure, FCI_{pre} and T respectively. In the ICS network one interpretation of the linear model is that FCI_{Pre}functions as a measure of the individuals understanding of physics, while T and $\mathcal{H}$ seem to measure different aspects of student participation in learning activities. This model seems like a hybrid of the models which best predicted SoG in the PS and CD networks. This hybrid interpretation is similarly reflected in the correlation network, shown in Figure 7. Seen in this way, the ICS network could serve as a proxy for the PS and the CD networks, which preserves some of the features of both *Problem Solving* and *Concept Discussion.*

The linear model for the ICS network allows us to consider that students engage in social interactions, both when solving problems and when discussing concepts (something which may or may not happen at the same time). Thus, when we ask students to name who they remember communicating with socially in the classroom context, it is not surprising that students include the names of people who they engaged with during both PS and CD activities. That is, we suspect that if you remember working together to solve a problem or discussing a physics concept, you also remember communicating socially. This may indicate that engaging in social interactions and disciplinary interactions are not easily seen as separate. They may interact or share a common, more general, underlying predictive variable. This line of thinking supports current research which combines classical cognitive thinking with socio-cultural theory.

C. Limitations of the study and further developments

One obvious limitation is that the study is situated in a Danish university and considers only physics majors. Replication of this study at different universities and in different cultural contexts would improve the generalizability of these results. Further, the study considers only the class of one particular year.

Additionally, we have to consider the validity of the survey itself. The validity issues are that we cannot be sure that students answer truthfully, and we cannot know why they chose as they did. They may even have talked about who they should name, because they wanted to give an accurate account of their actions. This goes against the premise that they only list students who they remember having talked to.

There are also limitations with regards to the network structure. Since we only use within course student-student interaction, we miss out on the impact of lecturers, teachers, and interactions outside the course. Finally, since not all students answered all surveys, we have an incomplete data set from which to create the networks, and this may be a serious source of error [37].

Linear models involving network measures account for some of the variance in the data. However, the rationale for using linear models is mostly one of convenience. Many of the techniques for handling quantitative data in PER studies have been imported from sociological statistics. Further research could look in to the possibility that other functional forms are relevant to predict non-network variables from network variables.

VII. CONCLUSION:

With this study we have undertaken an analysis of networked measures with student grades. We find that correlations ($0.3 < r \leq 0.45$,$p < 0.001$) exist between network centrality measures for students at one point in time and their grades at a later point in time. The measures yielding the highest correlations are all probabilistic measures taking the whole network into account.

The *In Class Social* network correlation network can be seen as a hybrid of the *Problem Solving (PS)* and *Concept Discussion (CD)* correlation networks, as it seems that the ICS captures both problem solving and concept discussion interactions in addition to social interactions.

With the ICS network data, we can create a linear model with FCI_{Pre} , target entropy, and hide as variables from the first block of students' study course to predict the sum of grades of the courses in the next block of classes. This model is significantly better at predicting grades than a model using only FCI_{Pre}.

We argue that social interactions seem to correlate positively with physics learning at least for some students. The counter-argument claiming that there are other underlying variables responsible for both network positions and grades need to incorporate how these variables affect both social interactions and grades in the context. We would like to investigate mechanisms for becoming central in academic (PS and CD) and social (ICS) networks and to further investigate attributes which are to correlated students' academic success as measures by grades.

ACKNOWLEDGMENTS

We would like to thank Professor Ian Bearden for his tireless work and enthusiasm in the data collection process. It is safe to say that without this, there would not have been any data to analyze. Also, we wish to thank Morten Brydensholt for his contribution to the development of the network data recording tool.

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Appendix 4 - Representing cognitive schemata with networks of student free text answers to conceptual problems: Justification and first steps towards a method

Representing cognitive schemata with networks of student free text answers to conceptual problems: Justification and first steps towards a method

Jesper Bruun

Basic cognitive structures such as image schemata or p-prims may be vital in coordinating our phenomenological experience of being in the world, and may thus form a link between language and the ability to do, for example, physics. The purpose of this paper is (1) to make a theoretical link between networks of student writings about how students believe a physical system behaves and students' cognitive schemata related to that system, and (2) outline the first steps in a method for analyzing the such networks. Student written answers to open-ended online conceptual questions are converted to linguistic networks describing how students associate between terms. Analysis of linguistic network structure coupled with cognitive schema theoretical considerations is then used to suggest that networks from student writing can represent some degree of physics understanding. For example, if nodes represent concepts relevant to physics problems and links represent processes of association between concept, signifying the use of an image schema. Finally, the paper outlines three issues which later research should resolve before linguistic networks become useful for understanding physics learning.

1 Introduction

The link between cognitive ability of physics students and how students perform, collaborate, and think about physics is a well established field of research which among other things draws on cognitive schema theory (Derry, 1996). It studies learning in physics as a form of acquisition of knowledge (Sfard, 1998), where knowledge - or rather knowledge structures - are represented using concept maps (Novak and Cañas, 2008), mental models (Gilbert and Boulter, 2000), hypothetico-deductive reasoning schemes (Lawson, 2003), or mental schemata.

One type of schemata investigated by cognitive schema theories are basic structures like image schemata (Johnson, 1987) and p-prims (diSessa, 1993). These are considered irreducible mental units that help structure our actions or sense of mechanism. Image schemata, in particular, are hypothetical prelinguistic structures that form the basis of conceptual metaphors used in language. Moreover, these basic cognitive structures are vital in coordinating our phenomenological experience of being in the world (Derry, 1996), and may thus form a link between language and the ability to do, for example, physics. As the basic building blocks, basic cognitive structures are combined to form more complex cognitive schemata like mental models (Derry, 1996) and coordination classes (diSessa, 2002).

Image schemata, p-prims, and higher level structures have been accessed through, for example, clinical interviews (diSessa, 1993), video observation of student interactions (Roth and Lawless, 2002), and student writing (Hein, 1999). Recently, studies in physics education research has shown that networks can be used to gauge the coherency of student knowledge structures (Koponen and Pehkonen, 2010) and the development of student epistemic framing (Bodin, 2012). These two studies show that networks offer a way of visualizing and analyzing data, which may reveal hidden structures in that data.

These studies offer insight into particular students' understanding and use of physics. However, Bodin (2012) focuses on students epistemic framing of problems in physics, while Koponen and Pehkonen (2010) investigates how students link a predefined set of concepts. They do not consider how students use natural language in connection with physics concepts. If natural language as expressed in writing is connected to embodied mental structures as proposed by image schema theory (Lakoff, 1987, 1993; Lakoff and Johnson, 1980; Roth and Lawless, 2002; Johnson, 1987), then patterns that network analysis of natural language can reveal may be linked to students' ways of understanding and use of physics.

The purpose of this paper is (1) to make a theoretical link between networks of student writings about how they believe a physical system behaves and students' cognitive schemata related to that system, and (2) outline the first steps in a method for analyzing the such networks. The theoretical link is proposed in Section 2, while the following two sections outlines a method for collecting (Section 3) and analyzing (Section 4) networks of student writing. Section 5 discusses some of the issues further research needs to resolve if linguistic networks are to be useful tools for understanding physics learning.

2 Linking cognitive schemata to networks

The idea of coupling image schemata, p-prims, or as termed by Derry (1996), memory objects, to networks stems from the way the authors have presented the coordination these schemata. For example, Johnson (1987, pp. 109-111) uses the work of Gentner and Gentner (1983) as an example of how image schemata provide metaphorical constraints on reasoning, and the analogical models he refers to are depicted as networks (graphs) by Gentner and Gentner (1983). diSessa (2002) speaks of coordination classes as a particular way in which p-prims can be structured with respect to cuing, and this is accompanied by a drawing of graphs. Derry (1996) conceptualizes mental models as "particular organizations of memory objects that constitute a specific event interpretation." Since networks constitute a way of organizing entities (nodes) with constraining relations (links) (Costa et al., 2007; Sneppen, 2012), it seems natural to think about the coordination of memory objects in terms of networks.

The benefit of describing image schemata as networks would be to achieve a quantitative understanding of how image schemata work to constrain meaning - in this case in physics. For this to happen, nodes and links need to be defined consistently and based in some kind of theoretical understanding. This would allow researchers to interpret the results of network analysis in a meaningful way to produce hypotheses that can be tested quantitatively.

To lend meaning to nodes and links in networks, consider how Johnson (1987) argues that our embodied understanding of the world is represented in the metaphorical projections we use in language. Johnson (1987) describes understanding as being in the world, as our "bodily capacities and skills, our values, our moods and attitudes, our entire cultural tradition, the way in which we are bound up with a linguistic community, our aesthetic sensibilities, and so forth" (Johnson, 1987, p. 102). [1] If image schemata structure all these very different things, it is no wonder that they also permeate our languages. The link between networks and cognitive schema theory investigated here is through written language. In principle, it could be any representation of language and even body language in the form of gestures, facial expressions and actions could be considered.

While Lakoff and Johnson (Lakoff, 1987; Lakoff and Johnson, 1980; Johnson, 1987) are interested in embodiment, language, culture etc. in general, diSessa (2002; 1983; 1993) is interested in a *sense of mechanism* in physics and how this is coordinated through phenomenological primitives (p-prims). While p-prims seem to be much like image schemata in that they are irreducible units that coordinate understanding, p-prims are targeted at physics rather than everyday life. However, students engaging with physics may use both image schemata, p-prims and other mental structures to structure their understanding.

This work is concerned with the connection between of embodiment, language, culture, and physics. When a student writes about physics, he will use both physics words and non-physics words to explain what he means. According to image schema theory many of the non-physics words will be markers of a coordinating image schema (Johnson, 1987). For example, in English, "gone a long way" is an indicator of the path schema (Johnson, 1987, p. 115).

In a simple linguistic network (Masucci and Rodgers, 2006) mapping of writing, all the words in the preceding phrase

[1] This relation between understanding and a way of being in the worlds is reminiscent of the German concept of Bildung (Klafki, 1996), which is the basis of a competency based view of what it means learn science (Dolin et al., 2003). This link is noted but not further pursued here.

could be represented as nodes. Adopting the view that a concept "is a perceived regularity in events or objects, or records of events or objects, designated by a label" (Novak and Cañas, 2008), then nodes in these linguistic networks are not concepts per default.

However nodes may represent some kind of entities beyond mere words. The conjecture of this paper is that working with networks offer a way of converting the entities in the form of nodes to links in the form of processes. The idea of nodes as entities comes from Ogborn et al. (1996), who use the term entities to describe the things science teachers use to build explanations. The idea links as processes stem from McCormick et al. (2011), who use links as processes to inter human engagement as interaction processes involving meaning making. Here, the links are viewed as intra human (Vygotsky, 1978) processes, specifically processes of association. That is, when a student writes an explanation, he will associate in a manner constrained by, for example, image schemata and p-prims, between the concepts available. This association process should then constitute some level of his understanding of physics.

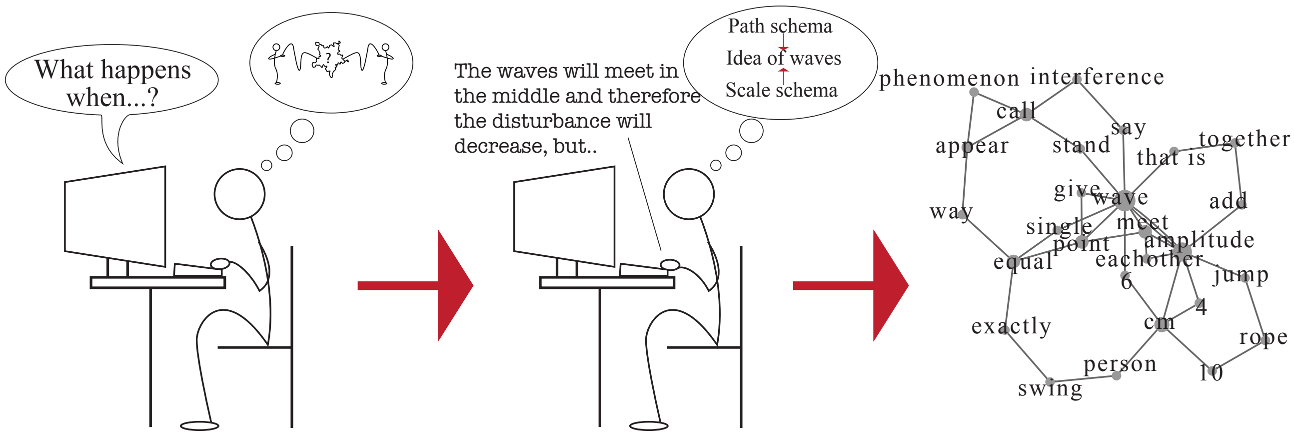
The purpose of the rest of the paper is to outline how physics understanding in the sense described here may be captured, if only partially, by networks of student writings in physics. The questions students answer are physics questions; the describe how real physical objects would behave. "Wave" is not used as an abstract metaphor like "a wave of silence", it is used to describe the observable phenomenon that is a wave on a string. Still, thinking about waves on a string and describing them, should invoke image schemata and p-prims.

3 Using online questions to capture student writing for linguistic networks

The data used in Section 4 have been collected by following a physics class of upper secondary students for sixteen weeks during a fall semester. On several occasions students have been asked to write answers to conceptual questions, but in this paper only the answers from two students are treated in terms of network analysis. Moreover, this study only investigates one question, although the analysis investigates student answers to this question from four different occasions.

Figure 1 shows how linguistic networks have been made in this study. Students were given time in class to answer a question. The question was formulated to make students explain what would happen to the system in a given situation. Upon seeing a figure accompanying the question text and reading the text, the hypothesis is that students activate different relevant image schemata to coordinate their understanding of the physics in the system. The students then use these coordinations to produce a text, which is recorded by the computer (and stored in a Learning Management System). In terms of representational forms, students express themselves using conceptual and phenomenological forms (Roth, 1995) to explain their understanding. Finally, the text is converted to a network as explained in Section 4.

The question used in this study, is depicted in Figure 2. It was developed in collaboration with the class' physics teacher. The curriculum centered around waves during the time of data



Data collection session *Data collection session* *Network representations*

Figure 1: Students read an open-ended question online and answers it online. They have access to books, pen & paper, but are asked not to discuss their answer with others. The hypothesized mechanism is that while students are writing image schemata are activated and used, resulting in a written text. Finally, the text is converted to a network based on the methods described in the paper.

collection. At the time of the first student answer, students had just engaged with waves on a string. Thus it seemed appropriate to ask the students to write about a system that they could understand in terms of a string.

The students answered the question on four different occasions. There were two reasons for this. The first reason was to see if students changed their ways of engaging with the question as they progressed through the teaching plan. The second reason was to compare the networks of student answers to see either traces of learning or variations due to different formulations of an answer.

The reason for formulating the question in an open-ended manner was to bring about as many associations as possible, while not restricting the students to a predefined set of concepts. The danger in such an approach is that some students will not make the connection to the problem or will not risk

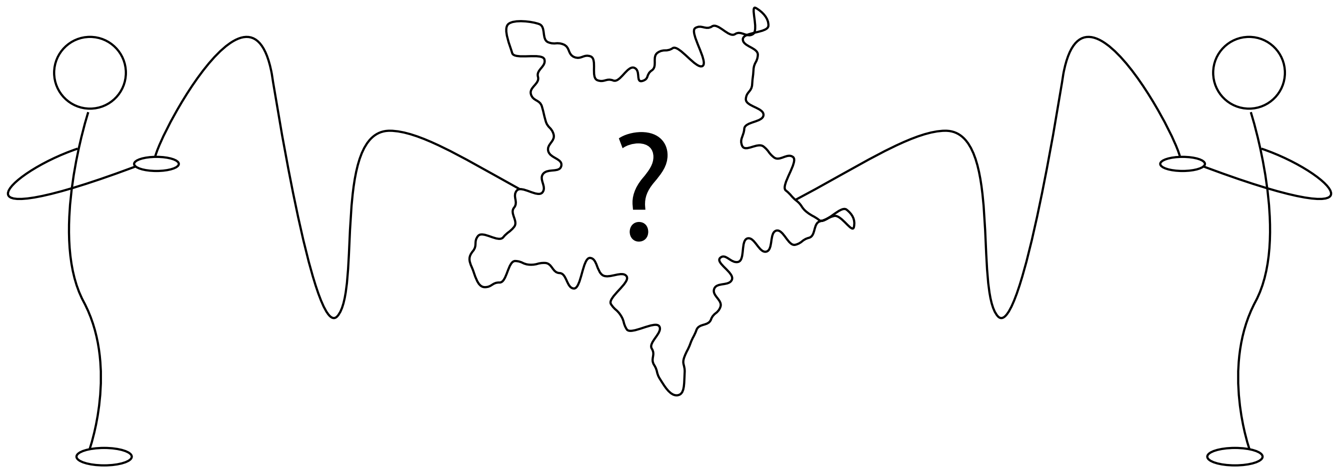


Figure 2: Supporting drawing following the question that were administered to the students. The question text read: "Here you see a drawing of two people moving a jump rope up and down. The drawing should give you an impression of what is going on, but the form of the jump rope on the drawing may not resemble its form in the real world. Explain what happens in the middle of the rope."

making wrong statements when writing. Some of the students write this explicitly. As one student writes: "I have no idea" (Hildur). Or as another student explains: "I was not in class when we were taught about waves, so I do not know" (Helle). However, the two students in this study did come up with predictions and explanations for them.

This way of collecting data does not try to control the learning situation as an experiment, but rather seeks to incorporate research design with teaching as it takes place in a typical learning situation, which in this case happens to be Danish. The idea is that the resulting research is developed in close connection with the object of study, teaching and learning physics. In a research perspective, this makes it more difficult to attain data of a sufficient quality.

One of the difficulties is that physics teaching at the secondary level is usually centered around problem solving, presentations by the teacher and the students, and laboratory exercises. Newer teaching practices like peer instruction (Mazur, 2009) and modeling (Hestenes, 2006), advocate learning activities where students verbalize their thoughts or collectively work on problems, thus developing shared models in a community of learners. Even so, students in physics classes rarely get to write about how they believe a physics system behaves. Writing in physics education is often associated with writing a lab report or writing down short arguments between equations. Thus, a researcher can not assume that students are used to the form of representation that students need to use to provide the study with data. Students will need practice to comfortably write out their thoughts.

Another difficulty for this particular research design is that students might grow bored or fatigued with answering the same question again and again. Although students were reminded that they might have changed their opinions or thoughts during data collection, some students expressed that they felt they had already answered the question and thus did not see the point in answering it again. The two students in this study answered the survey four times, although the answers do seem to reflect some of the difficulties listed above.

4 Generating linguistic networks and investigating them with cognitive linguistics

The process of creating a linguistic network from a piece of writing involves techniques derived from text mining (Feldman and Sanger, 2007) and linguistic network analysis (Masucci and Rodgers, 2009; Masucci et al., 2011; Masucci and Rodgers, 2006). This paragraph describes the procedure used for the texts of this study. The procedure utilized the R programming environment[2] with the tm package (Feinerer et al., 2008) for text mining purposes, and the iGraph package (Csardi and Nepusz, 2006) for network analysis.

First, the words in the raw text were reduced to their stem form. This means the words like “waves”, “the wave”, and “the waves” would be reduced to “wave”. The verb “to wave” and derived forms (“(he) waves”,”waved”, “waving”) were reduced to “to wave”. Thus even if a verb or noun share a common stem, they are kept as separate entities. Furthermore, some synonyms, which was considered obvious, were employed. For example the conversion of “(they are) equal” and “(they are) the same” to just one phrase: “(they are) equal”. While it is possible to train computer programs to do these processes, for the purposes of this study, it was done manually by search an replace. The final text mining step before converting to the non-reduced networks was to remove punctuation, white space, and convert upper case to lower case.

Linguistic networks are made on the basis of adjacency in texts (Masucci and Rodgers, 2006). For example, the phrase “waves will meet” (Figure 1) will result in three nodes and two connections (“wave”$\rightarrow$”will”, “will”$\rightarrow$”meet”). In this work, if “wave” and “will” are adjacent n times a link of strength n is created between the two words. The results are directed networks, which can be analyzed using network measures like the number of nodes (here entities), the search information - a measure of navigability in a network (Rosvall et al., 2005) - and the target entropy - a measure of the predictability of a

[2] http://www.r-project.org/

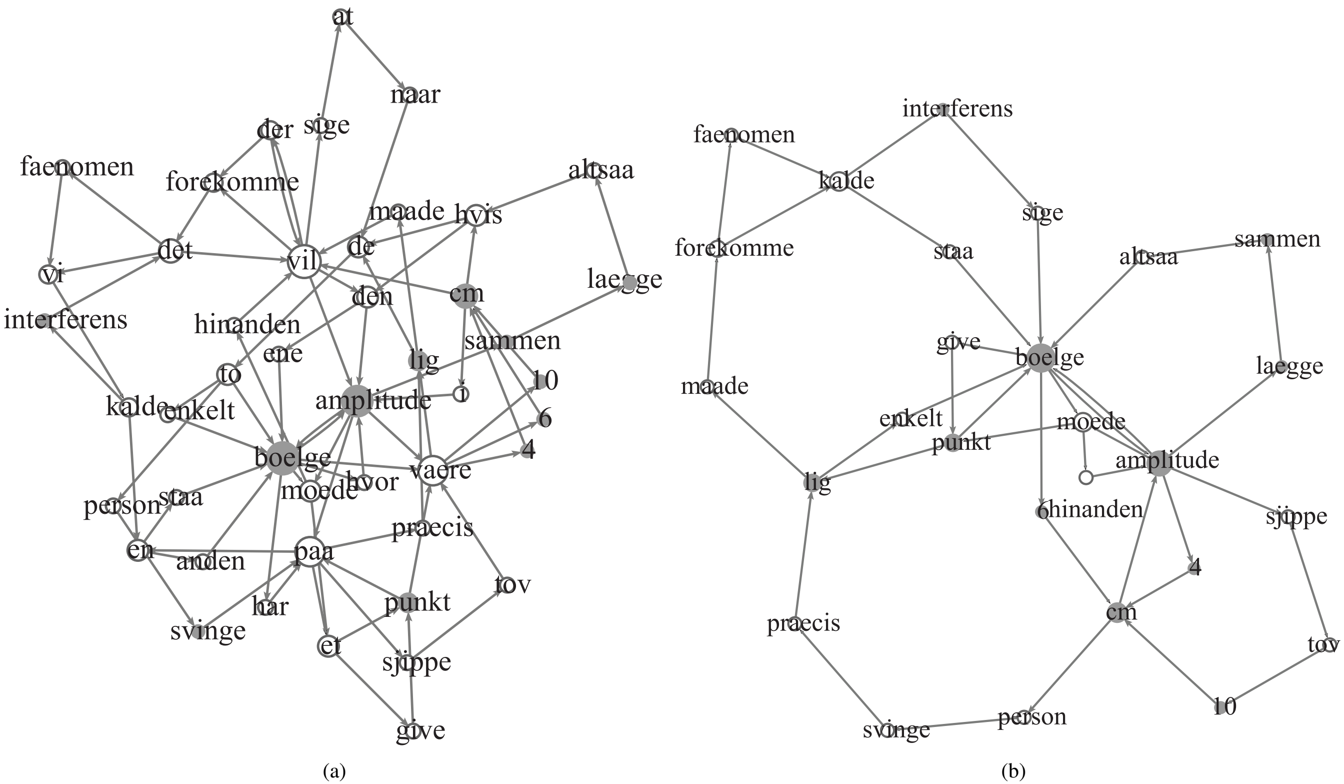


Figure 3: Two linguistic networks based on reduction of terms to their stem. (a) shows the network with no stop words removed. (b) shows the network where common Danish words have been removed. The grey nodes are entities that the author connects with physics in this context.

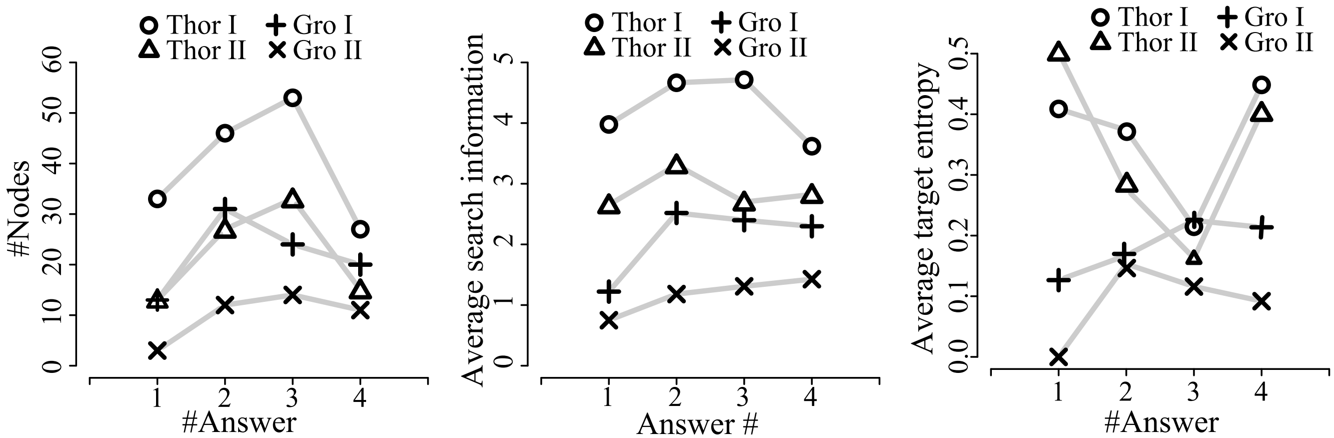
network Sneppen et al. (2005).

The verb *to be* is usually connected to propositions in language and metaphors (A is B). Thus it might be a way of "propositionalizing" image schemata. Removing this word from a text might reveal structures that were not there before for two reasons. First of all, if *is* is a common word, then it will have a large amount of connections, thus dominating the picture and obscuring other connections between words that might be more significant. Second, removing *is* might let the image schematic coordinations behind the text come to the forefront. In general, removing the common words corresponds to transforming them into links. Thus, in a linguistic *association* networks, image schemata structure physics relevant entities because they are links (and the propositional *is* becomes one way in which image schemata can structure entities). Since all common words are removed here, the analysis makes the crude assumption that common words are generally linked to image schemata, while non-common words are not or to a lesser degree. What it then revealed is the way all the activated image schemata structure the physics relevant entities. The price for this maneuver is that image schemata are not distinguishable.

Removing the common words leaves roughly half the words as entities in each of the networks. This is shown in Figure 4(a), where the number of unique entities in the networks are shown. The number of entities show a lot of variation according to both the two students and the answer number. However for each type of network, the search information (Figure 4(b)) is much more constant for each type of network. It shows a clear ranking both of the students (Thor's networks have higher S_A's than Gro's), and also that the reduced networks - the networks without common words - have a lower *per path* search information than the non-reduced networks. This means that on average it is easier to navigate to a reachable node in the reduced networks, and this can not be explained just by there being fewer entities in these networks[3]. It can be explained by the fact that the words removed are used in many different connections, and thus will have many connections in the network. Thus, navigating through them is difficult. In the same way, image schemata are used in a variety of connections, so even if it was known what image schema was in use, it would be hard to know what it was used for. In the reduced network, if the entities represent concepts, and links represent processes of associations then a low search information would signify an efficient mental structure, with many association paths connecting concepts. This might correspond to many ways to reach an meaningful answer.

The target entropy (Figure 4(c)) is difficult to interpret from these graphs. It is a measure of predictability, and both of Thor's networks start with a high target entropy (low predictability). They reach a minimum at the third answers becoming more predictable, only to rise at the end. A low target entropy means that as seen from a given node, it is easy to predict where the next message comes from (Sneppen et al., 2005). If messages are analogous to associations in these networks, then a low target entropy would mean that the associations follow a set path. For example, a string of words would not contribute to the target entropy, because it would be com-

[3]For example Thor's non-reduced network 4 has fewer entities than his reduced network 3, but a higher S_A



(a) (b) (c)

Figure 4: Three graphs comparing different structural aspects of networks of two students' writings. (a) The number of unique words, or entities, present in the networks. (b) The average per path search information, that is the number of yes/no questions needed to navigate from a given node to another node using an existing path. (c) The average target entropy, that is the predictability of the network. A low target entropy signifies a high predictability.

pletely predictable where it ended. A network showing a high target entropy would signify that many entities are reachable from many places. If the entities represent physics concepts and links represent association processes, this would signify that a student could associate between many concepts when answering a conceptual question.

It is tempting to say that the good student's reduced networks are characterized by a large number of entities, a low search information, and a high target entropy. This would require that such networks could be coupled to the general physics competency of the student, for example, the students ability to use physics in different situations alone and with others. The next section outlines some of the issues that further research needs to address to make such a connection.

5 Further development of the connection between cognitive schemata and linguistic association networks

The development of networks to represent how image schemata structure physics knowledge needs to address the following issues.

- Investigate how it affects the structure and interpretation of the network to convert words associated with different kinds of image schema to links (for example, by removing them from the text).
- Find out if and how it is possible to reliably identify meaningful words with regards to specific physics problems.
- Compare linguistic association networks to other ways of evaluating student competency in the context of research and teaching practice.

The first bullet anticipates that some image schematic markers are better viewed as processes of association in terms of networks - meaning that they should become link (and perhaps different types of links). However some might not be. The *links schema* (Johnson, 1987, p. 117-119) seems like an obvious candidate to be viewed as an association process, marked by words like “and”, “but”, “or”, and “if-then” (Johnson, 1987) - but it could also be “the same”. The *scale* schema (Johnson, 1987, 121-124) is a candidate for a schema that might not be well represented as a link in these networks as they apply to physics. The reason is that the markers would overlap with markers that could resemble relationships between physics quantities where it is important to know, which way the scale goes; whether something is larger or smaller than something else, and how much larger or smaller, can be important distinctions in physics. This could be solved by having different types of links, for example, a larger than smaller than type of link.

The second bullet calls attention to the fact that language is a dynamic entity in itself. Language changes as it is used. Thus student may refer to something as “that thing” meaning, for example, the wavelength of a wave. The question is, what kinds of words should be represented in the network as words with a physics-related meaning. Distinguishing between words with a defined meaning in physics and the use of more common words to describe such meanings might give insight into the development of language. Maybe one would be able to see how common words transform over time to physics words, changing both the term and it’s connections with other terms.

The final and perhaps most pressing issue is to establish the extent to which linguistic association networks are useful for evaluating student competencies. This can be done in a number of ways, for example, by relating student linguistic association networks to students’ use of other representational forms as one can investigate them by means of, for example, video observation, analysis of other types of products, or interviews of different kinds.

6 Summing up

This paper has proposed to represent pre-linguistic mental structures called image schemata as nodes or links in linguistic association networks. Specific words can function as markers of image schemata and it is suggested that they could signify association processes when represented in linguistic association networks. Physics terms in linguistic association networks represent concepts since they represent some perceived regularity. Such a projection of image schemata and physics concepts may capture important aspects of how students understand physical systems in specific situations.

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