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PhD Thesis

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Japanese Mathematics Teachers' Professional Knowledge

International case studies based on praxeological analysis

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Title and subtitle: Japanese mathematics teachers' professional knowledge.
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Topic description: The thesis describes and studies the Japanese mathematics teachers' professional knowledge and its dissemination. This theme is investigated in several concrete situations: the knowledge of teaching practice of school mathematics taught in the teacher education, the relation between the educational goals described in the national curriculum and concrete teaching methods discussed among the Japanese lower secondary teachers in service. The thesis investigates also critical phenomena, which arise in an attempt to transfer the Japanese teaching practice in a different teaching context.

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Abstract

What kind of knowledge is required for the teaching of school mathematics at a given level depends on which teaching and learning culture one belongs to, since such knowledge is constructed under the constant institutional influence of the society. This PhD thesis explores Japanese mathematics teachers' professional knowledge and its dissemination from the perspectives offered by the anthropological theory of the didactic (ATD). "Professional" knowledge here means the body of sound theoretical knowledge that is explicitly built and deliberately developed by the institution of teachers who shares it. To expose the characteristics of their knowledge in detail, we analyse case studies from teacher education, reflections among the teachers in service, and the international transfer of a specific Japanese teaching approach. In the study that aims to elucidate the knowledge taught in primary school mathematics teacher education, both the process of the lessons and the teacher educator's teaching methods are explicitly described and analysed. It turns out that the more striking feature of the Japanese teacher educator's is the theorised teaching approach which is further highlighted through the comparison with similar lessons observed in Finnish and Swedish teacher education.— One process of producing such theorised knowledge is analysed through the comments of the Japanese lower secondary teachers in service during a post-lesson reflection in the context of an open lesson. Teachers' comments observed at the reflection meeting are categorised and modelled from an institutional perspective. The analysis shows how the teachers' foci regarding the teaching techniques of the observed lesson are related to different aims of the teaching of the mathematics. We analyse what kinds of elements influence the teachers' different foci, and how the elements facilitate the realisation of generic educational goals by way of concrete teaching techniques. Finally, the thesis investigates critical phenomena, which arise in an attempt to transfer Japanese teaching practices and resources to a new context (Swedish lower secondary school) with a different teaching environment, in which teachers have different scripts for mathematics lessons. The analysis, based on a reference model that make explicit the researcher's criteria about the different teaching techniques related to different didactic goals, shows which of the Japanese teaching techniques the Swedish teacher had difficulties to implement. The model supplied by the ATD is further used to explain the conditions and constraints, which brought about these difficulties. The three studies together explore central aspects of the complex process of establishing the Japanese teachers' professional knowledge, and how it disseminates in their communities. The correlation between the dissemination of such

theorised shared knowledge, and well-established systems that provide opportunities and resources for designing and reflecting on lessons, is also discussed.

Abstract in Danish

Hvilke former for viden som kræves for at undervise i matematik på et givet klassetrin afhænger af den undervisnings- og læringskultur man befinder sig i, eftersom denne viden konstrueres under samfundets konstante indflydelse. Denne PhD-afhandling udforsker japanske matematiklæreres professionelle viden og dens formidling blandt lærerne, ud fra et perspektiv som er funderet i den antropologiske teori om det didaktiske (ATD). ”Professionel” viden refererer her til sund teoretisk viden som eksplicit og bevidst udvikles af den institution af lærere som deler den. For at fremstille detaljerne i denne viden analyserer vi case studier fra læreruddannelse, refleksioner som deles af praktiserende lærere, og den internationale formidling af en specifik japanske undervisningstilgang. I det studium, som sigter på at belyse den viden som formidles i matematiklæreruddannelse for grundskolen, gives en eksplicit beskrivelse og analyse af såvel lektionsprocessen som læreruddannerens undervisningsmetoder. Det viser sig, at det mest iøjnefaldende karakteristikum er en teoretisk tilgang, hvilket yderligere bestyrkes af en sammenligning med tilsvarende lektioner som blev observeret i finsk og svensk læreruddannelse. Vi analyserer dernæst én af de måder, hvorpå en sådan teoretisk viden produceres, gennem japanske mellemskolelæreres kommentarer under et refleksionsmøde i forbindelse med en åben lektion. Lærernes kommentarer kategoriseres og modelleres fra et institutionelt perspektiv. Analysen viser hvordan lærernes fokuspunkter vedrørende undervisningsteknikker i den observerede lektion knytter sig til forskellige mål for matematikundervisningen. Vi analyserer hvilke forhold som påvirker lærernes forskellige fokuspunkter, og hvordan disse forhold fremmer realiseringen af overordnede mål for undervisningen gennem konkrete undervisningsteknikker. Endelig udforsker afhandlingen visse kritiske fænomener som opstod i forbindelse med et forsøg på at overføre japanske undervisningspraksisser og –ressourcer til en ny kontekst med et andet undervisningsmiljø (svensk mellemskole), hvor lærerne har andre drejebøger for matematiklektioner. Analysen er baseret på en referencemodel som ekspliciterer forskerens kriterier for de forskellige undervisningsteknikker i relation til forskellige didaktiske formål, og viser hvilke af de japanske undervisningsteknikker som den svenske lærere havde vanskeligt ved at bringe i anvendelse. ATD-modellen bruges endvidere til at forklare de betingelser og begrænsninger som gav anledning til disse fænomener. Tilsammen udforsker de tre studier centrale aspekter af den kritiske proces, som japanske læreres professionsviden etableres gennem, og hvordan denne viden finder udbredelse i deres professionelle miljø. Vi

diskuterer også sammenhængen mellem udbredelsen af denne teoretiske, delte viden, og veletablerede systemer som muliggør og nærer design af lektioner og refleksioner over dem.

List of papers

Paper I: Comparing mathematics education lessons for primary school teachers: case studies from Japan, Finland and Sweden

Yukiko Asami-Johansson, Iris Attorps & Carl Winsløw

To appear in International Journal of Mathematical Education in Science and Technology

Paper II: The Didactic Notion of “Mathematical Activity” in Japanese Teachers’ Professional Scholarship: A case study of an open lesson

Yukiko Asami-Johansson

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Paper III: Conditions and constraints for transferring Japanese structured problem solving to Swedish mathematics classroom

Yukiko Asami-Johansson

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Asami-Johansson, Y., & Attorps, I. (2019). Japanese and Swedish Mathematics Teacher Educators’ Pedagogical Content Knowledge – An Institutional Perspective. *Educação Matemática Pesquissa 21*. El paradigma del cuestionamiento del mundo en la investigación y en la enseñanza, 157-170. DOI:10.23925/1983-3156.2019v21i4p157-170.

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Asami-Johansson, Y. (2018). Applying japanese didactic practices in a swedish classroom. In Bergqvist, E., Österholm, M., Granberg, C., & Sumpter, L. (Eds.). (2018). *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education (Vol. 5)*. Umeå, Sweden: PME. P. 13 (Oral communication).

Otaki, K., Asami-Johansson, Y., & Bahn, J. (2019). Questioning the paradidactic ecology: Internationally shared constraints on lesson study? In *Pre-proceedings of CERME 11*. Utrecht: The Nederland.

1. Introduction

The core of the thesis is three papers that examine different components of and perspectives on Japanese teacher's professional knowledge and its dissemination. Together with the papers, the thesis contains an introduction or, in Swedish, a *kappa*. Kappa actually means "coat" in English; it aims to show how the different themes of the papers contribute to a common, overarching research *problématique*, and it also builds up a coherent and even deepened conclusion of the thesis in this wider perspective. The first chapter of the thesis is thus constituted by this kappa, in which I present the following. Section 1.1 outlines the motivation and origins of the research project, where which I explain the personal context of my interest in the subject. Section 1.2 and 1.3 present the objectives of the PhD project and the relation of the papers to these objectives. Section 1.4 offers a literature review on the relevant research to locate the thesis in a wider perspective. In Section 1.5, I introduce the general tools on which the thesis is based, mainly the theoretical framework *Anthropological theory of the didactic*. Section 1.6 introduces the overall research questions and the methodology, which also explains how I applied the theoretical framework for the analysis of the various data that have been collected. Section 1.7 outlines the main results of the three papers and how they fit into the big picture, and finally, section 1.8, states the overall conclusion of the thesis and some suggestions for further and future research.

1.1 Background

1.1.1 Teaching as semi-profession and *The Teaching Gap*

The idea of “teaching as semi-profession”, coined by the American sociologist Amitai Etzioni in 1969 (Etzioni, 1969), was for me the beginning of many reflections regarding mathematics teachers’ professional knowledge. Etzioni claims that teaching is currently not considered as a proper profession, at the same level as the professions of medical doctors and lawyers. I encountered Etzioni’s ideas through some of Yves Chevallard’s articles exploring the institutional conditions that form (mathematics) teachers’ profession (e.g. Chevallard, 2003; 2006a; 2010). He explains that the phenomenon is related to the fact that unlike the “real” professions like medicine and law, teachers have not explicitly built a shared “a body of sound theoretical knowledge” (the original text: un corpus de connaissances théoriques fermes) (Chevallard, 2003, p. 11) as a basis of their practice. For instance, if a medical doctor has a cancer patient he cannot cure, he would not regard that it is his personal responsibility to find the cure. Indeed, it is a matter of extending the scientific knowledge shared by the whole profession of medicine. On the contrary, if a mathematics teacher has a pupil who does not master, say, the procedures of calculating with rational numbers, it is considered that the responsibility lies with the individual teacher. Then some questions arise: what kinds of knowledge could be considered as shared and solid professional knowledge for the profession of mathematics teaching in general, and how could such knowledge be developed and disseminated within the community of mathematics teachers?

This subject was already thoroughly discussed in Stigler and Hiebert’s best-selling book “The Teaching Gap—Best ideas from the world’s teachers for improving education in the classroom” (Stigler & Hiebert, 1999). This text is based in part on the TIMSS video study, where researchers made comparative observations on mathematics teachers’ practice in Japan, Germany and the United States. After a detailed description of the differences between teaching practices in Japan and the United States, the authors discuss how teaching can become a “true profession” (ibid., p. 177). They conclude that according to their observations, a key to develop professional knowledge on the teaching of mathematics is teachers’ collaborative work in direct connection to their teaching practice:

The star teachers of the twenty-first century will be those who work together to infuse the best ideas into standard practice. They will be teachers who collaborate to build a system that has the goal of improving students’ learning in the “average” classroom, who work to gradually improve standard classroom practices. In a true profession, the wisdom of the profession’s members finds its way into the most common methods.

The best that we know becomes the standard way of doing something. The star teachers of the twenty-first century will be teachers who work every day to improve teaching—not only their own but that of the whole profession (Stigler & Hiebert, 1999, p.179).

Stigler and Hiebert consider that teaching is a cultural activity. They claim that within a culture, there exists some shared assumptions about how a lesson must proceed, and this picture they call a *script*. It is formed by our own experiences as students (p. 87), e.g. by American teachers and students, and similarly in other countries. Pupils become teachers and then hand the script on to new generations. According to Stigler and Hiebert, this script is a mental model of the common patterns of teaching in each country. They described and compared the national patterns of teaching in the USA, Germany and Japan. A main conclusion from the observations of classrooms and students' results appears to be that the Japanese mathematics teachers' patterns of teaching are the most effective, both in terms of the structure of mathematical content and as regards the teachers' approaches to frame students' activities. For instance, lessons are constructed like stories with a red thread throughout the lesson, the presented mathematical contents and concepts have clear connections to the previous lessons, and teachers have common ideas and methods (such as structured problem solving) to let the students engage autonomously with mathematics. The next step for the authors was to seek the system that makes the Japanese mathematics teachers share such knowledge on teaching practices. Accordingly, the last few parts of the book were spent to discuss the procedures of *lesson study* in Japan – a complex system with well-established methods for the development of in-service teachers' shared practice and knowledge. It is also related to powerful tools for disseminating the knowledge developed by smaller groups of teachers. A lesson study can be shortly described as constituted by teachers' collaborative activities of planning, implementation, observation, evaluation of a lesson, and is usually followed by dissemination of the new knowledge constructed through these activities. (Fernandez & Yoshida, 2004).

The TIMSS video study was epoch-making in research on mathematics education, since it revealed the existence of the homogeneous but radically different cultural script in mathematics teaching, based on a large body of empirical data. It also helped understanding how these differences strongly influence the learning of students. In particular, Stigler and Hiebert's book carefully describes the different aspects of the Japanese lesson study, and forcefully argues for its impact as a tool for constructing, maintaining and sharing teachers' professional knowledge.

1.1.2 Perspective as a teacher, perspective as a researcher

The first time I read *The Teaching Gap* was when I were about to write my bachelor's thesis in 2005, in order to become a mathematics teacher in upper secondary school in Sweden. Combined with my Japanese background, the book made me interested in knowing more about the Japanese teachers' practice. I started to search for Japanese literature on different topics such as task design, problem based teaching approaches, blackboard organisation, and so on. It was surprised to find that there exists in fact a huge literature dealing with the theory and practice mathematics teaching, written by Japanese teachers and teacher educators. In particular, I discovered Kazuo Souma's books about his version of the so-called *structured problem solving approach* (Souma, 1995; 1997; 2000). At that time, I began working as a mathematics teacher in an upper secondary school in Sweden. I remember how I was fascinated (as a math teacher) by the construction of his tasks and the clear descriptions of how to let students work collectively and autonomously with mathematical tasks. I tested his method in my classes and wrote my bachelor's thesis based on these experiments.

All these impressions, beginning with *The Teaching Gap*, directed my interest towards investigating the nature of professional knowledge of mathematics teachers, and in particular to learn more about Japanese teachers' knowledge as a basis for their own teaching practice. Also, I wanted to investigate the possibility of transferring this knowledge to another teaching context, as it was suggested at the end of *The Teaching Gap*. The authors of this book, however, seemed to hold strong normative views of teaching and the teaching profession. Certain expressions cited above, such as "true profession", "the star teachers of the twenty-first century" and "the wisdom of the profession's members" show that their way of describing the teaching is also based on strong personal convictions, according to which the Japanese teachers' practices such as lesson study are considered an ideal reality to be exported and replicated elsewhere. I acknowledge my receptivity to such absolute convictions from the period I worked as a mathematics teacher, and how I could be easily convinced that I saw some positive effects on the students' work when I applied the Japanese structured problem solving approach in my classes. As a researcher however, if any expertise exists in the Japanese teachers' practice, I would like to clarify more explicitly what knowledge they actually use and for what purposes, and exactly what parts of that knowledge are disseminated through lesson study and other activities that the Japanese teachers carry out together, outside of and in connection to their classrooms.

In more recent years, I became engaged in research based on a theoretical research programme of French origin, the *anthropological theory of the didactic* – ATD. ATD is

developed by Chevallard and by other researchers in many different countries, and is still developing with the aim of supporting research on the dissemination of knowledge in institutions. Obviously, a research programme gives a direction in one's research efforts, and my basic assumption is that in order to investigate teacher knowledge, and how it disseminates, one needs to maintain a certain critical distance from the point of view of teachers, by explicitly modelling teachers' knowledge. To elucidate teachers' professional knowledge and its diffusion in their communities, I must first explicitly model – not merely describe – the phenomena to be observed within their teaching practice, and investigate the external conditions and constraints that influence these phenomena. By employing a praxeological analysis (which I will explain in later sections), I attempt to provide a detailed and objective analysis of Japanese teachers' professional knowledge, and use this analysis to investigate the possibilities to transfer the knowledge to other contexts with different teaching cultures.

1.2 PhD project objectives

The general objectives of this research project is twofold. Firstly, describing and analysing Japanese mathematics teachers' professional knowledge and its dissemination from an anthropological perspective. Secondly, to investigate didactic phenomena which arise from attempts to transfer Japanese teaching practice and resources for teaching, to a new context with different teaching environment, including different “cultural scripts” for mathematics lessons. In order to describe and analyse Japanese mathematics teachers' professional knowledge, one first have to identify them. Thus, the presented objectives include providing a model, which allows for a clear identification of methods to accomplish mathematical and didactic tasks. When it comes to investigating how such knowledge is shared among the teachers as established theoretical knowledge, the anthropological perspective allows us to elucidate the conditions that generates such sharing, and also the constraints for transferring the Japanese teaching practice and resources to other contexts. In the next section, I will explain how the three papers presented in the thesis together contribute to achieve these general research objectives.

1.3 PhD project overview

All three papers are concerned with the aim to identify and analyse Japanese mathematics teachers' professional knowledge in relation to their teaching practice, especially, concerning teachers' use of the structured problem solving approach, how the knowledge becomes *shared* and *theoretical* knowledge; we also consider how it relates and transfers to teacher knowledge in other countries.

Paper I investigates the *knowledge taught* concerning the teaching practice of school mathematics within courses in primary teacher education. While our primary interest is in the Japanese case, this paper gives an international perspective through comparison between Japan, Finland and Sweden. We compare both *how* and *what* the teacher educators in each country teach about mathematics and teaching practice in school. In so doing, the paper examines a part of the actual contents of the teacher knowledge and the teacher educators' instructions for the implementation of the knowledge in the classroom. The paper discusses institutional circumstances that formed the teacher educators' activities in each country.

The next stage is investigating how the Japanese in-service teachers apply their knowledge in classroom practice, and work collectively to produce and share practical knowledge, and

subsequently put and share it in more general, theoretical forms. In paper II, we study a public lesson by a Japanese teacher and the following reflection session by all participants, carried out in a so-called *open lesson* within a lower secondary school in Japan. Such open lessons are one of the essential components of Japanese lesson study tradition, which was briefly mentioned in section 1.1.1. The paper describes the teacher's teaching practice based on the structured problem solving approach. The teacher's teaching techniques are modelled and analysed in detail, and the participants' focuses, as observed in their comments regarding the teacher's use of these methods, are analysed in order to identify instances of transforming practical experience into theoretical knowledge. Further, the paper discusses what kind of conditions have generated or framed the participants' focal points, as observed in the comments.

It is apparent that Paper I and II concern especially the first part of the PhD objectives.

Paper III addresses mainly the second objective of the thesis – studying didactic phenomena that arise when attempting to transfer Japanese teaching practices to other contexts, *in case* the transfer of a particular Japanese structured problem solving approach to Swedish lower secondary school. In the study, teaching methods of the implemented Japanese teaching approach, corresponding to several didactic considerations was modelled. Thereafter, based on the model, how the Swedish teacher dealt with these Japanese methods are analysed. Paper III also discusses the institutional conditions and constraints that brought about the didactic phenomena regarding Swedish teacher's use of the Japanese teaching practice.

1.4 Literature review

This section is devoted to a discussion of previous related research, with the purpose of positioning my research in the wider field of research on mathematics teacher knowledge. To provide the review of research literature relevant to the present project, I have considered following four themes, related to the three papers described in the previous section:

1. Theorising teachers' activities for developing their professional knowledge through reflecting and improving their own work
2. Modelling the teacher knowledge taught within teacher education
3. Theorising Japanese teachers' collective work for the improvement of classroom teaching, focusing on the structured problem solving approach in the context of lesson study
4. The theoretical possibility and known experiments related to transferring specific teaching practices from one setting to another one, with a different cultural script for teaching.

In the review, I seek to examine what contributions those studies have provided to the above themes, and also what issues have not been investigated yet. Given the wealth of literature, particularly on theme 1 and 2, and the aims of the present thesis, I have prioritised studies that involve Japanese mathematics teachers' practice and their work to develop and disseminate shared knowledge about this practice.

1.4.1 Theorising the mathematics teacher knowledge

Pedagogic content knowledge and mathematical knowledge for teaching

Throughout the past three decades, a considerable number of frameworks and models have been produced to explain the structure and processes related to teachers' professional knowledge, with or without a specific focus on mathematics. Shulman's model (Schulman, 1986; 1987) of the *pedagogic content knowledge (PCK)* gave rise to the development of more fine-grained conceptualisations of teacher knowledge of the teaching of mathematics (Silverman & Thompson, 2008), especially in the category of teachers' mathematical knowledge for teaching. The notion of *Mathematical knowledge for teaching (MKT)* (Ball, 1990; Ball & Bass, 2000; Hill, Ball & Schilling, 2008) is a subsequent milestone, which explicitly emphasises teachers' *analytical* knowledge about pupils' learning of mathematics. The notion of MKT as a whole constitutes *criteria* for the teacher knowledge required for the teaching of mathematics, and it is formed by the combination of several forms of teacher knowledge in different dimensions (c.f.

Ball, Thames, & Phelps, 2008). The notions of PCK and MKT remained the cornerstones on which new models started to develop, in order to explicitly describe the different aspect of the teacher knowledge. This includes, in particular, the *mathematics content knowledge (MCK)* and the *mathematics pedagogical content knowledge (MPCK)* of prospective teachers, as it is deliberately pursued by *the teacher education and development study in mathematics (TEDS-M)* (Tatoo, 2013). I will describe the detail of the knowledge which the TEDS-M study has developed in the section of *modelling of the content of teacher education*.

Models for teachers' work based on French didactics

I describe now three models based on French didactics, which are relevant to my studies. The models regard theorising teachers' activities for developing their practice, through *reflecting* and *improving* their own work inside and outside the classroom.

Teachers' *observational didactic knowledge*, developed by Margolinas, Coulange and Bessot (2005), describes the processes of individual teachers' learning through reflecting on their interaction with students' work. Their model is based on Brousseau's (1997) *theory of didactic situations (TDS)*, where the students' activities and teachers' interaction with them - e.g. teachers' actions to frame students' interaction with mathematical problems - are described as teachers' didactic *milieu*. The model is constructed based on five different levels (- 1 to + 3) of teachers' reflections on the pupils' activity: from level -1 that concerns pupils' actual activities, to level +3 that concerns more generic issues such as values and conceptions about learning and teaching. The model was applied for the analysis of two teachers' reflections on their teaching activities and in particular the interactions with students during the lessons. The model interprets the data (transcript from the lessons, teachers' comments from the interviews) from different levels in parallel. Thus one could in principle use it to study the process of teachers' learning in a wider perspective – not only about the teachers' direct reflections on their actual interactions with the students, but also in terms of their wider intentions to establish students' learning, and the institutional environment that affects those intentions.

Gueudet & Trouche (2009) theorised the concept of teachers' *documentation work* to capture teachers' implicit knowledge, focusing on how teachers conduct their out-of-class activities, such as task design or planning the details of their lessons, through interacting with various *resources*. The notion of resources for the teachers' work used here contains the artefact in a material sense (a text book, teachers' guide, an interactive board, etc.) and also less tangible resources of teachers' activity (e.g. a discussion with a colleague). Their model further focuses on the outcome of teachers' use of different resources, as they rework and develop resources to

become a *document*, through the production of personal tools based on the resources, such as list of exercises for students, or revised lesson plans. Gueudet and Trouche propose that there exists a dialectical relationship between resources and documents in the context of the continuous process of teachers' documentation work, which they illustrate as a spiral (the set of resources 1 is used to create document 1, and the set of resources 2 entails document 2, and so on). This continuity is enveloped in a *documentation system*, a structured set of the documents that the teacher have developed.

By using this framework, Gueudet, Pepin & Trouche, (2013) examined the collective dimensions of teachers' work – teachers' interactions with their colleagues though resources. They question how resources function to organise the collective work of a teacher or group of teachers, to foster collective work, and to support the development of teacher communities of practice in schools (ibid., p. 1004). To answer these questions, the authors studied two teachers' work within different institutional conditions, and concluded that both teachers share resources with their colleagues as part of their ordinary work. The collective preparation of lessons, as well as task design, was supported by specific resources. This collective preparation work also “required and contributed to, the development of shared professional knowledge” (p. 1014).

In a review of research literature on teachers' work and interactions with resources, Pepin, Gueudet & Trouche (2013) stipulated that the research literature investigating the role of lesson study (implemented in different forms in different countries) is an example of teachers' collective use of particular resources for their professional development. An important characteristic of the resources involved in lesson study is that they are strongly connected to the mathematical content (ibid., pp. 936-937). The authors explain that point by referring Winsløw's (2011) indication of the resources reported in a case of lesson study; “rich resources, including a short and a detailed version of the lesson plan, but also very precise objectives, different possible ways for introducing the ratio, elements about students' skills and potential difficulties” (Pepin, Gueudet & Trouche, 2013, p. 937).

The frameworks of observational didactic knowledge and documentational approach can both be used to model individual teachers' activities of reflecting on and improving their own teaching practice. Using the terms of these approaches, the Japanese teachers' didactic environment comprise an enormous variety of resources or teachers' milieus (educational books available at bookshops, educational conferences for teachers, governmental/regional mathematics developers in every district, etc.) and several systems, such as lesson study and open lessons (Miyakawa and Winsløw, 2013), which in turn draw on the various resources. For

the analysis of the complexity of these systems with different phases, where teachers *collectively* study and *share* their experiences, Winsløw (2011) coined the term *paradidactic infrastructure*. The term is based on Chevallard's notion of *didactic infrastructure* (Chevallard, 2009), and is defined as the totality of conditions for the teachers' work outside of the classroom. Since the teacher is normally alone in the classroom and is at any rate focused on the students in this setting, the activity of sharing observations and reflections on teaching with other teachers must happen *outside* the classroom (Miyakawa & Winsløw, 2013). The notion of paradidactic infrastructure contributed to identify the theoretical distinction between teachers' activities in their classroom (didactic system) and out of the classroom (paradidactic system). I will return to the notion of paradidactic infrastructure in later sections.

1.4.2 Modelling the content of teacher education

Quite a few empirical studies have attempted to measure school teachers' mathematical knowledge for teaching, and to examine its relationship with student outcomes (Hill, Rowan & Ball, 2001; Hill, Ball, Blunk, Goffney & Rowan, 2007; Baumert, et al., 2010). Subsequently, several international studies concern the relationship between prospective mathematics teachers' mathematical knowledge and their knowledge about teaching mathematics (e.g. Li, Ma & Pang, 2008; Durand-Guerrier, Winsløw & Yoshida, 2010; Senk, Tatto, Reckase, Rowley, Peck, et al, 2014). The teacher education and development study in mathematics (TEDS-M) is a large-scale international study of the preparation of primary and lower-secondary in 17 countries from Asia, Africa, Europe, North America and South America. The detailed results from this study are not considered here, as they do not involve Japan. But as was mentioned in the previous section, TEDS-M has developed a framework to model prospective teachers' knowledge for the teaching of mathematics, namely, MCK and MPCK (Tattoo, 2013); it is clearly related to our purposes. MCK stands for "mathematics content knowledge", and MPCK stands for "mathematics pedagogical content knowledge". The MCK contains the domains of *content*, *cognitive* and *curriculum* (ibid. pp. 32-34):

1. *Content*

Subdomains: number and operations, geometry and measurement, algebra and functions, data and chance

2. *Cognitive*

Subdomains: *knowing* (ability to recall, recognize, compute, retrieve, measure, classify/order);
applying (ability to select, represent, model, implement, solve routine problems);

reasoning (ability to analyse, generalize, synthesize/integrate, justify, solve non-routine problems)

3. *Curricular knowledge*

Subdomains: *Novice* (mathematics content that is typically taught at the grades the future teacher is preparing to teach). *Intermediate* (mathematics content that is typically taught one or two grades beyond the highest grade the future teacher is preparing to teach). *Advanced* (mathematics content that is typically taught three or more years beyond the highest grade the future teacher is preparing to teach)

The MPCK contains the domains of *content*, *MPCK-specific* and *curricular* knowledge. The subdomains of MPCK-specific are following (ibid., pp. 34-35):

1. *Mathematical curricular knowledge*

Elaboration: Establishing appropriate learning goals; knowing different assessment forms; selecting possible pathways and seeing connections within the curriculum; identifying the key ideas in learning programs; knowing the mathematics curriculum

2. *Knowledge of planning for mathematics teaching and learning (pre-active)*

Elaboration: Planning and selecting appropriate activities; predicting typical student responses, including misconceptions; planning appropriate methods for representing mathematical ideas; linking didactical methods and the instructional designs; identifying different approaches for solving mathematical problems; planning mathematical lessons

3. *Enacting mathematics*

Elaboration: Analysing or evaluating students' mathematical solutions or arguments; analysing the content of students' questions; diagnosing typical students responses, including misconceptions; Explaining or representing mathematical concepts or procedures; generating fruitful questions; responding to unexpected mathematical issues; providing appropriate feedback

The subdomains of the MPCK described above can be interpreted as detailed categories that considers required teacher knowledge for teaching mathematics in classroom proficiently. It is striking that subdomains 2 and 3 above are quite similar to the characteristics of the Japanese structured problem solving approach (see following sections). I will discuss this issue in the later sections of this introduction.

Li, Ma and Pang (2008) point out, what influence prospective teachers' learning about the teaching of mathematics during teacher education, is not only *what* they learn but also *how* they

are taught. However, there is actually little research that describes how teacher educators normally teach the contents of their courses within mathematics teacher education; in particular, there is “limited information available about instructional approaches used in courses for prospective teachers in East Asia...” (Li, Ma & Pang, 2008, p. 49).

There are of course some studies available on the issue. The 15th international ICMI study, on “the Professional Education and Development of Teachers of Mathematics” (Even & Ball, 2009), presented several themes regarding mathematics teacher preparation and in-service development of teachers. The general question stated in the introduction of the section with the theme “the preparation of teachers” is “*What professional skills, what attitudes are to be acquired for the teaching of mathematics?*” (ibid., p. 13). In the text, the following statement demonstrates the lack of clarity which prevails in general, regarding the concrete content required for the preparation of teachers:

Learning to teach (...) requires a balance between teachers’ theoretical and practical knowledge and skills including knowledge of mathematics, knowledge of teaching mathematics, and knowledge of psychology and pedagogy. These components are only general; they do not answer the basic question about the content and extent of the knowledge required from future teachers (p. 13).

In order to identify at least some overall structure in this area, Liljedahl et al. (2009) discuss the different dimensions of teacher knowledge within the teacher training. They present a possible ideal structure of initial mathematics teacher education, in an analogy with a braid; in the beginning of the braid, three different types of knowledge (mathematical, pedagogical, and didactical contents) are presented as discrete strands. Then, later on in the educational programmes, the discrete knowledges begin to integrate, and finally form a united fibre. The *didactical knowledge* mentioned here is defined according to Winsløw and Durand-Guerrier (2007): the knowledge that concerns the social conditions of mathematics teaching and learning, and the design of didactical situations (in the sense of Brousseau, 1997) corresponding to specific target knowledge (ibid., p.7).

Liljedahl et al.’s (2009) view is developed subsequently to describe a phenomenon which is widely observed within teacher education, and which Bergsten and Grevholm (2004) named the *didactic divide* – the disconnection, in teacher education, between subject matter courses and courses on pedagogical knowledge (and possibly even more domains or disciplines, such as psychology, ethics, etc.). Earlier, Ball and Bass (2000) have pointed out the issue of the current situation in which mathematics teacher education is delivered, with components that do not connect well:

This divide has many traces. Sometimes it appears in institutional structures as the gulf between universities and schools (Lagemann, 1996). Sometimes the divide appears as fissures in the prevailing curriculum of teacher education, separated into domains of knowledge, complemented by “experience”–supervised practica, student teaching, practice itself. In all of these, the gap between subject matter and pedagogy fragments teacher education by fragmenting teaching. (Ball & Bass, 2000, p. 85)

As the text indicates, the phenomena of the didactic divide is related to the different institutional bodies in and around teacher education. Nevertheless, I suspect that the lack of “didactical knowledge”, (Winsløw & Durand-Guerrier, 2007), plays a crucial role in bringing about the phenomenon. However, there are only few research studies (such as TEDS-M) that systematically investigate what kind of didactical knowledge (in the sense of Winsløw and Durand-Guerrier, 2007) the teacher educators teach in the courses of the teacher training, and how they do so.

1.4.3 Structured problem solving and lesson study

This thesis investigates the Japanese teachers’ professional knowledge, especially their shared theoretical knowledge about the use of the structured problem solving approach. Shimizu (1999) described the basic flow of the Japanese structured problem solving approach with the Japanese didactical terms (pp.109-111): 1. *Hatsumon*: to ask a key question that provokes students’ thinking at a particular point in the lesson. 2. *Kikan-shido*: teachers’ instruction at students’ desk. Scanning by the teacher of students’ individual problem solving process. 3. *Neriage*: whole-class discussions. A metaphor for the process of polishing students’ ideas and of developing an integrated mathematical idea through whole-class discussions. 4. *Matome*: summing up. The teacher reviews what students have discussed in the whole-class discussion and summarizes what they have learned during the lesson. These terms describe teachers’ key roles, and are used by Japanese mathematics teachers on a daily bases. Japanese problem solving approach had appeared with similar structures in several variations (Hino, 2007). One of the such variations is *Open-ended approach* with open-ended problems (Becker and Shimada, 1977). An open-ended problem is a conditional or incomplete problem. Since it does not contain a single *correct answer*, the process of searching for an answer develops different methods and leads to dialogues between the students (ibid.). The *open approach method* is a similar approach with focus on students’ interest in participating in mathematical activities and at the same time to foster their mathematical thinking (Nohda, 1991; 1995).

International research concerned with Japanese teachers' collective work to develop their proficiency on and knowledge about structured problem solving, has mainly appeared within the field of the research on Japanese lesson study. The reason for this is that the establishment and development work (both by teachers in service and researchers) of the structured problem solving approach has always been carried out in the context of the studies of research lessons (or open lessons) implemented within a lesson study (Hino, 2007). Fuji (2018) explains the relationship between lesson study and teaching mathematics through problem solving as “two wheels of the same cart” (p. 2). He explains the normative aspects of the problem solving approach, which implies that Japanese teachers teach “not only content but also processes (of learning mathematics)” (p. 19).

The phenomenon of Japanese lesson study has been studied by researchers outside Japan for several decades now. One of the early examples is Lewis and Tsuchida's study from 1997. They reported on Japanese teachers' systematic collaborative work, where the teachers' “research groups” implement “research lessons” that are open to observation by other teachers, how the teachers “borrow other teachers' teaching ideas” in their collaboration, make “self-critical reflection” and share the developed professional knowledge about teaching. There were few such studies about Japanese lesson study at the end of 1990's (Winsløw, Bahn & Rasmussen, 2018). The breakthrough of Stigler and Hiebert's *The Teaching Gap* (1999) gave huge publicity to Japanese lesson study, and ignited many researchers' interest in studying the impact of this collegial learning system of teachers around the world. Winsløw, Bahn and Rasmussen (2018) categorised different types of such research related to lesson study, which were carried out by both Japanese and non-Japanese researchers during the last three decades, (p. 126):

1. Papers describing and analyzing what lesson study is in Japan
2. Papers describing and analyzing what lesson study is or could be in other countries
3. Papers reporting on experimental research using lesson study as a method to investigate specific questions related to mathematics education

For instance, the above-mentioned study by Lewis and Tsuchida (1997), as well as several other studies by Lewis (e.g. 2000; 2016), Fernandez and Yoshida (2004), Elipane (2012, doctoral thesis), and Takahashi (2014) belong to the first category. The literature falling mainly within the second category are very numerous (e.g. Isoda, Stephens, Ohara, & Miyakawa, 2007; Hart, Alston & Murata, 2011; Inprasitha, Isoda, Wang-Iverson & Yeap, 2015; Quaresma, Winsløw, Clivaz, da Ponte, Ní Shúilleabháin & Takahashi, 2018; Huang, Takahashi & da Ponte, 2019) and

there are also special issues of international journals focused in this direction (e.g. Huang & Shimizu, 2016), as well as a relatively new journal more or less devoted to this theme (*International Journal on Lesson and Learning Study*). Further, there are clearly two tendencies among the studies which have appeared in recent years: first, descriptive research with lesson study as a *research object*; and secondly, intervention research, which uses lesson study as a *method* (Winsløw, Bahn & Rasmussen, 2018, p. 127). Crossing these classifications, research that is located nearest to my study (specifically paper II) is descriptive research that analyse Japanese lesson study as a research object, without intervention on the part of researchers. In their chapter, Winsløw, Bahn and Rasmussen (ibid.) emphasize the importance of theoretical precision in all research related to lesson study. They argue that implementation of lesson study is strongly related to cultural and institutional features, but lesson study itself cannot be realised with just descriptions of procedures, which teams of teachers, facilitators and researchers can follow. Instructional manuals just describing the process of lesson study is not sufficient to realise its essential goals: to promote the participants' learning during different stages of lesson study.

(...) a theoretical framework (with explicitly defined categories and terms) is needed to move the analysis of mechanisms and principles of lesson study away from the culturally contingent narratives about lesson study, with which the literature abounds (...). In scientific terms, this requires more precise models of what lesson study is and is about—based on theoretical frameworks, which are shared and developed by researchers. (ibid., p. 126)

Huang and Shimizu (2016) reviewed research papers within two theoretical perspectives on research on lesson study – research examining the entire process of lesson study, and the core component of lesson study. Regardless of categories, they found a tendency of researchers to use cognitive theories and socio-cultural theories, or some combination of these two kind of frameworks. Now I take a closer look on the research that refer to the first category.

Lewis, Perry and Hurd, (2009) modelled the mechanisms of lesson study in terms of four lesson study phases or cycles (investigation, planning, research lesson, and reflection) that appear in chronological order, and three categories of participants' instructional improvement (changes in teachers' knowledge and beliefs; changes in professional community; and changes in teaching–learning resources). The model was applied as criteria to a long-term case study of North American lesson study to show an “existence proof” of the effectiveness of lesson study outside Japan (ibid., p. 302). According to the model, the result shows that lesson study led to

distinct improvements of the participants' mathematical content and pedagogical knowledge, and their knowledge about student thinking, by "building teacher professional community, and by improving teaching materials" (ibid., p. 302). Lewis & Lee (2016) applied the notion of *lesson study ecology* to investigate the conditions of the establishment and development of the lesson study outside Japan in educational, social and school political context. The authors also reported statistical data on the implementation of lesson study in Hong Kong and Singapore. They concluded that the stakeholders' (teachers, school and district administrator, researchers, textbook publishers and national policy makers) initiative and organisational support crucially promote the function of lesson study as "an activity valued as professional learning, as research, and as policy enactment and study" (p. 201).

The notion of *paradidactic infrastructure* was used by Miyakawa and Winsløw (2013) to model the wider conditions for Japanese teachers' learning and the distribution of their professional knowledge concerning the teaching practice of mathematics. Their model is based on ATD, and studies one case of the Japanese teachers' open lesson in depth. The teachers' activities is modelled in the stage of preparing of the lesson, evaluating and revising of the lesson (post-lesson reflection), and the whole activity including both stages as a whole (open lesson). The authors investigate the lesson plan, the implemented lesson and participants' observations, and the comments from the discussions by the participants during the post-lesson reflection session. Miyakawa and Winsløw conclude that the open lessons, as a shared paradidactic practice, support Japanese teachers to develop common theoretical knowledge related to didactic practices.

1.4.4 International transferring of professional scholarship

Studying foreign teaching practice, which is supposed to underlie successful performance within big-scale international studies such as PISA and TIMSS, is an intriguing endeavor for researchers and educators. Implementing such foreign teaching approaches implies transferring teachers' instructions to countries with different cultural scripts for teaching. It requires careful considerations to organise curriculum and lessons based on foreign approaches because of the different conditions in different countries. There are a number of studies exploring e.g. Singaporean, Chinese, and Taiwanese ways of teaching mathematics as they appear in the countries of origin (for instance the 13th ICMI Study). However, longitudinal studies that apply and carry through those foreign approaches in another country are seldomly found. One exception is in fact the implementation of the use of Japanese approaches – usually intertwined

with lesson study – to American teachers’ practice in the United States. On main impetus for this line of research was in fact Catherine Lewis’ work to disseminate observations made in Japan, including widely distributed lesson videos. But these studies usually do not have intention to question the constraints or obstacles that make the implementation of lesson study or structured problem solving inconvenient.

Indeed, I found only a few papers having the foci on the limitations or constraints of applying the Japanese lesson study abroad. In particular, Groves, Doig, Vale and Widjaja (2016) report certain limitations for the implementation of Japanese lesson study in Australia with the structured problem solving in focus. They described how the Australian teachers’ beliefs, in which they value students’ individual or group work on the task, rather than whole-class activities, different physical layouts of classrooms, in particular the lack of a large blackboard in Australian classrooms, make it difficult to carry through methods commonly employed by Japanese teachers. Also, discrepancies between the curricula of Japan and Australia caused difficulties in finding suitable tasks for Australian curriculum within Japanese resources such as lesson plans. Takahashi (2011) and Fujii (2014) stated the conditions and constraints regarding the implementing of the Japanese lesson study outside Japan. Similarly, Takahashi (2011) discusses the underlying reason that establishing lesson study takes long-term commitment by the teachers and schools in USA, and needs strong supports of lesson study expert from outside schools to understand and improve the impact of lesson study on teaching and learning. He identifies these factors as: a lack of experienced lesson study practitioners outside Japan, and the complex construction of a lesson study that includes both a new teaching approach (problem solving approach), and a new form of professional development. He explains that for the Japanese teacher, learning the process of lesson study is easy, since they experienced lesson study with problem solving as a teaching approach since they were pre-service teachers, and have a great amount of opportunities to participate in lesson study and in different types of open lessons as a novice teacher.

In his case studies from two African countries, Fujii (2014) discusses more directly the issue of transferring the structured problem solving approach, and identifies several aspect that a different from those found by Takahashi. He points out African teachers’ “misconceptions” of the use of the structured problem solving approach. They had “grasped the Japanese approach *superficially*” (ibid., 73) and essential features of the Japanese structured problem solving were consequently missing in African teachers’ lessons. Fujii stresses the “value” of the tasks repeatedly in his paper; “it is meaningless for students to be able to complete a task if the task

itself is not a valuable thing” (p. 79). He states also that examining and investigating the values of the tasks through meticulous *kyozai-kenkyu* (teachers’ study of teaching resources and construction of new ones) is a crucial factor for the implementation of lesson study. “Value” refers, here, to the aims of mathematics education:

(...) the heart of lesson study is the consideration of the educational value or aim. In fact, we always see things in lesson study from the educational value viewpoint. This proposition is not to be proved. It may be an axiom of Japanese lesson study. People outside Japan have made us realise the critical features and value of lesson study. This lack of awareness is to some extent a flaw. When people do good things without awareness, the most regrettable case is that people lose it without hesitation.

Therefore it is of benefit to both Japanese and foreign educators for us to identify the authentic nature of lesson study. (p. 81).

Fujii’s argument explains that Japanese teachers apply the structured problem solving and lesson study to realise an educational goal. In the article, using the metaphor of “two wheels”, he also argues the Japanese teachers “unconsciously” use the problem solving approach to achieve the educational goals, and states that the international research around lesson study made apparent the needs of the identification of this normative aspect of the approach:

We could say that Japanese educators have been implementing the “teaching mathematics through problem solving” approach to the curriculum without a defined theory or consciousness. However, this new insight demands that the concept of “teaching mathematics through problem solving” be fully described, along with lesson study, as the two concepts are two wheels of the same cart. (Fujii, 2018, p. 19)

Catherine Lewis, already in the middle 1990s, had described how Japanese educators work to translate educational policy into actual teaching and learning practice. She paid great attention to the Japanese mathematics teachers’ ways of improving their pupils’ collective learning of science and mathematics in preschool and elementary schools, and named the Japanese school class a *community of learners* (Lewis, 1995). She describes Japanese teachers’ endeavour to ensure that pupils will respond “supportively to one another’s thoughts and feelings” (ibid., 176), eliciting the pupils’ own ideas, helping them reflect on other pupils’ ideas, and so on. Lewis and Tsuchida (2004) point out that “Japanese national goals focus on the whole child (social, ethical and intellectual development), a breadth which, we speculate, may reduce the kind educational policies.” (p. 313).

While Groves et al (2016) and Takahashi (2011) described critical factors for the transfer of rather practical aspects of lesson study, Fujii (2014) and Lewis (e.g. 1995) discuss wider

social and educational aspects, there is in fact little research that studies the phenomena emerging from attempts to transfer new (but well established abroad) teaching practice from the perspective of a *science of didactic phenomena* (Bosch & Gascón, 2014), based on a strong theoretical framework as suggested by Winsløw et al (2018). Brousseau, in his book *Theory of Didactical Situations* (Brousseau, 1997), identifies several didactic phenomena, such as *didactic contract*, *Topaz effect*, *Jordain effect* (ibid.) that appear in the interactions of teacher and student. Brousseau attempts to construct theory-based intervention or *didactic engineering* to provide a methodology to overcome such phenomena in the context of mathematics teaching and learning. He was the first who “postulated the existence of didactic phenomena which appears in the forms of unintentional regularities in the processes of generation and diffusion of mathematics in social institutions and are irreducible to the corresponding cognitive, sociologist or linguistic ones” (Bosch & Gascón, 2006, p. 54). One initial phenomenon that led Brousseau to create the theory of didactical situations was called *Dienes’ effect* (Sierpiska, 1999). Brousseau studied the implementation of Hungarian mathematician Zoltan Pal Dienes’ teaching approach in French schools, which was popular in many countries in the 1970’s. He observed that this well-established teaching approach did not give expected result if the teacher simply rely on the original structure of Dienes’ approach, and does not engage to adapt and process the details of the activities for her class. Brousseau explains:

The more the teacher is assured of success by means of effects that are independent of her personal investment, the more she is likely to fail! We call this phenomenon, which shows the necessity of integrating the teacher-student connection in any didactical theory, the *Dienes effect*. (Brousseau, 1997, p. 37)

Brousseau here discusses individual teacher’s *epistemological assumptions* – and more widely, teachers’ mathematical and didactical knowledge – and their importance for the teacher to learn and control a new practice. However, the institutional conditions and constraints that could contribute the phenomena of failure, were not Brousseau’s main concern. In the next section, I will describe the framework that I applied in this thesis in order to focus on this aspect.

1.5 The anthropological theory of the didactic

This section will serve for the presentation of the theoretical framework of the thesis, namely, the anthropological theory of the didactic (*Théorie anthropologique du didactique* in French, cf. Chevallard, 1999a). As is customary we use the abbreviation ATD. The development of the ATD was initiated by Yves Chevallard in the 1980s, beginning with the *theory of didactic transposition*. Since then, ATD has been advanced and offered several different tools, methods and results to study various problems regarding the dissemination of knowledge in different forms of institutions, and that is why this framework is in fact a *research program*. ATD follows the tradition of a science of didactic phenomena, and aims to study the diffusion of any kind of knowledge (Bosch, & Gascón, 2014). In this section, I will describe how and which of the tools provided by ATD was applied to investigate didactic phenomena within various *didactic systems* (Chevallard, 2019), observed in my empirical studies. First, I begin with the basic standpoint of studying phenomena related to teaching and learning of mathematics from the anthropological perspective.

1.5.1 Didactic transposition

The anthropological attempt to study the mathematical knowledge begins from a simple question: *What is the thing you call (a piece of) knowledge?* (Chevallard, 2019). Chevallard considers that knowledge is “a changing reality” (2007). It takes distinct forms – *transposed* in different ways, in different institutions and educational systems. However, the reality of such different versions of knowledge is not recognised by the majority of us. French sociologists used to refer to such phenomena as the “*illusion of transparency*” – the tendency that people believe that knowledge somehow reflects and belong to an unproblematic, nature-like reality of the world. In the context of the teaching of mathematics, it appears, for instance, as a teacher believes that the knowledge contained in the school mathematics does not differ essentially from the mathematics as scientific disciplinary knowledge, and he does not question exactly what knowledge is or should be taught in the classroom, assuming that the prescribed knowledge to be taught is simply an excerpt of scientific knowledge (Chevallard, 1992).

Chevallard’s *anthropology of knowledge (anthropologie des savoirs)* is “an extended epistemology, which recognises that knowledge usually is an object that is to be used and taught (Sierpiska & Lerman, 1996, p. 856). Chevallard calls the scientific disciplinary knowledge produced by e.g. scientists as *scholarly knowledge*, and considers that such knowledge is nothing else but the *used* knowledge (Chevallard, 1989). The scholarly knowledge will be transposed

when it applies within different educational systems. Chevallard explains the difference between *used knowledge* and *taught knowledge* as following:

As long as you only use knowledge in doing something, you need not justify nor even acknowledge the used knowledge in order to endow your activity with social meaning. Its meaningfulness derives from its outcome, judged by pragmatic standards. Knowing something in this case, is close to, and even inseparable from, knowing *how to do* something. Knowledge and know-how enjoy the status of means to an end, which is the standard by which their *relevance* as tools of the trade will be judged. In contrast, teaching requires the social acknowledgement and legitimation of the knowledge taught. In going from used knowledge to taught knowledge, relevance gives way to *legitimacy*. Teaching some body of knowledge cannot be justified only on the grounds that the knowledge taught could be useful in such and such social activities. (Chevallard, 1989, p.59)

The process of adapting the scholarly knowledge for make it teachable within a given institution is called a *didactic transposition* (ibid.). The concept itself was introduced in the 1970s by the French sociologist Michel Verret, as in the term of *transposition didactique* (Verret 1975 quoted in Bergsten, Jablonka & Klisinska, 2010). Verret emphasises that the knowledge produced and *used* in the scientific community cannot be taught in educational institutions (even universities!) in the same form as it had there (ibid.). Chevallard emphasises that “transposition” does not mean something simply changed its place. For the better understanding for us readers, he gives a metaphor of the transposition in music (Chevallard, 1999b); one can transpose one key to another, and it still has the same melody, but many things have to adapted. Using this metaphor, he explains how knowledge transposes within educational systems:

Knowledge is *not* a *substance* which has to be transferred from one place to another; it is a world of experience which, through a creative process, has to be... transposed, to be adapted to a different ‘key’ – the child – and to a new ‘instrument’ – the classroom (1999b, p. 7).

Figure 1.5.1 illustrates the whole process of the didactic transposition. The first knowledge transposition from the scholarly institution to the educational institution is called *external didactic transposition*, since it is the step of which the used knowledge will be transposed, outside of the educational institution, into *knowledge to be taught* – for instance into a national curriculum for the school subject of mathematics. The rest of the transposition within the institutions of school systems is called *internal didactic transposition*. Teachers step into the transposition between the knowledge to be taught to the *taught knowledge*. They, for instance, interpret the national curriculum and make the mathematical content teachable in their classes, with particular students. It constitutes a complex work of the teachers – making lesson plans, studying the national curriculum through discussions with other teachers, exploring different

resources, and so on. The final step of the transposition takes place in the interaction between the teachers' and the students' actions, namely, transposition between the taught knowledge and *learnt knowledge*. Teachers' didactic work for the teaching and the students' activities for the learning produce the students' knowledge *actually learnt*.

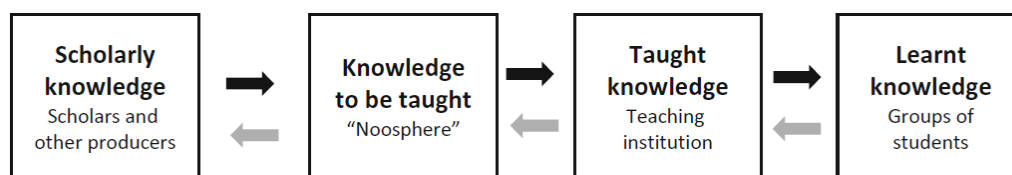


Figure 1.5.1. The process of didactic transposition (Bosch & Gascón, 2014, p. 70)

I did not directly apply the didactic transposition theory in the papers presented in the thesis. However, since one of the aims of the thesis is investigating the Japanese teachers' professional knowledge and its dissemination, it is important to relate the different elements of the teacher knowledge, discussed in the papers, to the process of the didactic transposition. In paper I, the discrepancy between the contents of the national curricula found in Japan, Finland and Sweden was investigated. The observed phenomena are related to both external and internal didactic transposition. How national curricula support the teacher educators' design of their lessons is related to the internal transposition in teacher education institutions. In paper II, the Japanese teachers' design of the lessons and his use of teaching techniques, and the participants' foci on the demonstrated lesson, concerns mainly the last two steps of the transposition. The discussion on the distinction of the basic view regarding the relationship between the educational aims and teaching practice expressed in the national curricula in Japan and Sweden in paper III concerns both the external and internal transpositions.

In the next section, I will describe the *theory of praxeologies*, which was the main tool used in the thesis. The section mainly draws on "*Introducing the anthropological theory of the didactic: An attempt at a principled approach*" (Chevallard, 2019).

1.5.2 Praxeological analysis

While the anthropological question related to the theory of didactic transposition was "what is knowledge?" the anthropological question related to the *theory of praxeologies* is "*What is knowing?*" (Chevallard, 2019, p. 77). The notion of *praxeology* was introduced to model

mathematics learning, teaching, disseminating and transposing of the knowledge as ordinary human activities (Bosch, & Guscón, 2014). A praxeology is, according to French anthropologist Marcel Mauss (1872-1950), a *social idiosyncrasy* which amounts to “an organised way of doing and thinking contrived within a given society – people don’t walk, let alone blow their nose, the same way around the world” (Chevallard, 2006b, p. 23).

The starting point of the anthropological postulate of the theory of praxeologies is that every human activity (e.g. “to watch a movie”, “to drive a car”, “to take care of children”, “to do a homework”, “to work out a subtraction”...) splits into a number of basic “parts” associated to *tasks* (Chevallard, 2019, p. 83). The second postulate is that there exist some “way of doing” classes of such, referred to as tasks of type T; such a way of doing is called a *technique*, written as τ (ibid.). The pair of a type of tasks T and a technique τ to realise the tasks $t_1, t_2, t_3...$ (which build the subset of the set T) $[T / \tau]$ is called a *praxis block*. It is the *know-how* component of “knowledge” – in another word, a *skill*. It lies in the nature of human beings that we ask why such a technique τ_1 is required to solve the task t_1 . The *technology* is a “discourse” on τ that explains τ and justifies its legitimacy. “The aim of a technology θ is to make the technique τ intelligible, to explain why it is what it is—and why it is as it is and not otherwise—even if there exist “competing” techniques τ' for T” (ibid., p. 87). The *theory*, denoted as Θ , is in turn the discourse on the technologies, and justifies a given technology. Chevallard explains that the notion of mathematical theory in the sense of ATD includes other unanalysed “anthropological” (thus nonmathematical) elements, which mathematicians ignores, and an associated theory Θ always exist for any triplet of $[T / \tau / \theta]$;

This tenet implies, for example, that when one brushes one’s teeth, this person’s behaviour is partly determined by the theoretical “ideas” the person holds about dental hygiene, and these ideas may in turn be determined by the person’s relations to many objects (toothbrush and toothpaste, obedience to parents in the case of a young child, a sense of the fragility of life, daily rituals, etc.). As a rule, a theory Θ in the sense of ATD has thus to do with a host of objects which, through the intermediary of praxeologies, contribute to shape persons’ and institutions’ behaviours. (ibid., p. 91)

The pair of a technology θ and a theory Θ $[\theta / \Theta]$ is called a *logos block* (*logos* is Greek for “word”, “speech”, “statement”, and “discourse”), or *theory block*. It constitutes *know-why* component of knowledge, hence, according to ATD, “knowledge” is the *dialectical union* of logos and praxis (ibid.). The quadruplet $[T / \tau / \theta / \Theta]$ thus constitutes a praxeology.

A set of praxeologies of *mathematical* “knowledge” is also called a *mathematical organisation* (MO) (Barbé, Bosch, Espinoza & Gascón, 2005), or simply, *mathematical*

praxeologies. The quadruplet $[T / \tau / \theta / \Theta]$ of a mathematical organization consequently deal with mathematical types of tasks, techniques, technologies and theories. Technologies of the MO can be explained e.g. tools for discourse on and justification of the techniques for solving mathematical tasks, like theorems that legitimate the accuracy of the techniques. Theories of a formal MO include discursive entities such as definitions and proofs that may also help to connect a mathematical praxeology with other mathematical praxeologies.

To organise the activities of teaching and learning, teachers constitute a *didactic organisation (DO)*, or *didactic praxeologies*. The internal didactic transpositions are mediated by teachers' DOs (Winsløw and Møller Madsen, 2008), since it is the teachers' work to process and adapt (transpose) the knowledge to be taught to the taught knowledge for the learning in the classroom. The *didactic tasks* of a didactic organisation thus aim to achieve the planned mathematical organisation. Teachers' *didactic techniques* include mainly the use of different resources and tools, such as mathematical problems, questions, blackboard, ICT, orchestration of mathematical discussions of students, etc. The logos block of the didactic organisations explains the legitimacy of the DO praxis. Didactic technologies is thus shared (consciously or unconsciously) by the teachers in their communities, and didactic theory "contributes to shape institutions' behaviours" (see the citation of Chevallard, 2019 above).

In this thesis, I employed the praxeological analysis of the Japanese teachers' didactic organisations in all three papers. The focus of the analysis was to investigate the theory block of teachers/teacher educators' didactic praxeologies. When we could identify the didactic theories that have shaped the Japanese and Scandinavian teachers' behaviours (as a member of their respective institutions), we can then study the social, traditional and cultural backgrounds that brought up those didactic theories, and thereby we are able to find the conditions and constraints which generate several didactic phenomena observed within the empirical studies.

In paper III, another tool from ATD, namely, *reference epistemological model*, was employed as criteria to investigate the Swedish teacher's use of the didactic techniques which were (attempted to be) transferred from Japanese structured problem solving approach. Bosch and Gascón (2006) state the necessity for researchers to elaborate their own particular epistemological model as a reference point, in order to study the body of knowledge, which rose to the surface by the analysis of didactic transposition. Bosch and Gascón argue that using "scholarly knowledge" for the didactic research on the epistemology of mathematics (e.g. "what is mathematics?", "what is algebra in elementary school?") is no longer valid, since the scholarly knowledge itself has become an empirical object to study. Figure 1.5.2 shows the position of the

researchers of didactics (the didactician). The reference epistemological model (REM) constructed by the didactician should be considered as a “working hypotheses” which has “certain specific features” (Ruiz-Munzón, Bosch & Gascón, 2013) stemming from the purpose and position of the researcher. That means, the model needs to be developed continuously by the community of researchers by their epistemological analysis, and the empirical data taken to build the reference model come from different institutions (e.g. scholar mathematicians, educational policy makers, school class...). “It is important that REM do not uncritically assume any of the viewpoints that are dominant in these institutions” (ibid., p. 2872). The empirical data used in the reference model made in paper III is based on the praxeological analysis on the didactic organisation of the applied Japanese teaching approach.

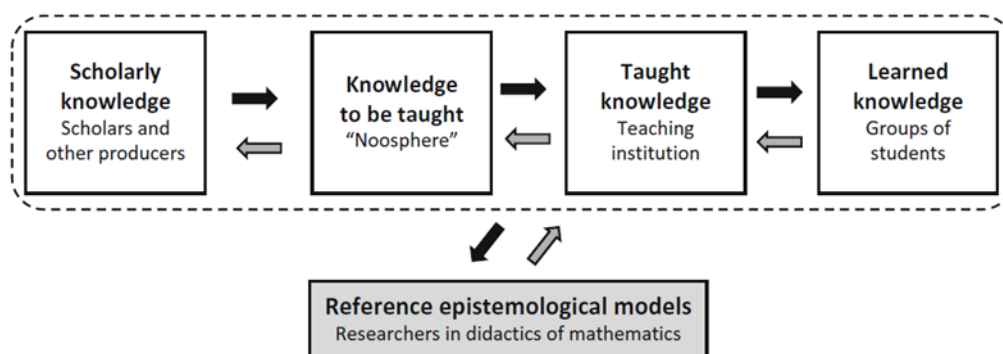


Figure 1.5.2 The external position of researchers (Bosch & Gascón, 2014, p. 71)

1.5.3 The level of didactic co-determination

When we consider a praxeology as a model of a human activity of “knowing” or “learning” something, essentially, there are always learners (e.g. students), actors who help the learners (e.g. teachers) and a so called *didactic stake* – something to be learned by the learners. Such a system is called a *didactic system* (Bosche & Gascón, 2014). To what extent the learners’ mathematical praxeologies around the didactic stake will be constructed depends on the teachers’ didactic praxeology, and vice versa. The dynamics of this mutual relation of two organisations in didactic system is called *didactic co-determination*. Mathematical and didactical organisations therefore cannot be separated because of their mutual interaction (Dorier & Garcia, 2013).

The complexity of the praxeologies depends on the *ecology* of these mathematical and didactical praxeologies, that is, conditions of all kinds and origins, that form a given praxeology. Chevallard (2019) models the hierarchy of such conditions as a *scale of levels of didactic co-*

determination, where the didactic system is located in the lowest. The didactic conditions that shape didactic stakes (taught knowledge) is displayed as five-tiers of different degrees of mathematical knowledge: *discipline*, *domain*, *sector*, *theme*, and *subject* (Fig. 1.5.3). Each level seen in the figure is a “seat of conditions” specific to that level (Chevallard, 2019, p. 95). The model allow us to explain how conditions from various levels outside the didactic systems influence the realities of the praxeologies at any levels of the didactic system.

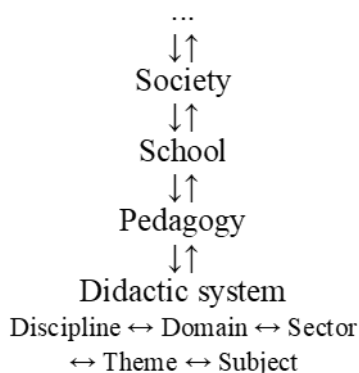


Figure 1. 5. 3 The scale of levels of didactic co-determination

In this thesis, the scale of levels of didactic co-determination was applied to investigate what levels the Japanese teachers focus on when they discuss the use of didactic techniques within a lesson (paper II). In paper III, the model was used to explain how educational and mathematical considerations regarding the teaching practice differ between the Japanese and Swedish teachers.

1.5.4 Paradidactic infrastructure

Teachers’ work does not remain exclusively inside the didactic systems. Their work extend even *outside* the didactic systems – to *paradidactic systems* (Winsløw, 2011). Teacher’s paradidactic systems exist to produce knowledge and resources for and about didactic systems, and the range of paradidactics is potentially quite wide. The first aim one associate with teachers’ activity (or work) in paradidactic system is to design a didactic system – i.e. (in most cases) the dynamics of a lesson. However, as we can see in Margolinas et al. (2005), Gueudet & Trouche (2009), teachers also *reflect* on their own didactic systems, for instance to develop future ones. Moreover, teachers do this reflecting work even by observing other didactic systems in different settings, as it appears in open lessons (Miyakawa & Winsløw, 2013) in Japanese lesson study, or within their *practice research* (Miyakawa & Winsløw, 2019). A typical paradidactic system which is a subset of lesson study (which itself is a bigger paradidactic system) is, as it is

described in section 1.4.4, *kyozai-kenkyu* (Watanabe, Takahashi, & Yoshida, 2008) – exploring different resources for use in didactic systems. For instance, plethora of educational books written by researchers and teachers provide resources for Japanese teachers to carry out *kyozai kenkyu*. All these large and small paradidactic systems are conditioned by the *paradidactic infrastructure* (Winsløw, 2011). “Infrastructure” itself is a general concept. It refers to “the underlying base needed to develop any determined reality” (Chevallard, 2019. P. 84). In the thesis, paradidactic infrastructures in Japan and Scandinavian countries were described to explain the different conditions that exist in these countries for the dissemination of established teacher knowledge.

1.6 Research questions and methodology

1.6.1 Research questions

As stated in section 1.2, the PhD project has two general objectives: describing and analysing Japanese mathematics teachers' professional knowledge of their teaching practice and its dissemination, and to investigate didactic phenomena, which arise by the transfer of Japanese teaching practice and resources for teaching to a context with different didactic and paradidactic infrastructure, in particular with different “cultural scripts”. These objectives have led to the following general research questions (RQ):

- RQ1. What are the main characteristics of the Japanese mathematics teachers' professional knowledge?
- RQ2. In what way can the theoretical and methodological tools from the ATD be used and developed to identify and analyse Japanese teachers' professional knowledge of teaching and its dissemination in their community?

In section 1.7, I will elaborate how these research questions are then specified and elaborated in the individual papers. All papers address more or less both questions.

1.6.2 Methodology

Remarks on the research process

This thesis is the result of a long research process which involves two institutions and a number of choices and change of perspective which have contributed to the maturation and shape of the final result. The process did not follow a standard recommended procedure of PhD-projects, where data collection is only begun once fully mature research questions are formulated. This may in part be due to circumstances but I also believe that the complexity of the research object – the professional knowledge of Japanese mathematics teachers – necessitated, for the researcher, a more dialectic process in which data collection *interacted* with progressive sharpening and finalization of the research perspective.

The empirical data on which this thesis is based were collected in the period 2010-2016, which includes most of the time during which I prepared my licentiate thesis at the University of

Linköping (Asami-Johansson, 2015). The focus of my licentiate project (2010-2015) was in fact on the adaptation and implementation of Kazuo Souma's version of the structured problem solving in Swedish schools. The data related to this intervention project were collected from 2010 to 2011 in a Swedish secondary school, and these data are also analysed with completely different methods and perspectives in paper III.. I started collecting the empirical data for paper II in 2011 and later paper I between 2014 to 2016. Thus, the chronological order of data collection was totally opposite to the order of writing the papers. And as a matter of fact, while I developed most of the basic interests and aspirations behind the thesis prior to 2010, the actual research questions and results presented in paper I-III were only shaped and carried out during my time as a PhD-student at the University of Copenhagen.

Data collection

Paper I

To highlight the characteristics of the Japanese teacher educator's mathematical and didactic praxeologies of lesson, and later to identify the conditions and constraints that form these praxeologies, we applied an international comparative method for paper I. As contexts of comparison, we chose mathematics teacher education lessons in Sweden and Finland. The choice of Swedish teacher education was very natural, since the idea of the paper was based on reflections about my own lessons in teacher education courses in Sweden, where there are few standard resources available for teacher education – teacher educators generally design their teaching in personal ways, based on personal teaching materials and compendiums, and only occasionally share practices and resources. The reason that I recognized this lack of common methods was that I had previously had the opportunity to observe method courses for prospective teachers in different Japanese universities. There the teacher educators gave very similar lessons; thus I inferred that they had to possess some kind of shared resources for the design of their lessons. I also became interested to study the actual discrepancy in the contents taught in mathematics methods courses in Japan and Sweden, and the conditions, which could explain both the contents and common form of the Japanese courses.

To put the Swedish and Japanese in further perspective – and in particular to go beyond cultural contexts with which I have been directly engaged – I decided to include a case from a second Western country, which could also help to investigate whether the Swedish teacher education is in fact an extreme case for instance concerning its lack of shared resources. We chose Finland, since Finland had always was significantly higher result of the PISA survey than

Sweden and the OECD average (OECD, 2013), and also because the second author in paper I had several contact with Finnish teacher educators and was involved in my project from a very early stage. For conducting an effective comparison, we observed lessons where the educators treated more or less the same subject matter, namely “polygon area” from a methods course.

We started collecting data in Japan 2014, in Sweden 2015, and finally in Finland 2016. All three lessons were video recorded. The specific subject of the lessons were as follows: ‘Quantity and Measurement’ (Japan, with 53 students), ‘Area of Polygons’ (Finland, with 34 students) and ‘Area and Perimeter’ (Sweden, with 20 students). We also collected supplementary data: the syllabi (course description) of all three courses of which the recorded lessons formed part: national curricula in mathematics for elementary school in Japan, Finland and Sweden; the textbook (Kanemoto, et al., 2010) of the method course for the Japanese prospective teachers; the compendium for the lectures and workshops written by the Finnish teacher educator; the lesson schedules for the whole course written by the Swedish teacher educator. Finally, during the lessons, we took observation notes, and the teachers of the lessons were asked to fill a questionnaire about their teaching practice in this lesson and more generally. The material (including video transcripts, except Swedish) were translated from Japanese and Finnish into English, since not all authors understood all languages of origin.

Paper II

The empirical data analysed in paper II were collected in 2011, when I still was working with my licentiate project, in which investigating Kazuo Souma’s structured problem solving oriented teaching approach was the main focus. At that period, Souma served both as a professor in teacher education at Hokkaido University of Education and as a principal of the Asahikawa lower secondary school in Hokkaido, which is a so-called “attached school” affiliated school to the university. One day he invited me to observe an open lesson held in the school. The mathematics lesson on that occasion was conducted by Mr. Yachimoto, who did his master degree in didactics of mathematics under the supervision of Souma. Yachimoto had actually studied and practiced Souma’s approach for many years, and constantly used it in his daily lessons. I did not use the data collected during Yachimoto’s open lesson in my licentiate project, since I wrote a monograph in which the focus was on implementing Souma’s method in Sweden. But I have frequently used excerpts from the video of Yachimoto’s lesson to introduce the method to Swedish teachers.

The demonstration lesson and the post lesson reflection session described in paper II, was a part of an annual one-day “research conference” of the school. The lesson was given to 7th grade class (age around 13) of 41 students. The data used for the analysis in paper II were: video-recordings of Yachimoto’s lesson and the following post-lesson discussion, the lesson plan written by Yachimoto, a compendium that describes the “study theme” of the school in the year of the observation, the guidelines for the national Japanese curriculum for lower secondary mathematics, and my observation notes. The number of attendants to the open lesson was about 65, including 25 mathematics teacher students. The latter observed the lesson through direct video transmission from the classroom (that could not hold all of the observers in addition to the students and the teacher). After the lesson, the post-lesson reflection session was held (as is customary in Japan) in the same classroom, and the prospective teachers also joined to the discussions (in presence). The procedure of the reflection session was following the normal script for reflection sessions in Japanese open lessons: 1. The teacher’s reflections about the lesson, 2. Attendants’ questions and discussions, 3. Advisor’s (it can happen that reflection session provides more than one advisor) comments about the lesson, 4 Advisor’s comments about the discussion as a whole, and “conclusion” of the open lesson. The analysis of the lesson and the discussions from the post-lesson reflection were transcribed and translated into English, in order to enable discussions with my two thesis supervisors.

Paper III

The data of this longitudinal study were collected in a Swedish lower secondary school from 2010 to 2011. When I started collecting the data for my licentiate project, the project had a character of an intervention study for “implementing” Souma’s method in Swedish classrooms. The aim of my licentiate thesis was thus to investigate the “viability” of Souma’s teaching approach as a design tool for mathematics lessons in Sweden, and study in what way this Japanese method can contribute to “acquire both mathematical knowledge and a positive attitude for participating in the lessons” (Asami-Johansson, 2015, p. 8). In the licentiate thesis, I made a praxeological analysis using the theory of praxeology from the ATD, and described the “rich” mathematical organisation the approach can present, and how the teacher’s didactic organisation “succeeded” to achieve a-didactic situations (in the sense of Brousseau), by using a rather naturalistic and uncritical analysis focusing on what succeeded. However, I eventually noticed that although the Swedish teacher managed some aspects of the Japanese teaching practice well, some other parts remained as unused, so that the teacher did not succeed to handle as the certain

critical intentions found in Souma's works. Studying this phenomenon was not the aim of my licentiate project, and I did not have the capacity to conceive or carry out such an analysis at that time. The empirical data used for paper III consists of video-recorded lesson observations, my observation notes, lesson plans written by the Swedish teacher, who carried out the project in 10 months with me the Swedish national curriculum for lower secondary school in mathematics, and the guidelines for the Japanese national curriculum in mathematics. In the licentiate thesis, I used also data from interviews with the students and the teacher, which were not used in paper III.

Before we started the project, I translated excerpts from Souma's books, as well as examples of his lesson tasks (Souma, 1995; 1997; 2002), and described the flow of a typical lesson and the main didactic methods (how to state the initial questions, kikan-shido, use of the blackboard, textbooks, and so on). We planned the initial tasks and process of every lesson together. My role in the classroom was to be a silent observer, and I usually did not share my reflections about the lesson with the teacher.

Analytical methods

As it described in section 1.5, I employed praxeological analysis in all three papers, however, the analysis processes slightly differs from one study to another. Following, I will describe how the methodology was carried out for each paper.

Paper I

To answer one of the research questions of paper, I identified the main elements of each teacher educator's didactic praxeologies as observed in their lessons, and as well as the main differences. The central didactic task of a method course is to provide the prospective teachers with relevant disciplinary knowledge in mathematics and with corresponding teaching approaches. Teacher educators try to support prospective teachers to learn how to construct the mathematical and didactic praxeologies of their future lessons. That means the teacher educators' didactic praxeology (we denote it DO_{TE}) aims to support the prospective teachers' learning of the mathematical and didactic organisations of school lessons (we denote them MO_{SCH} and DO_{SCH}). These two organisations belong to a different institution from DO_{TE} (namely, they have their real existence in the School), but the two also form the teaching/learning object of the DO_{TE} (see Figure 1.6.1).

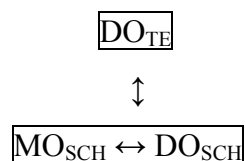


Figure 1.6.1: the complex of praxeologies handled within the teacher education

Thus, when we identify the main elements of DO_{TE} , we must consider this special structure that involves two entities from another institution. We identified didactic techniques used by the teacher educator, in particular the types of tasks from DO_{TE} which the teacher educators aimed to realise, as well as the techniques they employed.

The second research question was to investigate the institutional conditions and constraints that brought about the differences of the characteristics of the DO_{TES} . We firstly studied the description regarding the subject of measurements and area of polygons in the national curricula in Japan, Finland and Sweden. Then we needed to identify the didactic theories that support the observed didactic techniques of each educator. We let the educators answer a questionnaire to investigate what kinds of educational considerations were underlying the construction of their didactic organisations. The structure of the questionnaire was inspired by the Content Representation (CoRe) model (Loughran, et al., 2006), which originally aimed to the development of Physics teachers' understanding of "pedagogical content knowledge". From the outcome of the questionnaire, we identified the didactic theories held by the teacher educators, and described the social, traditional and cultural backgrounds that brought about those didactic theories. Finally, we displayed the task types, techniques, and theories of each DO_{TES} in a table to show the contrast of the characteristics of the DO_{TE} of the three teacher educators. To understand the background of these characteristics, we also explored the wider paradidactic infrastructure of each country.

Paper II

The first research question in paper II concerns the identification of the components of didactic knowledge which appeared in the lesson observers' comments during the post-lesson reflection, and also the investigation of how the notion of mathematical activities appeared in the teachers' foci in the reflection comments. Firstly, I studied the general definition of the notion of mathematical activities as it appears in the guidelines for the national curriculum, and also its specific appearance in the description of the mathematical content of the lesson. In so doing, I analysed to which levels of didactic co-determination the notion is related. In the description of the participants' comments, I emphasized how the notion is exposed.

The second research question concerns the process of establishing the didactical knowledge developed during the reflection session, and its relation to the different levels of co-determination. Firstly, I outlined the core episodes from the open lesson, and made a praxeological analysis of Yachimotos' demonstrated lesson to better situate the comments of the participants during the post-lesson reflection. I identified his didactic techniques corresponding to the mathematical praxeologies actually developed in the lesson. Secondly, I categorised the comments from Yachimoto, the observing teachers, and the advisor, at three major levels with respect to the co-determination model: 1. reflections regarding the generic didactic theory, 2. reflections regarding the specific didactic organisations, and 3. reflections regarding the generic theory applied to specific techniques and technologies. In the description of how the different institutional levels are related to those comments, I explained how the notion of mathematical activities enables participants to connect different foci, concerning both the generic educational aims and more specific didactic techniques.

Paper III

I carried out a praxeological analysis to answer the first research question about the central didactic techniques and technologies in Souma's version of the structured problem solving approach, and the crucial conditions for realising such praxeologies in Japan. In the analysis, I identified the main didactic techniques corresponding to the target knowledge of the mathematical organisations as described in Souma's books (Souma, 1995; 1997; 200) and in the lesson plan collections (Kunimune & Souma, 2009a; 2009b). To investigate the conditions, which shaped the didactic techniques of the approach, I needed to identify the theory block of Souma's approach. Thus I studied the commentary of the lesson plan collection book (ibid.) that describes how the didactic theory of Souma's praxeologies shaped the didactic practice. As the title of the book "*Practical lesson plan collection for mathematical activities*" indicates, the books propose ideal lesson plan sequences by applying different kind of mathematical activities. Using the results from paper II, I described how the notion of mathematical activities justifies the praxis of Souma's praxeologies as a crucial element of didactic theory.

To study to what extent the Swedish teacher realised Souma's central didactic techniques, which is the focus of the research question 2, I elaborated an epistemological reference model based on above mentioned praxeological analysis of Souma's method. In the reference model, I presented 15 different didactic techniques related to different didactic tasks, which originated from both mathematical, social, and pedagogical considerations. I studied 16 lessons given by the Swedish teacher, and categorised what kinds of didactic techniques she used, and what kinds

she did not use, in relation to the reference model. Then I applied the levels of didactic co-determination model to investigate the conditions and constraints that are likely to explain the difficulties for transferring the totality of the Japanese teaching practice to the Swedish context.

1.7 Discussion of papers

In this section, I will present summaries of the three papers and their main results as they pertain to the overall research questions. At the end of this section, the contributions of each paper, and the common thread running through the papers are discussed.

1.7.1 Paper I

Title: Comparing mathematics education lessons for primary school teachers: case studies from Japan, Finland and Sweden. To appear in *International Journal of Mathematical Education in Science and Technology*.

Paper I aims to uncover crucial conditions and constraints which form the practices of Japanese primary mathematics teacher education. In the paper, a lesson on polygon area from a “methods course” in Japanese primary mathematics teacher education is taken as a case. The paper investigates in detail what kinds of mathematical and didactic knowledge for the teaching of school mathematics is taught, and *how* they are taught. As stated by Li, Ma and Pang (2008), for knowing what components influence the prospective teachers’ learning of teaching mathematics, studying both the contents of the course and teacher educators’ instructional approach was needed because of the particular *nested* structure of the lessons of teacher education, i.e. both the contents and modalities of teacher education lessons is supposed to influence how and what prospective teachers will teach in their professional future. To identify and highlight the more remarkable features of the Japanese teacher educator’s lesson, a comparison was carried out with lessons on the same subject, from similar courses in two different Scandinavian teacher educations (in Finland and Sweden) . The specific research questions stated in paper I are following:

- RQ1. What are the main elements of each teacher educator’s didactic praxeologies in the lessons? In particular, (how) do they relate the didactic organisation (DO_{TE}) of each lesson to the mathematical and didactic organisation ($MO_{SCH} \leftrightarrow DO_{SCH}$) aimed for lessons concerning the determination of polygon area in school?
- RQ2. What are the main differences between the three lessons, concerning research question 1?
- RQ3. What institutional or social conditions and constraints can provide wider explanations for these differences?

The praxeological analysis based on ATD identifies the characteristics of each teacher educator's techniques to relate their "theoretical knowledge" to the didactic organisations of school mathematics teachers. In particular, significant differences are identified between the countries, which helped to identify the characteristics of the Japanese teacher educator's didactic praxeologies. The approach to the subject of the Japanese educator was highly theorised in the sense of using well-established didactical special terms and principles to expose the school mathematical subject and its teaching through the structured problem solving approach, which is a well established and shared didactic theory for both the Japanese teachers and teacher educators. We found that the Japanese model is the most 'university-like' and characterised it as '*theoretical*' or '*academic*' in comparison to the approached found in Finland and Sweden, which – in quite different ways – focused more on exemplary praxis.

In the Finnish lesson, the compendium prepared for workshop, which explains the basic properties of geometrical figures and practical teaching approach regarding these objects, gives a specific set of techniques both for the mathematical and didactic organisations of the subject in question. By performing a role-play as a teacher, the Finnish prospective teachers *rehearse* the proposed teaching method to be used in school classes. The didactic technology is supported by the generic "inductive way of learning", and a relatively broad didactic theory of educational principle of "learning by practicing". As a consequence of these observations and of the teacher educators confirming this as their normal practice, we call the Finnish model a '*rehearsal model*'.

Swedish teacher educators' didactic organisations aims to let the prospective teachers *experience* a model of *specific* mathematical and didactic organisation of lessons of the school mathematics; in a sense, they are taught the subject as if they were school pupils. The author of the thesis (a teacher educator in Sweden) recognizes this as a common practice. We call this approach the '*immersion model*' as teacher students are immersed in a supposedly exemplary episode of school teaching. The educator's techniques are based on the didactic theories that are related to pedagogical considerations such as raising students' self-efficacy, but these are not explicitly taught to the students. In terms of contents it is therefore the one that offers the least in terms of didactical theory to support the delivery of the specific subject or teaching methods in general.

The discrepancy between the paradidactic infrastructures that exist in the three countries explains some of the conditions for the observed phenomena. The well-established Japanese paradidactic infrastructure provides a broadly known didactical literature (to a large extent based

on teachers' experiments) pertaining to the teaching of school mathematics. In Finland, the paradidactic infrastructure does not supply didactical literature to the same extent as it has been in Japan. However, there is well-documented praxis level of didactic organisations within the teaching guides. In addition, the explicit structure of the teacher education compendium that gives instructions on the target knowledge with the didactic theory of inductive way of learning, indicates existence of standard didactic knowledge. The Swedish paradidactic infrastructure does not offer a similar literature and there a few means for teachers to exchange systematic observations and productions related to their teaching. The latter fact also influences the establishing of shared knowledge related to teaching practice, as the possibility of the dissemination hardly exists.

1.7.2 Paper II

Title: The didactic notion of “mathematical activity” in Japanese teachers’ professional scholarship: case study of an open lesson. Accepted for *Journal of Research in Mathematics Education*, pending revision.

This paper studies the notion of “mathematical activity” and how it affects Japanese mathematics teachers’ practice and professional knowledge development. We do so by observing its impact during a case of one of the essential components of the Japanese paradidactic infrastructure, namely, open lessons. In the paper, we focus on participants’ comments stated during so-called post-lesson reflection meeting. The participants consisted of mainly lower secondary mathematics teachers from different schools in the Hokkaido region of Northern Japan. They have collectively observed a demonstration lesson about the calculation of surface area of cones, conducted by an experienced teacher. The focal point of this study was how the specific didactic notion of *mathematical activities* provide an interface between the discussion of specific teaching techniques and more generic pedagogical objectives of mathematics teaching, as observed within the discussions of the participants. The research questions of this paper were as following:

- RQ1. What are the teachers’ paradidactic foci? In other words, what components of didactic knowledge can appear or develop during the post-lesson reflection in an open lesson in Japan? In particular, what is the role of the notion of mathematical activity?

RQ2. How is the teachers' knowledge of didactic practice shaped during the post-lesson reflections, and how do the discussions relate to components of the different levels of didactic co-determination?

The notion of mathematical activity has developed since 1950s in the context of the endeavour of providing students' *autonomy* and *socialisation* through their learning of mathematics (Ikeda, 2008). The actual contents this notion holds are expressed in the guidelines to the National curriculum as below: "various activities related to mathematics where students engage willingly and purposefully. (...). Mathematical activities may also include engaging in trials and errors, collecting and organizing data, observing, manipulating and experimenting" (MEXT, 2008, in CRICED, 2010, p. 16). Many educational books written by Japanese teachers in service, as well as papers by researchers, explore the possible teaching and learning practice through mathematical activities. Here, "practice through the mathematical activities" means using mathematics activities as a method to create lessons in which e.g. students *engage willingly and purposefully* (see the citation above) with the learning of mathematics. Consequently, designing lessons that achieve to engage pupils in mathematical activities has become a common *paradidactic stake* (or lens) to nourish the reflection in many post-lesson discussions.

The praxeological analysis showed that the participants' comments are categorised in three different types: 1. Comments regarding logos part of the teachers didactic praxeologies, e.g. how general educational aim such as "to improve students' abilities of expressing themselves" is treated during the lesson. 2. Comments regarding specific praxis part of the didactic praxeologies, e.g. how the teacher managed letting the students notice that the area of a circular sector is proportional to the length of the arc. 3. Comments regarding the combination of generic logos and specific didactic technologies and techniques, such as how the teacher organized the blackboard disposition to support the students' development of their capacity to express themselves mathematically.

Applying the notion of *levels of didactic co-determination*, the analysis indicates that the notion of mathematical activities reflects crucial aspects of both the objectives of mathematics education, and the concrete teaching practice observed in the lesson. Together with the structured problem-solving approach, which as stated by Fujii (2014; 2018) also emphasises the socialisation related to pupils' exchange and validation of mathematical ideas, the notion of mathematical activity allows the Japanese teachers to develop an acute awareness of the dialectic

between the generic educational aims and the practical teaching methods which aims to develop the specific target knowledge.

1.7.3 Paper III

Title: Conditions and constraints for transferring Japanese structured problem solving to Swedish mathematics classroom. Submitted to: *Recherches en Didactique des Mathématiques*

Paper III addresses mainly the second aim of the thesis – studying didactic phenomena that arose when attempting to transfer a particular Japanese problem solving oriented teaching approach to a Swedish teaching environment. The research questions in paper III are following:

- RQ1. What are the central didactic techniques and technologies in Souma's version of the structured problem solving approach, and what conditions are crucial for its realisation in Japan?
- RQ2. To what extent did the Swedish teacher realize Souma's central didactic techniques described in RQ1? How can this be explained by a wider view based on the levels of didactic co-determinacy, and on differences in paradidactic infrastructure?

The first part of the paper is dedicated to establish a reference epistemological model for the structure of the didactic organisations proposed in Kazuo Souma's teaching approach based on the structured problem solving. The description of cultural background that formed the structured solving approach in Japan explains the strong normative aspect of the general problem solving approach which is prescribed by the Japanese national curriculum. The normative view of the national curriculum in turn brings up Japanese mathematics teachers' background for connecting the use of the problem solving approach to the achievement of educational and pedagogical aims. In this context, as an elaboration of points established in paper II, the influence of the notion of mathematical activities on the components of Souma's structured problem solving approach is discussed; it provides the interaction between the concrete teaching techniques related to the lower levels of the didactic system and the generic pedagogical objectives related to the higher levels of the didactic co-determination.

The reference epistemological model helped to identify the didactic techniques the Swedish teacher eventually realised, those which were not. She managed to use the techniques regarding creating and stating initial questions, which subsequently form the basis for pupils'

learning of new mathematical knowledge; involving the pupils in whole-class discussions of the questions and possible answers; and institutionalising the achieved knowledge at the end of the lesson. The techniques she did not manage to use were: realising a process in which the students by themselves formulate the core task based on the initial tasks; and organising the board in a way which shows the whole process of the implemented lesson as a record. Consequently, the institutionalising moment was never based on looking back at the learning process based on common work with everything at the board, as suggested by Souma.

Thereafter, the levels of didactic co-determination point to the essential differences between the countries in terms of the didactic theories that justify (or could justify) the teachers' didactic techniques. In Japan, this concerns didactic theories, which help to relate generic educational aims with specific techniques to teach a piece of mathematical knowledge. The Swedish teachers' lack of familiarity with such theoretical tools made it difficult for her to make use of some of the central techniques with its original functions. For instance, teaching techniques related to board management seem totally obscure in a Western lesson script in which "board teaching" has acquired a somewhat pejorative sense, as "teacher centred". The notion of paradidactic infrastructure again explains the accessibility of such knowledge for the Japanese teachers.

1.7.3 Discussion

Investigating and modelling the body of mathematical and didactical knowledge taught in teacher education is crucial for elucidating the Japanese teachers' fundamental assumptions, underpinning the rest of their professional knowledge. The contribution of paper I was that the study investigated both the contents and teacher educators' actual practice in methods courses, with a deliberate focus on the nested structure of didactic knowledge as taught in teacher education. We recall from the literature review that although this aspect was important, not many researchers has focused on this issue, and studied in detail how teacher educators teach the fundamentals of teaching practice as institutional knowledge, according to their cultural script. One of the findings from Paper I – the Japanese teacher educator's design of his didactic organisation is strongly supported by the well-developed Japanese paradidactic infrastructure – is related to both paper II and III, where this infrastructure appears in relation to the development and sharing of professional knowledge among teachers in service.

The questions stated in paper II is inspired by a result stated by Miyakawa and Winsl w (2013), that Japanese teachers' paradidactic practices (in this case, open lessons) do not primarily aim at revising or reflecting on specific didactic practice, but to develop the shared theoretical

blocks of didactic practice in a much wider sense (p. 204). Accordingly, paper II identified the factual components and mechanisms of such development and dissemination of the theoretical part of teacher knowledge, by using the model of the levels of didactic co-determination. The analysis of the role of the Japanese didactic notion of “mathematical activities” in paper II was also crucial to provide the central idea of the analysis presented in paper III. The analysis of the compatibility of teachers’ foci on a specific teaching techniques and general pedagogical issues in participants’ discussion (shown in paper II) endorses the analysis on the nature of Souma’s teaching techniques. Further, this analysis also explains the difficulties that arose for our endeavours to transfer certain praxis blocks of Souma’s didactic praxeologies to a Swedish classroom, as studied in paper III. In that sense, one of the contributions of the paper II and III was that this didactic research gave an explanation to a didactic phenomenon identified by Fujii (2014; 2018). This concerns what is the “authentic nature” (Fujii, 2014, p. 81) of the normative aspect the structured problem solving approach contains, and how this core is actually reflected in the teaching practices from an anthropological perspective. The description in paper I and III of the contrast between Japan and the two Scandinavian countries as regards paradidactic infrastructures, also highlights the significance of the well-established Japanese paradidactic infrastructure as the driving force of the dissemination of teachers’ professional knowledge in Japan.

1.8 Conclusions and perspectives

This section is devoted to discuss the results of the papers in relation to the general research questions of the thesis stated in section 1.6.1. In the last part of the section, I will discuss some perspectives for the future research suggested by the present thesis. I will start with by discussing the answers found to the first research question.

RQ1. What are the main characteristics of the Japanese mathematics teachers’ professional knowledge?

The question addresses my very first reflection to Etzioni’s expression about “semi-profession”, described in the first section. “Real” profession in the context of the ATD refers to the institution, which possesses explicitly shared “a body of sound theoretical knowledge” (Chevallard, 2013). Thus, when we talk about “professional” knowledge, it must be “theorised” and “shared”. That is why, I have identified to what extent Japanese mathematics teachers’

professional knowledge is “theorised” and “shared” within the community of mathematics teachers. This argument can be split into to “what is theorised”, “how it theorised” and “how it is shared”.

Paper I shows how the Japanese teacher educator used the specialised “terms”, which represented several specific solving techniques. For instance, he mentioned several terms for the transforming techniques of a parallelogram (to a rectangle) for calculating its area. Those terms are already described in the guidelines for the national curriculum, and will be learned and used by the prospective teachers as concrete concepts when they in turn make the lesson plans or discuss about the didactic techniques with their colleagues. This process of *objectification* of mathematical techniques and technologies, and all those didactic procedures that are expressed such as *bansho*, *kyozai-kenkyu*, *hatsumon*, *kikan-shido*, *neriage*, *matome*, are crucial for the theorisation of knowledge and its sharing. Without the notions that name and define these procedures, it will be difficult to sustain and refine shared didactic technologies. Once those processes are recognised as “object”, they can be accepted as known existing techniques and technologies by the teachers. It can be compared to the use of notions such as “didactic transposition”, “didactical contract” “Topaze effect” help us to objectify and describe the concrete substance of those phenomena. Without objectifying the details of the process of the didactic transposition, we cannot study, or not even identify the phenomenon involves clearly. The existence of the specific terms of mathematical solving methods and different didactic moments of the structured problem solving approach indicate that the process of the approach was theorised in order to be shared by the teachers.

A significant characteristic of the structured problem solving is that the approach is strongly connected with the paradidactic and highly theorized notion of mathematical activities. The notion mediates the generic educational aim of mathematics education and concrete teaching practice, since the notion itself equipped with both nature of the didactic theories and didactic techniques. The case of the *bansho* “technology”, further described in paper III, is typical example, where the notion of mathematical activities caters to needs of teachers at both the lower and higher levels of the co-determinacy. The set of the *bansho* “techniques” itself deals with subject level – showing the initial task, recording students’ spontaneous mathematical techniques, their identification of core tasks, and the final comments that institutionalises the knowledge learnt. However, while having access to all the writings on the blackboard, students look back on the whole process they have gone through during the lesson, and reflect with those writings on every part of the process including their own interpretations, e.g. how a solving

process for the task developed, how one expresses a certain concept in a mathematical expression, etc. This purpose of the bansho technique is also related to more general levels in the didactic system than the subject level, and that is why bansho techniques constitute an important didactic technology in Japan. A didactic theory – mathematical activities – explains the functions of the bansho techniques, or in other words, the bansho techniques aim to visualise the process of the mathematical activities on the blackboard (Imazaki, 2017).

The second research question was stated as follows:

RQ2. In what way can the theoretical and methodological tools from the ATD be used and developed to identify and elaborate Japanese teachers' professional knowledge of teaching and its dissemination in their community?

The foremost advantage of applying the ATD is that the framework allow us to look at the Japanese teachers' lessons, their post-lesson reflections, and the Swedish teacher's lessons from the perspective of a *science of didactic phenomena* (Bosch & Gascón, 2014). Certainly, previous studies have focused on the Japanese teachers' theoretical knowledge of teaching (e.g. Jacob & Morita, 2002), but they lacked an independent reference model and thus remained naturalistic, and as such under the "illusion of transparency". Also, lacking the institutional perspective, they do not have a perspective to study what conditions brought about the refined theoretical discourse that is punctually referred to. The anthropological standpoint of the three papers makes it possible to generate general hypotheses based on the observed cases of the teacher educators, the discussions of the Japanese in-service teachers, and the longitudinal studies of the Swedish teacher. Certainly, they are not "proved" by the case studies but can, as other scientific hypotheses, be validated inductively through further case studies, either from the literature or through renewed experiments and observations. Indeed, the conditions and constraints that brought about the observed phenomena are social and cultural, and combined with the theoretical reference models, the hypotheses go much beyond what was found in the cases, by analysing the individual teachers' or teacher educators' activities.

Another possibility that the ATD suggests is recognising the mutual relation between the establishment and development of professional knowledge of teaching, and its dissemination. Especially, the theory of paradidactic infrastructure contributes to clarify this relation. Because the knowledge is institutional, and is a changing reality (Chevallard, 2006b), and without the paradidactic infrastructure, new knowledge will not disseminate, or more exactly, new

knowledge can develop among individual teachers, but cannot be established as shared. As it stated in Miyakawa & Winsløw (2013), the activity of sharing the knowledge with the colleagues must happen *outside* the classroom. This condition appeared clearly in the case of the Swedish teacher education, where the paradidactic infrastructure is in some sense less developed than in Finland and Japan, so that the knowledge that individual teachers possess remains individual knowledge. In that sense, the notion of paradidactic infrastructure opened many doors to explore the Japanese teachers' professional knowledge of teaching and its dissemination.

Further perspective

The different models applied in this thesis constitute, together, a comprehensive anthropological perspective on Japanese mathematics teachers' theorised, shared professional knowledge. One aspect which could be further exposed is, to what extent the "teacher's epistemology" (in the sense of Brousseau, 1997, p. 37) matters for the understanding and mastering of a specific teaching practice. Here, the teachers' epistemology roughly means their mathematical and logos blocks of didactical knowledge. In Japan, prospective teachers of elementary schools can choose a particular discipline as their specialty during teacher education. For instance, if a prospective elementary school teacher chose mathematics as specialization, he/she takes some additional credits in pure mathematics and didactics at the university, and then obtains the license for teaching at lower secondary levels as well. It is likely (although difficult to document) that most in-service Japanese elementary teachers who are active in practice research groups and who present research papers at mathematics teacher conferences, have in fact specialized in mathematics during their initial education. Assuming this is the case, it would be interesting to study if there is any evidential correlation between teachers' epistemology and their development of professional knowledge on the one hand, and their experience from university studies on the other. To carry out such a study would require a meticulously designed methodology. We consider it could be adequately based on the theory of paradidactic infrastructure. The structure of paradidactic systems can be shaped around various types of paradidactic stakes, thus, as suggested by Miyakawa and Winsløw (2013) concerning the functions of open lessons, we can develop new models to further explore the sources and functions of teachers' epistemologies.

To distinguish a didactic system and a paradidactic system is not a simple matter without having the notion "paradidactic", since teachers themselves do not have to recognize such differences. Teachers, who are acting in a paradidactic system (e.g. a post lesson reflection) always talk *about* a didactic system, but they never talk about the paradidactic systems they are

submerged in. So far, it is only researchers, who observe such systems, and to some extent open lesson advisors (who usually comment not only the lesson, but also on the post-lesson reflection session as a whole). Here, we notice that the advisors are often teacher educators and researchers. Similarly, when teacher educators implement lectures or workshops concerning teaching practice, they are talking about didactic systems, and what they are aiming at is a paradidactic stake. In that sense, I could have carried out the analysis of paper I by focusing on the teacher education lessons as paradidactic systems, and the bodies of the knowledge could have been analysed as paradidactic stakes for both educators and prospective teachers. All the specific didactic terms described in the previous sections actually belong to the paradidactic systems, since teachers (in principle) never use those terms in the classroom. The terms are supposed to be used only to *describe* didactic systems. This aspect could have enriched the analysis of paper I, and have added slightly different perspectives to the whole study. Another possibility of using more explicitly the focus on paradidactic knowledge is to capture different paradidactic phenomena, which may have been considered as didactic phenomena before the notion paradidactic existed; for example, a phenomenon like didactic divide (Otaki & Asami-Johansson in process). This concept is still young, and holds substantial potential for further refinement. Becoming more aware of the role and structure of paradidactic systems could also be important to those who seek to disseminate resources and practices among mathematics teachers internationally.

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Paper I

Comparing mathematics education lessons for primary school teachers: case studies from Japan, Finland and Sweden

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ABSTRACT

The aim of this paper is to investigate and compare lessons given in primary school teacher education in Japan, Finland and Sweden. We analyse one lesson from each country and compare them using a common framework. Chevallard's anthropological theory of the didactic (ATD) is used to frame this analysis and in particular to model teacher educators' didactic organization of the lessons. The focus is on how the didactic organizations of the teacher educators relate to the mathematical and didactic organizations of primary school. Based on official documents and viewpoints of the teacher educators, we also discuss how the contents and descriptions of the national curricula, and the different traditions of the teaching practices in each country, influence the didactic organizations found in the lessons.

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Teacher education; teaching knowledge; anthropological theory of the didactic; praxeology; mathematical organization; didactic organization; paradidactic infrastructure

1. Introduction

1.1. The use of comparative methods in research on teacher education

In the last few decades, international comparative studies in mathematics education, especially on classroom practices, have provided insight into differences in teaching cultures between Western and East Asian countries (e.g. [1–3]). Stigler and Perry [4] stress the importance, for researchers and educators, of cross cultural comparison for explicit understanding of pupils' learning of mathematics: 'Without comparison, we tend not to question our own traditional teaching practices and we may not even be aware of the choices we have made in constructing the educational process' [4,p.199]. Contributions and challenges of international comparisons of teacher education have appeared in recent years (e.g. [5]). Various cross-national studies have reported the main features of the mathematics teacher education in different countries (e.g. [6]), and a number of studies concern different aspects of student teachers' knowledge in pre-service teacher education (e.g. [7–9]).

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1.2. Studies regarding the provision of professional knowledge for prospective teachers

Considering providing the different kinds of professional knowledge for prospective teachers, several studies investigate the complexity of preparing them for the transition from being prospective teachers (hereafter, PTs) to becoming teachers (e.g. [10,11]). Winsløw et al. [12] have viewed the novice teachers' first years of teaching practice as a period of transition on mainly three interrelated levels: at an epistemological level: adapting their theoretical knowledge acquired in the pre-service education to the conditions of the practice of teaching; at an institutional level: passing from one institutional context (the university) to another (the school system); at a personal level: from being a student in a community of students to being a professional in a community of teachers (p.93).

The notion of the didactic divide is introduced by Bergsten and Grevholm [13] to illuminate the fundamental problem within teacher education in Sweden. They refer to Kilpatrick, Swafford and Findell [14] who state that teacher education needs to provide opportunities for PTs to connect different kinds of knowledge, and if certain connections are not realized, one may say there is a didactic divide between disciplinary and pedagogical knowledge¹ of mathematics. Bergsten and Grevholm also illustrate the situation in teacher education programmes, drawing on Ball and Bass's assertion that 'the teacher education across the twentieth century has consistently been severed by a persistent divide between subject matter knowledge and pedagogy', as 'the gap between subject matter knowledge and pedagogy fragments teacher education by fragmenting teaching' [15,p.85].

1.3. Teacher educators' teaching beliefs and teaching practices

Fewer studies concern teacher educators' teaching beliefs and teaching practices (e.g. Pope & Mewborn [16]). Concerning this issue, Hemmi and Ryve [17] made a comparison between Swedish and Finnish teacher educators' perception of 'effective mathematics teaching' by studying interview data with teacher educators (hereafter, TE/TEs) and school mentors. They reported that Swedish TEs tend to recommend PTs to adapt their teaching to individual pupils' thinking, and their everyday experiences. Finnish educators emphasize that mathematical teaching should connect to pupils' prior learned skills and should also balance the teaching focuses between routines, variation and homework. A comparative study conducted in Finland and Sweden [18] showed substantial differences of TEs' and teachers' views on the school-based teacher education between the countries.

1.4. Aim of this study

The aim of this paper is not to compare the teacher education programmes or teacher education contents in general. We compare how a certain subject is taught in what we can roughly call a methods course, that is a course on 'mathematics and its teaching' at the university – a kind of course which is distinct from both school-based teacher education and from normal mathematics courses; in many countries, such courses are meant as a kind of bridge between academic mathematics and teaching practice. As Liljedahl, Durand-Guerrier, Winsløw et al. [19] state, what is unique with teacher education is, 'what educators teach is also how educators teach, and what the prospective teachers learn is also

how they are learning' [19,p.29]. The task of teacher education is usually to make PTs learn the disciplinary knowledge in mathematics and, at the same time, its teaching approaches (thus, in fact, more than just mathematical knowledge and practice). Based on our review concerning primary mathematics teacher educators' practices, we found that there is little (if any) research on the contents and activities in methods courses focusing on this nested structure of didactic knowledge.

This study describes and compares the lessons in three different countries, to reveal crucial conditions and constraints forming the practices in methods courses at the primary school mathematics teacher education programmes. Our focus will be on the epistemological level [12] with the aim of investigating how the three TEs deal with the theoretical knowledge and practice of teaching mathematics. For a comparison to the Swedish context, we chose Finland which has significantly better results in mathematical literacy assessments [20]; and Japan, where the teaching culture in mathematics is reported to be more collective, compared to that of the US and Europe [21]. In this paper, we compare lessons from methods courses concerning the area of figures in the plane, given in the three aforementioned countries.

2. Theoretical framework and research questions

From the perspective of the anthropological theory of the didactic (ATD) developed by Chevallard and his colleagues, mathematics learning is considered as a construction of *praxeologies* [22] within social institutions where different levels of mathematical knowledge are required. A *praxeology* is a model of human activity and it provides both methods for the solution of a domain of problems (*praxis*) and a structure (*logos*) for the discourse on the methods and their relations to broader settings. Here, the *praxis* part consists of a *type of tasks* (T) and a *technique* (τ) to solve the (T), and the *logos* part includes a *technology* (θ), which explains and justifies the techniques, and a *theory* (Θ), which justifies and explains the technology more generally and formally. Two special kinds of praxeologies are also denoted, more specifically, *mathematical organizations* (MO) and *didactic organizations* (DO). A MO is a praxeology where the type of tasks is mathematical, and a DO is a praxeology where the type of tasks concerns the support of the learning or teaching of a MO. Thus those two kinds of organizations are mutually dependent or, as Chevallard [23] puts it, *co-determined*. The *praxis* part of the teacher educators (TEs)' DO is therefore consists of the types of didactic tasks (e.g. 'providing the prospective teachers (PTs) a certain teaching method of addition of 2-digits numbers'), and TEs' didactic techniques (e.g. 'giving PTs the task of writing a report regarding the teaching methods of addition'; 'letting the PTs demonstrate an example lesson during the class'). The use of the notion of praxeology make possible for researchers to recognize and categorize the actual components of the teaching and learning activities in every educational level (e.g. about analysis on the task including proportional relationships in elementary school level in Sweden [24]; Wijayanti & Winslow (2017), about analysis of the mathematical content in Indonesian textbooks in secondary school level [25]; about the difference of the type of tasks and associated techniques between university and secondary level [26]).

Our study adopts this tool to characterize TEs and PTs' activities in the teaching methods class, in order to make explicit what kinds of mathematical and didactic practice and knowledge are at stake there. From the viewpoint of the praxeology, the purpose of the

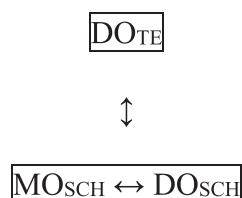


Figure 1. The complex of praxeologies handled within the teacher education.

regular teaching methods classes we meet on a daily basis is expressed as the type of tasks of the DO. The type of tasks of the DO of teacher education is, as it described in the introduction, to make PTs learn the disciplinary knowledge in mathematics and its teaching approaches. TEs' DOs promote PTs to learn how to construct the MO and DO of their future lessons. In this paper, we denote the didactic organization of the lessons in the teaching methods courses in pre-service teacher education by DO_{TE} . In addition, the school mathematical and didactic organizations demonstrated, or otherwise referred to during the lessons are denoted MO_{SCH} and DO_{SCH} . In the MO_{SCH} , there are types of tasks (T), which school pupils are supposed to solve, and schoolteachers' tasks (types contained in DO_{SCH}) are to support pupils to achieve the MO_{SCH} (T). These two organizations are thus intertwined with each other. The pair of the MO_{SCH} and DO_{SCH} belong to another institution (the School) than the teacher education institution, and together they form the object of the TE's praxeology DO_{TE} . (see Figure 1). We say that the didactic divide appears when the mutuality of the MO_{SCH} and the DO_{SCH} is not expressed explicitly by TEs in their DO_{TE} . TEs must thus handle this complex structure of the praxeologies: to promote the PTs to acquire the knowledge and methods necessary to construct their own future lessons where the MO_{SCH} and DO_{SCH} are interrelated.

To realize the aim of this paper, we address three research questions for a lesson in each of the three countries mentioned above:

RQ1. What are the main elements of each TE's didactic praxeologies in the lessons? In particular, (how) do they relate the didactic organisation (DO_{TE}) of each lesson to the mathematical and didactic organisation ($MO_{SCH} \leftrightarrow DO_{SCH}$) aimed for lessons concerning the determination of polygon area in school?

RQ2. What are the main differences between the three lessons, concerning research question 1?

RQ3. What institutional or social conditions and constraints can provide wider explanations for these differences?

In the next section, we present the methodology to address these research questions.

3. Methodology

In order to answer RQ1 and RQ2, we present episodes where the TEs treat a common subject matter, namely the determination of the polygon area. For each country we have selected episodes from the teaching where the TE and the PTs interact around similar aspects of this mathematical theme, to make them as comparable as possible, and at the same time represent characteristic features of the teaching in each country. We have analysed the elements (T, τ , θ , Θ) of the DO_{TE} , and the MO_{SCH} / DO_{SCH} which are presented

within the DO_{TE} of the lessons. Thereafter we highlight the characteristics of the DO_{TE} of the TEs in each country. Video recordings of the episodes were made: ‘Quantity and Measurement’ (Japan, with 53 students), ‘Area of Polygons’ (Finland, with 34 students) and ‘Area and Perimeter’ (Sweden, with 20 students). The names of the TEs are pseudonyms. The recorded lessons were transcribed in the original language, and then we translated the transcriptions into English together as we watched the videos. All analysis was made with the English transcriptions by all authors.

We should mention that unlike the Japanese and Swedish courses, the Finnish course consists of two separate sections: a lecture session and workshop session. During the lecture session, the educator mainly describes the mathematical contents, as a background for mathematics lessons in school. Then during the workshop session, which is carried out several days later, the PTs practice certain teaching scenarios meant for school pupils, but with each other in lieu of pupils. In this paper, we present one workshop session from the Finnish programme, where the PTs have opportunities to interact to each other and the TE.

The theory block of the DO_{TE} is not observable from one single lesson and therefore we asked each TE to answer some questions after they conducted their lessons. The design of this questionnaire was inspired by the Content Representation (CoRe) model [27], which was originally created as a methodological tool to develop science teachers’ pedagogical content knowledge [28]. The questionnaire consists of eight questions which may help to identify different components of the DO_{TE} and MO_{SCH}/DO_{SCH} . The data used in this analysis were from responses to the following questions:

- Q1. What do you intend the students to learn regarding this topic (area of polygons)?
- Q4. What kinds of difficulties/limitations are connected to teaching this topic?
- Q7. What teaching methods do you use to make your teaching on this topic engaging, and for what particular reasons?

Q1 is related to identifying the elements of the MO_{SCH} that were prioritized in the DO_{TE} -practice. The questions Q4 and Q7 help to identify institutional conditions of the praxeology as a whole. This is relevant to the RQ3 – investigating the wider explanations for the differences between the three countries’ lessons. To analyse the procedures of their daily lessons also ensures that the particular lessons we observed were not exceptional. The questionnaire was translated into Japanese, Finnish and Swedish, and the TEs answered it in their own languages. Then their answers were translated into English. The analysis of the TE’s answers was made jointly by the authors, using the English translations.

To support the investigation concerning the RQ3, we additionally made a small-scale comparison of each country’s national curricula, the curriculum guidelines and a few textbooks in the section concerning measurement. The curriculum is a result of the *didactic transposition* designed by different stakeholders within the education system [22]. During the process of formulating the curriculum, the original mathematical *scholarly knowledge* [22] created by the community of mathematicians, is disassembled and reconstructed into the *knowledge to be taught* [22] in a form which is more appropriate for teaching within the school systems of each country. Comparing the national curricula therefore lead us to distinguish the conditions of the construction of the DO_{TE} . We present the result of this comparison in the next section.

4. Results

4.1. National curricula and guidelines concerning measurement

In The Guidelines for the Japanese National Curriculum for grades 1–6 [29], the determination of length, area and volume is described in the chapter (of the Guidelines), Quantity and Measurements, positioned between the chapters of Arithmetic and Geometry. The content for each grade is described in detail with concrete teaching proposals. The Guidelines emphasize that the teaching methods are supposed to build on the pupils' previous knowledge and the pupils' various ways of solving problems. For that purpose, the Guidelines includes tables which presents the overview of the central content for grades 1–6 and for grades 7–9.

The Guidelines describes that children's learning process of measurements consists of four phases; direct comparison, indirect comparison, measurement using arbitrary objects as units, and measurement using standard units. This order is clearly followed by Japanese textbooks [30].

In the Finnish National Core Curriculum for Basic Education [31], the content regarding quantities, units and measurement for grades 1–2 is briefly described in the chapter Geometry and Measurement following the chapter Numbers and Calculations. For the grades 3–6, the content of perimeter and area is included in the chapter (of the curriculum) Geometry and Measurement following the chapter Numbers, Calculations and Algebra. In the Swedish curriculum, the content of quantities, units and measurement, perimeter and area are included in the chapter Geometry following the chapter Algebra for the grades 1–6 [32]. The descriptions of the contents consist of only a few lines. None of the two latter curricula give any practical guidelines for teaching the contents.

Unlike Japan, textbooks are not approved by a ministry in Finland and Sweden. The presentations of the contents of measurements in Swedish textbooks for grade 1 are often placed within sections covering Arithmetic (e.g. [33]), in spite of the fact that the Swedish national curriculum introduces the concepts within Geometry. In the Finnish textbooks, the concept of measurement for grade 1 is placed between the chapters of Arithmetic and Geometry (e.g. [34]). In Finland and Sweden, the four phases for the introduction of the concept of measurements are not present, unlike Japan. Some Swedish textbooks introduce direct comparison and measurement using standard units simultaneously (e.g. [33]), while some Finnish textbooks introduce the measurement using arbitrary units first, then the comparison using arbitrary objects as units, the direct comparison, indirect comparison and finally the standard units (cm) are presented (e.g. [34]). Potential tasks with an understanding of indirect comparison are not addressed in most Swedish textbooks.

The MO technologies of the four phases of measurements in the Japanese Guidelines are strongly connected to each other. For instance, the technology of using arbitrary objects as units in grade 1 is linked to the technology of area determination of rectangles in grade 4; the sum of the number of squares (which are arbitrary objects) expresses the quantity of the area. Therefore, to follow the 'correct order' of the four phases (firstly, pupils learn the direct comparison, secondly, indirect comparison, then measurement using arbitrary objects as units, and finally, standard units) is absolutely essential from the epistemological point of view. Hence, it would never happen that one introduced direct comparison and measurement using standard units at the same time or introduced measurement using

arbitrary units before direct comparison in the Japanese textbooks. One follows the order of the four phases that the Guidelines suggest.

Comparing these two contexts, we conclude that the Japanese curriculum does not give much space for different interpretations of its contents. For example, the four phases of measurement provide a suggestion for a uniform teaching approach for textbook authors and users. We assume the reason that many Swedish textbook authors position the section of measurements in the domain of arithmetic, is to enable a natural connection between area calculations and the basic arithmetical operations. This suggests that Swedish textbook authors may have different interpretations on the national curriculum, and consequently, different textbooks provide different teaching approaches in Sweden.

4.2. Lesson observation 'quantity and measurement' in Japan

The course *Elementary mathematics teaching methods* aims to provide the PTs with knowledge of the contents of elementary school mathematics and its teaching methods. The 12 lessons consist of the goal of mathematics education and elements of mathematics lessons, arithmetic, quantity and measurement, geometry, functions, lesson design (including problem solving) and principles for the *mathematical way of thinking*. Mr. Matsui is the lecturer of the course at a national university located in the middle part of Japan. He has worked as a mathematics teacher in lower secondary school for 14 years and as TE at the university for 12 years. This is the sixth lesson of 12 in total and it concerns the chapter on 'Quantity and Measurement'. Episode 1 represents the first half of the lesson and episode 2 represents the second half of the lesson.

4.2.1. Episode 1: the concept of area and area determination of rectangles

Mr. Matsui explains the four phases in the process of pupils' learning about measurement by referring to the Curriculum Guidelines and clarifies those different comparison methods for the class. Then he mentions the concept of area. In the following transcription, 'PT' means a prospective teacher, and 'M' means Mr. Matsui.

M: In grade 1, (referring to the contents overview in the Guidelines) they learn about area with direct comparison and then indirect comparison. Then it will be in grade 4 that they again learn about area. There they will compare areas using arbitrary units, and then standard units.

Mr. Matsui now demonstrates how grade 4 pupils learn the concepts of area and perimeter. He draws a rectangle (A) with grids of (6×4) and a square (B) (5×5) on the blackboard (see Figure 2).

He asks the PTs why some pupils in grade 4 think that the areas of (A) and (B) are the same. A PT answers that it depends on the sum of the width and heights, since 4 and 6, and 5 and 5 are equal; 10. Mr. Matsui remarks that most textbooks introduce the area of rectangles in this way: showing the two rectangles with same sums of perimeters and let the pupils to understand that it would not work to compare area by the perimeter. He then describes how the introduction of the standard units is usually carried out in textbooks and demonstrates a practical teaching approach:

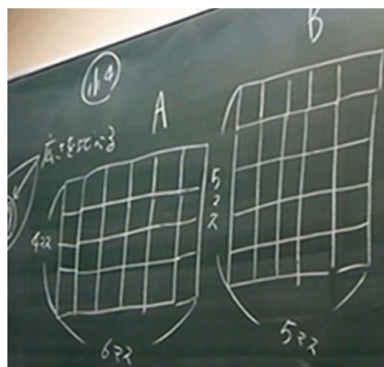


Figure 2. Mr. Matsui's figures of introduction for area determination of rectangles.

M: When they use an arbitrary units (squares in the rectangle), they count the number of the squares like this (writes down numbers 1,2,3 . . . in the grids). So counting the number of the squares is still an arbitrary measurement. Then the next stage (in the book) is to show that one square consists of the standard unit of 1 cm times 1 cm and to define it as 1 cm^2 . The introduction of the standard units concerning area is usually done in that way. Then the next stage is . . . we say 'isn't it a bit tough to count all the grids every time?' (writes 'tough') and we must encourage pupils to find out an 'easier' way to determinate (writes 'easier way'). They have already learned multiplication by grade 2, and understand that it would be determined by 6×4 and mentions that this is called 'formula' (writes 'formula'). This is supposed to be the first time pupils learn the notion of 'formula'.

4.2.2. Episode 2: area determination of parallelograms

The second half of the lesson is spent experiencing a short version of a structured problem solving approach. This approach emphasizes learners' active participation in mathematical activities, using challenging problems and collective reflections [1]. Mr. Matsui distributes to the class grid papers where a figure of a parallelogram of width 6 cm and height 4 cm is drawn, and lets the PTs find out several different methods for the determination of the area of these parallelograms which could be developed by pupils in grade 5:

M: Pupils in grade 5 have already learned direct/indirect comparison, measurement using arbitrary unit and standard units of area and the formula for area of rectangles/squares. Thus, it means that we will use all that knowledge and find out the formula for the area of a parallelogram.

Seven PTs draw pictures and explain their different solutions on the blackboard.

PT2: I moved this (pointing the right triangle on the left) here (on the right) and made a rectangle. Then the area will be 4×6 and 24 cm^2 (see Figure 3).

M: If we cut this triangle ABE and put here (the shaded section), is the area still the same? If we ask children of grade 1, they may argue that the area can be changed. This we call area preserving property. Some children in grade 1 do not understand it (writes down 'area preserving property').

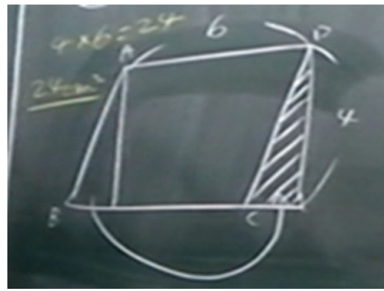


Figure 3. PT2's solution.

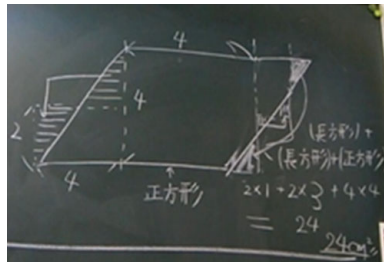


Figure 4. PT 3's solution.

Since PT2 talked about moving a part of the parallelogram and making a rectangle, Mr. Matsui explains the general and crucial property of *additivity* of quantities, referring to the Guidelines.

M: Additivity is another important property. For instance, if you put 100 g play-dough and 50 g play-dough together, some younger children think that the weights will be less than 150 g. Since, what you see when you put the two doughs together is a change of shape.

When another PT explains her solution method, as shown in Figure 4, Mr. Matsui asks her:

- M: You said after you made 4×4 square, you moved the top left to the bottom left, didn't you?
- PT3: Here? (pointing the top left triangle)
- M: Yes, there. To put this triangle to the left bottom, which kind of movement is needed?
- PT3: (turning the top-triangle down) turning over?
- M: Turning over? Then it sound like it was turned to up-side down.
- PT4: Point symmetry.

Mr. Matsui traces the two triangles by yellow and red chalks (see Figure 5) and confirms with the class that it turns 180° . He continues:

M: When it (the red triangle) turns 180 degrees then it fits on this (yellow triangle). Point symmetry is learned in grade 6. It is not necessary to use the proper term but if the pupils have experienced this kind of activity, the lesson on point

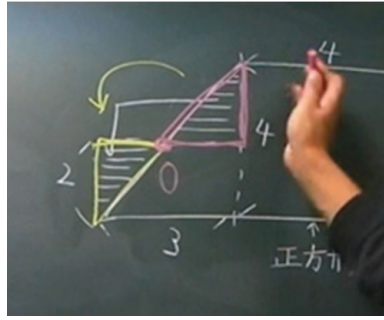


Figure 5. Mr. Matsui's explanation of point symmetry.

symmetry in grade 6 will be richer. In addition, rotational translation will be learned in grade 7. The future lesson will be more meaningful if you consciously apply and illuminate these related topics.

Thereafter, he compares the different kinds of *shifts* between PT3's *rotation* and PT2's *parallel translation*. Finally, he explains the formula for the area of parallelogram as height times length since the geometric transformations shows that the height and length of parallelograms corresponds to those of rectangles. In the same way, he gives a final task to determine the area of a trapezoid, using same didactical approach. Some of the PTs transformed trapezoids to a double size of the original parallelogram and made a rectangle. From this solution, the class concluded the formula for the area of a trapezoid to be $(a + b)h/2$.

4.2.3. Analysis of the episodes

The type of tasks of the DO_{TE} in episode 1 aim to help the PTs learn how to construct the praxeology of 'making the formula for area of rectangles'. Mr. Matsui's DO_{TE} includes several techniques, such as mentioning a typical teaching approach presented in textbooks. Yet, the most crucial technique of his DO_{TE} is to refer to the Guidelines. He describes the teaching/learning process from the direct comparison to the area of rectangles by referring to the contents overview. In so doing, he exemplifies the four phases of the measurement by a case, the area of rectangles. When he explains the process of establishing the formula, the specific terms from the Guidelines are described: direct/indirect comparison, arbitrary unit, additivity, which are components of the MO_{SCH} technology. School pupils do not have to master applying these terms, but the PTs do, in order to understand the whole construction of the MO_{SCH} better. The DO_{TE} technique of discussing the use of the different terms makes the technology of the MO_{SCH}/DO_{SCH} explicit. The main tasks of the DO_{TE} in episode 2 are: 1. helping the PTs learn the MO_{SCH}/DO_{SCH} of 'determination of area of a parallelogram and trapezoid', 2. letting them anticipate pupils' solution methods on this topic and examine the viability of such methods.

Mr. Matsui lets the PTs participate in a short version of an example lesson using the structured problem solving approach. This is a main technique of the DO_{TE} in this episode. Mr. Matsui lets the PTs follow up one of the most important techniques of the DO_{SCH} – whole-class discussions. The whole-class discussions lead to the discourse of several mathematical techniques and this in turn leads to the use and establishment of a richer technology and theory of the MO_{SCH}. It means, through discussing/comparing the various

solving methods, pupils recognize that the rigid transformation is a fundamental concept in order to reach an algebraic interpretation of area determination. In the episode, Mr. Matsui asks PT3: ‘which kind of movement is needed?’ This is a question to make the technology of the MO_{SCH} technique explicit: letting the PTs realize why such and such technique can be used.

These components of the DO_{SCH} promote the construction of a praxeology where the knowledge from the previous grades to the forthcoming grades are connected and re-established. Mr. Matsui refers to the Guidelines and describes how to make grade 6 lessons richer by, e.g. discussing the notion of rotation in the grade 5 lesson. This is a direct technique of the DO_{TE} , which support the PTs to grasp the DO_{SCH} technology– applying the statement of pupils’ previous experienced local MO_{SCH} .

4.2.4. Analysis of the theory block of the DO_{TE}

Regarding Q1, ‘What do you intend the students to learn regarding this topic (the making of formulas for area determination)?’ Mr. Matsui answered: ‘Areas of polygons can be determined in various ways by using pupils’ previous knowledge’. Also, he stressed that the PTs should be able to apply certain didactic terms: ‘The terms describe the various methods of area determination and help the PTs in understanding the pattern of the different solving methods’. Regarding Q7, ‘What teaching methods do you use to make your teaching on this topic engaging, and for what particular reasons?’ he emphasized ‘to consider having the pupils’ perspective’, ‘confirming the previously learned items’, ‘to consider having various solution methods’. He also remarked that it is important to let the PTs know intimately the flow of a lesson with a problem solving approach, which are: reason individually → discuss with neighbours → present the solutions in class → respond to comments from the lecturer. He described how he treats the simulated whole-class discussion with the PTs: he asks some of the PTs who use typical solution methods, to present and explain them to the class. During the presentations, he usually instructs them not directly but by his gestures, where/how they should stand by the blackboard, if the volume of their voice and speaking tempo are appropriate, etc.

These answers indicate, in line with the lesson observations, that the focus of his DO_{TE} is on making the PTs learn how to relate the local MO_{SCH} s of individual lessons on a larger time-scale and thus to construct a complex MO_{SCH} .

Mr. Matsui’s remarks about the importance of using the specific didactic terms and of knowing the flow of the structured problem lessons, indicate that these statements are crucial components of the theory level of the DO_{TE} , which are shared to a large extent within the teacher education. The purpose is to make the theory block of the DO_{SCH} explicit. Since the importance of using the specific terms and applying the problem solving are clearly stressed in the Guidelines, to stress these two issues for the PTs is indispensable. One of the authors have attended method courses in several other universities in different regions in Japan, and observed that every TE refers to the Guidelines and explains the didactic terms described there.

4.3. Lesson observation ‘geometry’ in Finland

The course *Didactics of Mathematics* for PTs for grades 1–6 in a state university located in southern Finland provides knowledge of the contents of elementary school mathematics

and its teaching methods to promote children's learning of mathematics. As described in the methodology section, the course is carried out with lecture respective workshop sessions. The 12 lecture sessions treat basic arithmetic, numbers, fraction and decimals, percentages, units and quantity, geometry, probability, inductive way of working, problem solving, curriculum and observation of pupils' way of thinking. The lecturer, Ms. Ahonen, has worked as a mathematics teacher in primary and lower secondary school for 5 years, thereafter as a TE for 16 years.

In the lecture session, 'lesson 8, Geometry', given several days before, the following concepts were explained: classification of geometrical figures, line symmetry and rotational symmetry, perimeter and the area of polygons, properties of the circle and concept of scale. In the workshop session discussed here, the PTs move between six different tables to work practically with the above-mentioned concepts. The PTs work in groups using a compendium (work sheets with descriptions) written by Ms. Ahonen. The compendium gives instructions on the target knowledge of related mathematical concepts of each table and its teaching methods including some tasks for school pupils. Ms. Ahonen moves between the tables to give advice to the PTs on how to solve the tasks the compendium suggests. Here we present the episodes from the workshop of 'area of polygons' and 'area and perimeter', since the topics treated in these workshops are highly relevant to the topic, which was dealt with in the Japanese lesson.

4.3.1. Episode 1: 'area of polygons'

The description of 'area of polygons' in the compendium starts with the following:

Area of polygons is learned in grades 5-6. The formula regarding area determination should be treated inductively. That is, by looking at few particular cases, one derives the general rules together with pupils.

In accordance with the description in the compendium, one PT in a group plays the 'teacher role'. The 'teacher' explains how to determine the area of rectangles by using grid paper with squares of 1cm^2 .

PT5: How many squares are there now? (points at rectangle of 5 grids in length and 3 in heights)

PT6: We have 15 squares.

PT5: We look at this one (she draws another rectangle of 3 squares in length and 2 in height)

PT5: You can count this here (points at the length 3) and here (the height 2), 3 times 2. When we multiply the width by the height, we get the area (of the rectangles).

Ms. Ahonen ('A' in the transcript) has been watching this group and remarks:

A: Here, we see that the different phases of how one teach the formulas using the inductive way of learning. It means, in reality, there are several cases. Thousands of different cases (of different rectangles) from those you have done here. After you have verified the formula, you can begin to apply this formula,

namely, letting pupils work with tasks from textbooks. So they practise applying the formula. Afterwards it is good to summarise what you have taught and ask yourselves: in which case can you apply this formula? For example, this method (formula) is not suitable for triangles.

The next task is to find out the formula for the area of a parallelogram. The compendium describes the method of parallel translation (however, the term parallel translation is not used in the lesson) and explains that one can use the same formula as for rectangles. PT6 reads aloud the text:

PT6: Pupils draw various parallelograms on the paper. By cutting, they will find out how to form parallelogram to a rectangle.

The PTs cut papers and transform the figure to make rectangles. PT6 continues reading aloud the heading '*The limits of the formula and special cases*' in the compendium to her peers:

PT6: 'One cannot transform a trapezoid back to a rectangle'.

They do not discuss the exact significance of 'the limits of the formula' such as, why One cannot transform a trapezoid back to a rectangle, and in what way then one can teach a method of calculating the area of a trapezoid – they move to the next task: Area of Triangles. They explain to each other the method of area determination by reflecting the instruction of the compendium:

Make a parallelogram by drawing two similar triangles and let pupils notice that the area of one of the triangles is half the area of the parallelogram

Additionally, the compendium describes the property of an area of right triangles under the heading 'The limits of the formula and special cases' as:

Right triangles are a special case where one can determine the area by its cathetus

When PT5 has read these descriptions, she wonders if one can use a rectangle and divide it into two right triangles, instead of using a parallelogram as the compendium suggests. She draws a diagonal in a rectangle and asks Ms. Ahonen:

PT5: Which is the smartest way to determine a triangle's area, starting from a parallelogram or a rectangle?

A: (Points at the rectangle PT 5 made). But the thing is that all triangles do not have right-angles.

PT5: Ok . . .

A: But it is good that pupils verify different ways that the area of triangle is defined by 'Base times height divided by two'.

4.3.2. Episode 2: 'area and perimeter'

The task at this table is to make different kinds of quadrangles having area 12 cm^2 using a Geo-board. Ms. Ahonen encourages the PTs to make even irregular quadrangles with

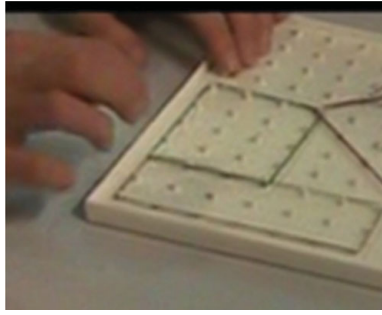


Figure 6. The different perimeters on Geo-board.

the same area. The PTs try making several different shapes of quadrangles and eventually notice that the perimeters do not need to be the same even if the areas are the same. Ms. Ahonen then asks the group:

- A: In which way do the forms of the figures influence the perimeters? What does a figure look like in order to have big perimeter?
- PT7: Like this (makes a long slim rectangle) (see Figure 6).
- PT8: Why does it work in that way? Are there any rules?
- A: It has to do with the inductive way of working in lower grades.

We can derive understanding toward this phenomenon through many single cases in the lower grades. That is good enough on these levels (lower grades).

4.3.3. Analysis of the episodes

In episode 1, the first task of the DOTE is to help the PTs know the ‘inductive way of learning’ which promotes pupils to find out the formula of area autonomously. The second task is to let the PTs learn a specific model of MO_{SCH}/DO_{SCH} of the area determination. There, the techniques of the DO_{TE} are using the compendium with exercises and using role-play. The compendium describes directly a part of the MO_{SCH}/DO_{SCH} . For instance, the MO_{SCH} for finding out the formula is, using figures, counting of the grids, and the multiplication. The statement of (not mathematical) induction is the most evident technology of the MO_{SCH} for justifying these techniques. The other MO_{SCH} technologies are standard units and commutative property of multiplication. Consequently, the technique of the DO_{SCH} , as the compendium suggests, is to let pupils try to count the number of the squares of different rectangles to find out the formula by themselves.

The compendium suggests that one should apply pupils’ previous knowledge to establish ways to compute the area of polygons: from the area of rectangles to parallelograms and then finally that of triangles. However, unlike the Japanese case of finding the formulae of parallelogram and trapezoid, the PTs in this workshop did not have an opportunity to discuss the validity of the formula: in the episode with PT6, who was reading aloud the description in the compendium One cannot transform a trapezoid back to a rectangle, PTs in this group did not have any discussion about why the method of area computation for parallelograms would not work for trapezoids. Also, in the episode with PT5, concerning the area of triangles, Ms. Ahonen does not discuss this epistemological connection. She remarks that ‘not all triangles have right-angles’ but does not emphasize the importance of

using a systematic approach in the MO_{SCH} to make PT5 realize why it is (according to the compendium) advisable to work with parallelograms and not rectangles.

These episodes indicate a limitation of the workshop: Even though the PTs are interested in learning more about the theoretic level of the MOs described in the compendium, the workshop lacks opportunities for discussions and institutionalization of the theory block of the MO_{SCH}/DO_{SCH} , since the TE's primary focus was on emulating the praxis block of the MO_{SCH}/DO_{SCH} . A similar phenomenon is observed in the dialogue concerning the perimeter of a quadrangle in episode 2. PT 8 wants to know more about the theory and technology regarding the area and perimeter computations. However, Ms. Ahonen's DO_{TE} techniques consistently aims to inform the PTs about the inductive way of learning, where an understanding of a phenomenon is absorbed from many single cases. It does not aim to construct a deeper technology of the DO_{SCH}/MO_{SCH} . On the other hand, we have not had an opportunity to observe Ms. Ahonen's corresponding lecture session (implemented some weeks ahead of the workshop), where theoretical (didactical and mathematical) perspectives on the DO_{SCH}/MO_{SCH} concerned by the workshop lessen had presented to the PTs. This might explain her focus on the praxis block of the MO_{SCH}/DO_{SCH} during the workshop.

4.3.4. Analysis of the theory block of the DO_{TE}

According to her responses to the questionnaire, Ms. Ahonen's intention, for the PTs learning regarding the determination of the area of polygons, is a hierarchical structure: the area of triangles is based on the area of parallelograms which in turn is based on the area of rectangles. Ms. Ahonen intends to give the PTs 'a teaching model' concerning the formula for area determination. She considers this section a good starting point for the PTs to learn the inductive rule for teaching, which is important for all mathematics teaching. In response to Q4: 'What kinds of difficulties/limitations are connected to teaching this topic?' she describes the PTs' fragmental knowledge about the formulas for area determination. They can apply the formulas but lack a deeper interpretation of why they work. During her lesson, she often discusses pupils' misconceptions of area and perimeter to let the PTs realize their own misconceptions. Regarding Q7 about the teaching procedures, she describes the combination of lectures, homework following the implementing of workshops with manipulatives and group discussion. She remarks, 'I attempt to emphasize those items which the PTs have difficulty with during my lecture. Some learn by doing and others learn by discussions with groupmates.'

Ms. Ahonen's answers above indicate that the application of the inductive way of working is a crucial component of the theory block of both the MO_{SCH} and DO_{SCH} , since she remarks that the inductive rule for teaching is important in all of the mathematics teaching. At the same time, we can state that the maxim of 'using an inductive way of learning within teacher education' is part of the theory (Θ) of the DO_{TE} that justifies the praxis block of the DO_{TE} . This statement is considered a crucial component of the course. Her remark that she provides a 'teaching model' for the PTs by using the compendium and workshops indicates that the theory of the DO_{TE} for the second task 'to let the PTs learn a specific model of MO_{SCH}/DO_{SCH} ' is a traditional statement of learning by practicing: the consideration that the PTs will learn the praxis of the DO_{SCH} by following the compendium and doing the role-play.

Neither during the observation nor in the questionnaire does Ms. Ahonen's DO_{TE} indicate that she wants to mediate any specific theory of the DO_{SCH} besides the inductive way. She states her concern about the PTs' fragmental knowledge of mathematics, but does not remark on what kind of mathematics the PTs are supposed to learn.

4.4. Lesson observation 'area and perimeter' in Sweden

The course *Mathematics and Learning for Primary School, Grades 4–6 Teachers II, Geometry*, in a state university located in the middle of Sweden, treats mathematical knowledge in geometry and mathematical education in relation to the current Swedish curriculum. Examples of content covered are: An historical perspective of geometry, mathematical terminology within geometry, analysis of pupils' knowledge in geometry, a didactical approach to teaching geometry from theoretic perspectives, different forms of representation in geometry and the importance of using mathematical expressions. The lecturer Ms. Nilsson has worked as a mathematics teacher in grades 4–7 for 13 years and as TE for 12 years. The subject of today's lesson is the concept of area and perimeter and area determination. We present here two episodes from the lesson, which lasted 150 min in total.

4.4.1. Episode 1: the concept of area

Ms. Nilsson ('N' in the transcript) starts the lesson with the definition of the polygons and lets her PTs consider their own interpretations of area and perimeters.

N: When you teach about new concepts, it is better if you reflect by yourself first. What do I know about this? So it will be a good starting point.

Thereafter, Ms. Nilsson gives the PTs five group-exercises concerning area and perimeter. The first exercise is to measure the area of the rectangular chair sheets using a covering by a grid of ice cream sticks. In this exercise, Ms. Nilsson lets the PTs discover the concept of the area by using arbitrary units and the formula for area determination.

N: It is very common that one starts with the formula when one learns a new concept. One might not understand where the formula actually comes from. You see the rectangular figure here; the area is the number of the squares on the one side (points at one side) multiplied to the number of squares on the other side (points at the other side). Then it will be a region, which is covered by x numbers of squares. So, if one counts the number of all squares, which can be quite many, then one may discover that it will be easier if one multiplies one side with the other side; we can say the length times width.

Thereafter, Ms. Nilsson shows the class statistical data from TIMSS 2007 for grade 4 and 8 about Swedish pupils' misconceptions on area determination. She concludes by referring to the Swedish national curriculum:

N: When we look at the curriculum. (Shows the text from the national curriculum on a slide) Here you see about grades 1–3. It says almost the same thing also for

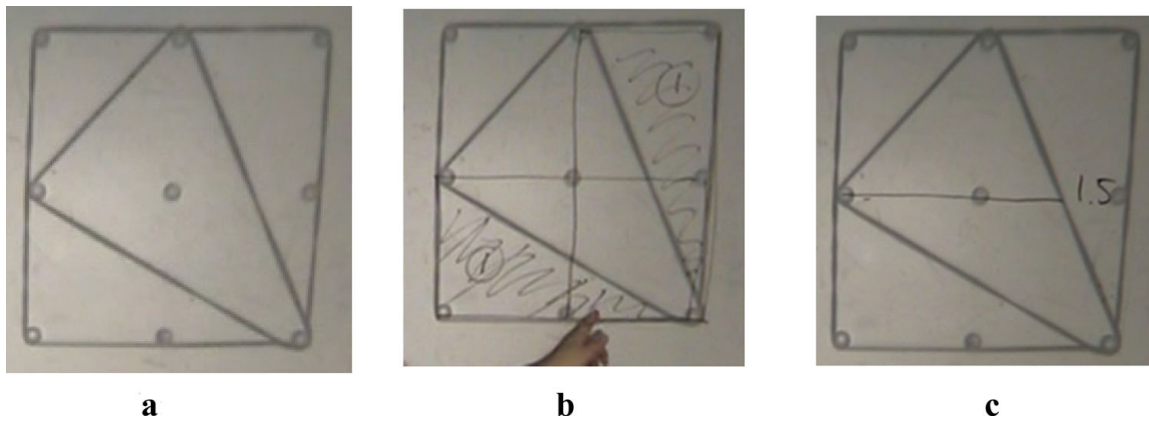


Figure 7. (a) an isosceles triangle (b) with an auxiliary line (c) PT 10's figure.

the grades 4–6. I will not read it out aloud, but you can see what the emphasis in the text is about.

4.4.2. Episode 2: area determination of polygons using Geo-board

The sixth exercise is to determine the area of geometrical figures by using a *Geo-board*. Ms. Nilsson demonstrates a method for area-determination of an isosceles triangle using a rubber band outlining the triangle. She divides the framing rectangle (a square) into four squares. Thus the sides of the triangle occur as diagonals of three rectangles within the frame. Now the PTs ponder the method for area-determination of another isosceles triangle in groups (see Figure 7a).

N: Think about the diagonal. The diagonal must go from the one corner to the other. Not the half way. (PT 9 raises his hand) Yes?

PT9: I use . . . the diagonal to determine the under triangle.

N: Ok. This part (the rectangle on the under part) is 2. 2 divided by 2 is 1 (writes $2/2 = 1$). What did we do here? We circumscribed the whole and made a rectangle. And the rectangle has 4 area units. Then we begin to take away this (triangle) part. We take away one. (writes $4-1$). Then we have a new rectangle here. And in the same way: $2/2$ is 1. I take away this part as well. So you see? We have already taken away those (points at the whole right triangles on the bottom and on the right). We do not take away those now (pointing at the small right triangles on the bottom and on the right) (see Figure 7b).

In the middle of Ms. Nilsson's description, PT 10 suddenly wonders if he can use another method:

PT10: I use this as the base, which is 1,5 (see Figure 7c). And determine the area of the two (upper and under) triangles and add them. The base is 1,5 and the heights are both 1. And I divide it by 2 (writes $1,5/2$). It is 0,75. So I add them. Then it is 1,5.

N: Thank you, (to the class) does it make sense?

PTs: Yes (some).

PTs: No, it does not (some others).

N: I can say, one (meaning ‘pupil’) can understand this method if one is more skilled in mathematics.

Then PT11 wanted to present some other method at the whiteboard. However, he could not completely explain his solution process to the class. Ms. Nilsson comments to PT11:

N: It is good that you have your own knowledge. We should have it. But we must start with something basic when we work with children, so that we do not lose them in the process.

PT12: Can you explain your method again?

N: The one I started with? Yes, let’s finish this. We start from the beginning.

Ms. Nilsson did not let PT11 complete presenting his *different* solution method to the class. However, she explains willingly *her* method again when PT12 asked her to do it.

4.4.3. Analysis of the episodes

The tasks of the DO_{TE} in episode 1 are: firstly, to encourage the PTs to establish their own (correct) perceptions of area and perimeter. Secondly, to inform the PTs of the technique in the MO_{SCH} of finding the formula for the area of rectangles by covering with squares. The technique to realize the first task is to let the PTs write down their current perception of the concept of area. Ms. Nilsson’s intention is that the PTs will validate their actual perception of the concept during and after the exercises.

The second task deals with exactly the same issue as that described in the Japanese and Finnish lessons – let pupils find the formula by counting the number of the squares in rectangles. Ms. Nilsson’s DO_{TE} technique is to describe the MO_{SCH} technique directly for the PTs: ‘it will be the region, which is covered by x numbers of squares . . . one count the number of all squares . . . one multiplies one side with the other side’. This technique does not promote the PTs understanding of the MO_{SCH} technology that justifies the valid MO_{SCH} technique. Neither does it illustrate the DO_{SCH} technique to use for teaching the MO_{SCH} technique to pupils.

In the second episode, the task of the DO_{TE} is to have the PTs experience a model of specific MO_{SCH}/DO_{SCH} of area determination using Geo-board. This technique of DO_{TE} – let the PTs experience a lesson ‘Geo-board with group discussions’ – generated more mathematical techniques than Mrs. Nilsson had expected. The sides of a rubber-band polygon on a geoboard occur as diagonals on rectangles with integer coordinates. Mrs. Nilsson’s intention was to train the PTs’ algorithmic skills with one technique based on the technological observation that the diagonals halves of the areas of these rectangles. As the technique by PT10 suggested, one can make other observations using the integer coordinates of the vertices. She let PT10 explain his alternative technique, but did not validate it by, say, verifying that the base is 1.5 length units as stated, e.g. using the similarity of triangles. She commented to the PT11 ‘we must start with something basic when we work with children’ when he wanted to explain his method. Her intention was not to discuss the viability of different mathematical techniques for grade 5 but to establish a certain MO_{SCH} technique which is possible for all PTs to manage.

4.4.4. Analysis of the theory block of the DO_{TE}

In her response to the first question (TE's intention for the PTs to learn on the area determination), she firstly states that the PTs should be aware of their own perceptions on the concept of area and perimeter: 'That they understand the concepts and methods by themselves is a prerequisite. They must be able to teach to give pupils understanding, in order to create interest and commitment in the classroom'. She strongly emphasizes her PTs' difficulties and limitations concerning geometry. Some of them have learnt the formulas for area determination superficially and sometimes incorrectly. Also, the PTs' perception that 'geometry is a difficult subject' blocks their learning process. Furthermore, the PTs have not developed mathematical terminology allowing them to explain their solutions properly. To deal with these difficulties, she uses manipulatives to give them concrete ideas about different mathematical concepts and train them to establish their own interpretation of the concepts. In order to enhance their mathematical communication skills, she uses group discussions with workshops.

Hemmi and Ryve's research [17] suggests that the 'Swedish discourse on classroom teaching builds on a rather extreme interpretation of constructivism' (p.516). Ms. Nilsson's remark that her first didactic task is to make the PTs be aware of their own perceptions of the concept the area and perimeter indicates that a constructivist theory of learning underlies the justification of her DO_{TE} technique. Further, her strong concern about the PTs' anxieties regarding learning geometry and her attempt to *nourish* the PTs' interest toward geometry, point to the influence from a psychological view of teaching, focusing on the development of students' self-efficacy [35].

Ms. Nilsson's didactic technique for supporting her goal that PTs become 'able to teach so as to give pupils an understanding in order to create their interest' is to demonstrate an 'ideal' lesson example. The DO_{TE} theory that justifies this praxis is a traditional statement of learning by practicing: one acquires a method by watching a demonstrated teaching approach.

5. Discussion

The overall task of the DO_{TE} of the three educators is more or less common: to help their PTs learn to construct the MO_{SCH} and DO_{SCH}. However, the three TEs' techniques for realizing their aim are quite different. We summarize here the results and compare the TEs' main DO_{TE} (see Table 1) to give the answer to the RQ1: *What are the main elements of each TE's DO_{TE} in their lessons? How do they relate the DO_{TE} to the MO_{SCH}/DO_{SCH}?* Also, RQ2: *What are the main differences, concerning the RQ1?*

A significant characteristic of the DO_{TE} technique of the Japanese TE which differs from the Finnish and the Swedish is the *theorizing* of the MO_{SCH}/DO_{SCH} by using *several technical terms* explained in the Guidelines. Regarding the formula for the area of rectangles/parallelograms, the Japanese TE uses the specific terms *direct/indirect comparison*, *arbitrary unit*, *additivity* to make the theory block of the MO_{SCH}/DO_{SCH} explicit, while the Finnish TE used the general didactic term *the inductive way of learning*, and the Swedish TE described the MO_{SCH} technique for the PTs without demonstrating the DO_{SCH} technique to achieve this MO_{SCH}. According to Iwasaki and Miyakawa's study [36] of the process of Japanese teachers' development, the teachers begin to use the technical terms at quite early

Table 1. Task, technique and the theory in three TEs' DO_{TE}.

	DO _{TE} Task (type) (T)	DO _{TE} Technique (τ)	DO _{TE} Theory (Θ)
Japan Academic/ theoretical model	T(a) To help the PTs learn a model of the MO _{SCH} /DO _{SCH}	τ (a) To address some solution methods and related issues in specific terms by studying the Guidelines	Θ (a, b) The shared statement: the specific didactic terms described in the Guidelines should be taught.
	T(b) To illustrate how to link the previously experienced MOs	τ (b) To refer to the Guidelines and textbooks	Θ (c) The shared statement: The structured problem solving approach should be introduced to PTs
	T(c) To anticipate pupils' way of solving problems, and examine the viability of the different solutions	τ (c) To demonstrate the structured problem solving as emulated in a 'short' lesson	
Finland Rehearsal model	T(d) To help the PTs experience 'inductive way of learning'	τ (d, e) To let the PTs follow the compendium by emulating the teaching approach applying a 'role play'	Θ (d) The inductive way of learning should be taught within the method course
	T(e) To help the PTs learn the suggested model of specific MO _{SCH} /DO _{SCH}		Θ (e) The educational principle of learning by practicing: by emulating a specific teaching method one acquires the method
Sweden Immersion model	T(f) To help the PTs experience a model of specific MO _{SCH} /DO _{SCH} of area determination	τ (f) To let the PTs experience a short version of the lesson and teach them a specific τ of the MO _{SCH}	Θ (f) The educational principle of learning to teach by experiencing a teaching model as a pupil: by experiencing a suggested teaching approach, one acquires the method.
	T(g) To help the PTs to establish their own (correct) perceptions of area and perimeter and validate its propriety	τ (g) To let the PTs write down their current perception of the concept of area	Θ (g) Constructivist theories
	T(i) To nourish the PTs' interest and skills in geometry	τ (i) Group-work, workshops	Θ (i) The theory of self-efficacy

stages in their career: 'These terms principally allow teachers to draw attention to significant facts – the nature of mathematical problems, teachers' acts, students' acts, etc. – in the complicated teaching and learning situation, and apply some labels to them' (p.91). Consequently, the use of this language makes it explicit for the PTs how the MO_{SCH} and the DO_{SCH} are *mutually connected*. In the Finnish case, the existence of an explicit technology of the DO_{SCH} (the inductive way of learning) indicates that the Finnish DO_{TE} also aims to theorize the MO_{SCH}/DO_{SCH} to some extent. It gives a particular method for constructing the practice block of the MO_{SCH}/DO_{SCH}, but without much focus on illuminating the mutuality of the MO_{SCH} and DO_{SCH}. Neither, does it demonstrate how to construct a sequence of epistemologically connected MO_{SCH}/DO_{SCH} in the long term. In the Swedish case, the practice block of the DO_{TE} is *individually* designed by the TE, since a collectively shared and generally adapted theory block of the DO_{TE} is absent. If a different TE would be in charge of this course, the structure of the lessons could be quite different even at the same university.

A similar technique shared between the Japanese and the Swedish DO_{TE}, which differs from Finland, is that of letting the PTs participate in a short version of an emulated lesson using problem solving. Both TEs aim to demonstrate a model of MO_{SCH}/DO_{SCH} for the PTs, and immerse them in it. The Japanese structured problem solving establishes

a complex MO_{SCH}/DO_{SCH} , by using well-constructed initial problems and the following whole-class discussions, while the Swedish correspondent contains a single task, since the Swedish TE's main focus is to train the PTs' algorithmic skills. Consequently, the Swedish TE's performance is more that of a schoolteacher rather than a TE, and thus the boundary between the DO_{TE} and the DO_{SCH} becomes unclear in the Swedish lesson.

Considering RQ3: *the institutional explanations for those differences*, we describe the notion of *paradidactic infrastructure* [21]. The paradidactic infrastructure is conditions that affect the teaching related practice outside the classroom praxeology. Japanese *lesson study* is a typical example of such a practice, since it is teachers' 'goal oriented long term collaboration beyond the classroom' [37,p.187]. Within the process of lesson study, Japanese teachers need specific terminologies to communicate with each other, and the most of these terms are clearly described in the Guidelines. Usually, most Japanese TEs participate as advisers/commentators for teachers during 'open lessons' [38] which often are held as a part of lesson study. Thus, teaching and adopting a common set of terms is a crucial component within the method courses in Japan. In the observations made for this paper, neither the Finnish nor the Swedish educators discussed the national school curricula to any significant extent. As was mentioned in the previous sections, both the Finnish and the Swedish national curricula describe the MO_{SCH} and DO_{SCH} in broader and less specific ways than the Japanese counterpart. These differences are also reflected in responses to the TE questionnaire. The Japanese TE focuses on very specific mathematical and didactical aims of his lesson, and how they relate to the school curriculum. The Swedish and Finnish TEs give broader aims, such as filling gaps in PT's mathematical knowledge, recognizing and overcoming their own mathematical misconceptions (Finland), and promoting students' self-efficacy (Sweden).

5.1. The types of DO_{TE}

The Japanese method courses provide established theories that are adopted nationally by universities and mathematics teachers. One reason behind this is the well-maintained paradidactic infrastructure shared by the community of the TEs. In this case, the Japanese model is the most 'university-like' and therefore can be characterized as theoretical or academic.

In the Finnish lesson, the compendium gives a specific set of techniques both for the MO_{SCH} and DO_{SCH} . This DO_{SCH} is then enacted when the PTs *perform as teachers* in the role-play in front of their PT-colleagues. They rehearse the proposed model during the workshop, as a kind of preparation to teach in real classrooms. This technique is justified by the classic educational principle 'learning by practicing'. Hence, we denote the Finnish case as a rehearsal model. However, several different explanations can be given from the viewpoint of the paradidactic infrastructure in Finland: as in Japan, the school-based teacher education [18] almost takes place within the teacher education institution, due to the cooperation with so called university practice schools [18,p.137–140]. Thus, the Finnish PTs have other opportunities to rehearse instances of DO_{SCH} in these schools. Secondly, active Finnish mathematics teachers frequently use a *Teacher's Guide* and its structure and main content are quite similar between different publishers [39]. Thus, those teacher's guides function as a crucial provider of the praxis part of the DO_{SCH} for Finnish teachers in service. Thirdly, the teaching traditions, like applying a balanced combination of lectures and

homework [17] are shared within the community of TEs. In that sense, it is predetermined for the Finnish TE what are the crucial components to be taught within Finnish teacher education.

Unlike Finland and Japan, university practice schools do not exist in Sweden, and the Swedish teachers in service do not generally use a teacher's guide for designing their lessons [40]. It can be stated that a paradidactic infrastructure is not explicitly shared by Swedish teachers. The Swedish TE makes the PTs experience the MO_{SCH}/DO_{SCH} without theoretical explanation and *immerses* them into the MO_{SCH}/DO_{SCH} techniques demonstrated during the simulated short version of the lesson. The Swedish PTs have no opportunities to rehearse a model DO_{SCH} -practice during their university based course. Instead, they experience something like acting as school pupils during Ms. Nilsson's model lesson. For that reason, we call the format found in the Swedish case an immersion model for a methods course. A main difference between this model and the Finish rehearsal model is the role which the PTs get to practice (pupils in the former and teacher in the latter).

6. Concluding remarks

In this paper, we have investigated the didactic praxeologies (DO_{TE}) realized in mathematics teacher education lessons in Japan, Finland and Sweden. In the Japanese lesson, the focus of the DO_{TE} is to convey and exemplify theoretical blocks of MO_{SCH} and DO_{SCH} which are to a large extent prescribed by the national curriculum. The theoretical content of the lesson is supported by the use of well-established technical terms to describe school mathematics and related *didactic phenomena*, and didactic theories such as structured problem solving which are widely shared by Japanese teachers.

The Finnish DO_{TE} is based on a prior lecture on the inductive way of learning mathematics, and lets the PTs practice 'inductive teaching' techniques (DO_{SCH}) by a kind of role-play where students act as both pupils and teachers, following given lesson scripts (compendium). In the Swedish case, the TE immerses the PTs in the demonstrated MO_{SCH} based on principles informally inspired by psychological ideas such as self-efficacy and constructivism. We contend that the presence, in Japan, of a rich, shared, documented and content-specific theory of MO_{SCH} and DO_{SCH} , makes it possible for the Japanese TE to engage in a relatively classical university model of teaching, in which these theories are taught directly, and only exemplified.

In both Finland and Sweden, the corresponding theories remain very general and difficult to relate to actual teaching tasks. However, in Finland, there is also a rich, shared and documented praxis level of DO_{SCH} , within teaching guides and teacher education compendiums; this leads to the model of rehearsing those practices. In Sweden, the teacher educator simply demonstrates, with the teacher students as 'pupils', what she considers good DO_{SCH} -practice. Thus, in all three countries, we find strong explanations for the different choices of DO_{TE} in the different paradidactic infrastructures and resources for mathematics teaching which are available in each country. Certainly, the empirical data of this study is very limited, but the alignments between the striking differences in DO_{TE} and similarly strong differences in the conditions and constraints of DO_{SCH} in the three countries, lend support to our hypothesis that the differences found are far from coincidental, and reveal deeper and more general differences in the

ways in which mathematics teacher education is done and conceived of in the three countries.

Note

1. Bergsten and Grevholm note that the term ‘pedagogical knowledge’ they use, includes Shulman’s [15] notion of pedagogical content knowledge, curriculum knowledge (knowledge of how to sequence topics and use materials in teaching) and knowledge of general issues in education.

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Paper II

The Didactic Notion of “Mathematical Activity” in Japanese Teachers’ Professional Scholarship: A Case Study of an Open Lesson

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Abstract

This paper investigates how Japanese mathematics teachers produce and share didactic knowledge together. It is a case study of a post-lesson reflection meeting so-called *open lesson*. The crucial idea of this study is the dialectic between the *specific* and *generic* level of *foci* of the participants’ reflections about the observed teaching practice; namely, about applied teacher’s specific didactic technique for achieving a specific mathematical goal, and more general pedagogical issues such as realisation of the objectives of mathematics education. This dialectic is mediated by the *meso*-level notion of *mathematical activity*, described in the guidelines for Japanese national curriculum. The application of the *scale of levels of didactic co-determination*, provided by the *anthropological theory of the didactic* into the analysis shows in what way the dialectic interplay between the teachers’ comments with focus of the specific and generic levels influences the development and establishment of the Japanese teachers’ shared professional scholarship.

Keywords: anthropological theory of the didactic, didactic praxeology, mathematical activity, paradidactic infrastructure, teacher knowledge

INTRODUCTION

Using Etzioni (1969)’s expression, the teaching profession is treated as a *semi-profession* in many countries. It means that teaching is not considered as a real profession, at the same level as medicine or law. One of the main reasons is the lack of explicit and justified knowledge that is clearly shared in the community of teachers, as a support to practice their profession (Chevallard, 2006). To explore the components of the knowledge required to become a “good” mathematics teacher, much international research has focused on the cultural scripts of the community of Japanese mathematics teachers (beginning with Stigler & Hiebert, 1999). Some studies focus on Japanese teachers’ widely shared theory about teaching practice (e.g. Jacobs & Morita, 2002) to pursue “effective teaching” and describes the characteristic of their practice (Corey, Peterson, Lewis, & Bukarau, 2010). A considerable number of studies concern Japanese *lesson study* as one of their crucial methods for sharing and developing teacher knowledge (Stigler & Hiebert, 1999; Lewis, 2002; Winsløw, 2011; Isoda, 2015). Several recent special issues and books focus on lesson study implemented outside of Japan (Groves & Doig, 2014; Quaresma, Winsløw, Clivaz, da Ponte, Ní Shúilleabháin & Takahashi, 2018). A number of these studies emphasise the value of teachers’ cooperative lesson planning and feedback receiving during the post-lesson reflection meeting.

Miyakawa and Winsløw (2013) analyse the conditions which support the construction and distribution of knowledge in relation to Japanese mathematics teachers' didactic practice. They present a case study of an open lesson and the following post-lesson discussion, and conclude that these activities enable Japanese teachers to develop and share theoretical knowledge about their teaching practice. Rasmussen (2015) investigates what impact the post-lesson reflection give to prospective teachers during the implementing of lesson study in a teacher education program in Denmark. He analyses the comments concerning the didactic practice observed by the participants during the discussions, and concludes that different institutional preferences (prospective teachers, teachers in service, teacher educators) in the post-lesson discussions are a source of new insight for the participants.

With these studies of post lesson discussions as a starting point, my study aims to investigate one specific and important case of theoretical knowledge, namely the notion of *mathematical activity*, as it appears in a reflection meeting following an open lesson. The importance of mathematical activities is strongly emphasized in the guidelines for the Japanese national curriculum both from 2008 and 2018, within the sections bearing on "objectives and contents". As I will explain further in the following sections, this notion is strongly linked to teachers' didactic techniques to organize students' autonomous learning practice in relation to specific mathematical tasks.

THEORETICAL FRAMEWORK

The analyses of this paper rely on several tools from the Anthropological Theory of the Didactic (Chevallard, 1999, quoted in Bosch & Gascón, 2006, hereafter ATD). The first tool is *praxeological modeling* (ibid.), which can in principle be used on human activity of any kind, by dissecting it in terms of *praxeologies*. A *praxeology* consists of two units; the practical block (*praxis*) and the theoretical block (*logos*). The *praxis* consists of *type of tasks* and *techniques*, which can solve the task. The *logos* "discourse about the praxis", contains two levels: *technology*, which is explanatory and unifying discourse about the techniques, and *theory* which provides a unifying and justifying discourse on the technology.

A *mathematical praxeology* (MP) is evidently one in which the tasks are somehow mathematical (more precisely, are considered mathematical by the institution in which they occur). A *didactic praxeology* (DP) is one in which the tasks concern the teaching of one or more MP. It is carried out by teachers and can have more or less shared logos. These two kinds of praxeology are *co-determined*; it means that a MP developed in the classroom depends on the teacher's DP, and the construction of the DP is depending on how the MP is described officially (in guidelines, curricula etc). We notice that didactic *theory* may be both private and shared by teachers, and it is often not questioned by the community of the teachers. The didactic theory includes "a certain conception of mathematics, the rational of teaching it and the mission of schools in society" (Bosch & Gascón, 2014, p. 79).

The extent to which the praxeologies is structured depends on *paradidactic infrastructure* (Winsløw, 2011), which is the second tool from ATD applied here. This notion is related to the didactic infrastructure (Chevallard, 2009), which describes the totality of conditions for the teachers' work in the classroom, that is, the *didactic praxis*. Paradidactic infrastructure is, similarly, the totality of conditions for teachers' work *outside* the classroom; this includes their efforts to share and develop didactic knowledge which could improve their teaching practice. Teachers' collective activities like lesson study, open lessons and practice research are called *paradidactic practices* (Miyakawa & Winsløw, in press) – they are all essential elements in the Japanese paradidactic infrastructure. Miyakawa and Winsløw (2013) further state that the Japanese paradidactic infrastructure supports teachers' development of the knowledge about the co-determination of DP and MP.

The third tool is the notion of *scale of levels of didactic co-determination* (Bosch & Gascón, 2006). This model was originally created to help analysing how DP and MP are shaped and sometimes defigured by condition and constraints at different institutional levels (from curricular specifications relating to a mathematical technique, to generic features of the school, society and so on). In this paper, I use the scale to situate the teachers' focus during paradidactic practice of post-lesson reflection, called *paradidactic foci*, which could help to identify “unintentional regularities” (ibid., p.54) of DP in the lesson. The levels of paradidactic foci are defined following to the co-determination model as below:

- civilisation (e.g. Oriental culture and ethos)
- society (e.g. Japanese national traits)
- school (e.g. Japanese lower secondary school, with its policies, goals etc.)
- pedagogy (e.g. generic teaching principles)
- discipline (here, mathematics)
- domain (e.g. algebra, geometry,...)
- sector (e.g. equations, similarity,...)
- theme (e.g. triangles, root,...) and
- subject (e.g. one simple type of task, and corresponding technique)

In this paper, the subject and theme levels are called as the *specific-level*, the levels from the sector to discipline as the *meso-level*, and the higher levels as the *generic-level*.

Idea of the study and research questions

Our empirical data come from a so-called “open lesson”, in which a number of teachers and other guests observe and discuss one particular mathematics lesson. In our case, the guests include an invited advisor from an educational university (this is quite common). All participants get a copy of the teachers' lesson plan before the lesson starts. The lesson plan describes the flow of the whole lesson, the students' prerequisite knowledge, the mathematical and didactic tasks of the lesson, and the teacher's ideas for solving the teaching task (e.g. Fernandez, Cannon & Chokshi, 2003; Isoda, 2015). Thus, during an open lesson, the participants observe mainly how the teacher applies explicitly described didactic techniques to realise the mathematical praxeology described in the lesson plan, and also the new MP students develop as a result.

What makes the following reflection session (*hanseikai*) significant is the *dialectic* between *specific* and more *generic* observations. Some participants comment on the realised DP of the lesson focusing on precise didactic techniques to support the students' learning. Others have a broader focus and evaluate the realised DP and MP of the observed lesson and sometimes even more general DPs and MPs within the school mathematics framed in terms against the goal based on certain didactic theories. Miyakawa and Winsløw (2013) also described such dialectic within the reflection session:

The discussion relates the lesson to more theoretical aspects of the mathematics curriculum as such, and even to more general pedagogical and societal aims of the school. This way, the discussion provides a space—an ‘ecology’ in the sense of Chevallard (1988, p. 99)—for developing teacher knowledge that is neither narrowly limited to teaching a particular lesson nor drifting into discussions of teaching philosophies which are more or less detached from the reality of schools and teaching” (p. 204).

Then questions arise: what else could grow in this ecology, in terms of DP logos? Can one find any explicit connections between generic didactic theories and the technologies? In other words, how can *generic* didactic theories, which are directed from the general pedagogical and societal aims of the school, help to organize and

validate the didactic technology that explains and informs teachers' *specific* didactic techniques? If they are connected within the participants' discourse, in what way, are they connected?

To sum up, the research questions of this study are as follows:

RQ1. What are the teachers' paradidactic foci? In other words, what components of didactic knowledge can appear or develop during the post-lesson reflection in an open lesson in Japan? In particular, what is the role of the notion of mathematical activity?

RQ2. How is the teachers' knowledge of didactic practice shaped during the post-lesson reflections, and how do the discussions relate to components of the different levels of didactic co-determination?

METHOD

In order to answer both research questions raised above, I first made a small scale analysis of the guidelines for the Japanese national curriculum (MEXT, 2008, translated into English by CRICED, 2010). To analyse the comments based on the generic didactic theories from the teacher who conducted the open lesson, and the participants, I studied the *objectives of mathematics* in the guidelines, since there, the fundamental aims of the mathematics education that teachers are supposed to realise are described. Considering RQ1, I studied how the guidelines defined the notion of mathematical activities, and how this notion relates to the actual mathematical contents in the guidelines. Secondly, I outlined the core episodes from the open lesson together with the analysis of the realised mathematical praxeology, and the praxis of the teacher's didactic praxeology (DP). The logos part of the DP that justifies the DP praxis is revealed by analysing the various comments of the teacher and the participants during the reflection session. There, all comments are characterised in three major patterns: 1. reflections regarding generic didactic theory, 2. reflections regarding the specific DP, and 3. reflections regarding the generic theory applied to specific techniques and technologies. In this paper, I analyse the comments of the teacher, the advisor and 4 of the participants, which are relevant to the topic described above. As each comment of the participants is described, I have emphasized how the notion of mathematical activities is exposed in their comments. To answer RQ2, I located the comments according to the scale of levels of didactic co-determination, to reveal how the different institutional levels are related in the participants' comments, and how the notion of mathematical activities functions connecting the participants' paradidactic foci at different levels.

The Context of the open lesson

The observed open lesson took place in June 2011 at a 7th grade class (age around 13) of 41 students at the Asahikawa lower secondary school in Northern Japan. This school is "attached" to Hokkaido University of Education (meaning, for instance, that it serves for preservice teachers practice). The school holds an annual one-day "research meeting" (*kenkyu-kai*), and invites hundreds of teachers from inside/outside of the region. Every second year, the school raises a "study theme" which is common for all disciplines in the school. For instance, the theme until the previous year was "raising students' ability to think" and from this year, it is "raising students' ability to express themselves with focus on *questioning*, which supports students' use of their language". The teachers plan and work with the lessons with this theme as focus. The annual research meeting is an important event where the teachers present the outcome of their daily efforts. The teachers in every discipline describe their achievements during the period and their texts are edited and presented in a booklet, which is distributed to all participants during the research meeting. Further, the teachers have an opportunity to improve their work by receiving reflections and advice from the participants from other schools, as well from researchers who are invited as "advisors" from other universities.

Yachimoto, who is teaching the open lesson we consider here, has worked as mathematics teacher for 16 years. The title of today's lesson (and lesson plan) is "determination of the surface area of a cone. In the previous lesson, the students have learned how to determine the area of a sector of a circular disk by using the central angle a (namely, $A = \pi r^2 \frac{a}{360}$). As is usual, the reflection session was held in the classroom immediately after the 50 minutes lesson. The number of attendants to the open lesson was about 65, whereof 25 were student teachers in mathematics. The open lesson and the post-lesson reflection session were video recorded and transcribed into English. The analysis work was done based on the English transcript, and thoroughly discussed with two fellow researchers in didactics of mathematics.

RESULT

The notion of mathematical activities

As it is mentioned in the introduction, the components of the notion of mathematical activities appears significantly within the participants' comments during the post-lesson reflection and plays a notable role to justify their argumentations. To give an insight into the phenomenon, I describe and analyse the notion as it is defined in the section "Objectives and Content of Mathematics" in the guidelines for the Japanese national curriculum (MEXT, 2008, translated by CRICED, 2010).

Normative aspect of the notion

The notion of mathematical activities figures in the Course of Study since 1998. Historically, this notion has been developed over a long period of time before gaining this official status (Isoda, 1999; Nagasaki, 2007). In the guidelines 2008, the notion appears first in the "Overall Objectives of Mathematics":

Through mathematical activities, to help students deepen their understanding of fundamental concepts, principles and rules regarding numbers, quantities, geometrical figures and so forth, to help students acquire the way of mathematical representation and processing, to develop their ability to think and represent phenomena mathematically, to help students enjoy their mathematical activities and appreciate the value of mathematics, and to foster their attitude toward to making use of the acquired mathematical understanding and ability for their thinking and judging. (MEXT, 2008, in CRICED, 2010, p. 15, emphases by the author).

The guidelines then provide further details related to every sentence marked in italics above. The sentence "Through mathematical activities" is described as below:

Mathematical activities are various activities related to mathematics where students engage willingly and purposefully. (...) Mathematical activities may also include engaging in trials and errors, collecting and organizing data, observing, manipulating and experimenting; however, simply listening to teachers' explanations or engaging in simple computational exercises will not be viewed as mathematical activities. (ibid., p. 16)

Ikeda (2008) describes two essential ideas which affected the development of the notion of mathematical activity; students' *autonomy* and *socialisation*. He emphasized the guidelines' phrasing cited above, "where students engage willingly and purposefully..." considering that the guidelines manifest here something *non-mathematical* as a conceptual provision of the notion of mathematical activity. Indeed, the aims described in Overall Objectives of Mathematics, such as "(Through mathematical activities) to appreciate the value of mathematics", "to foster their attitude toward to making use of the acquired mathematical understanding and ability for their thinking and judging" (p. 15), indicate strong normative prerogatives.

Mathematical activities as content and the relation to the structured problem solving

The Guidelines (MEXT, 2008, p. 16) describe three types of mathematical activities, which are particularly emphasized (numbering was added by the author for later reference):

- Type 1: activities to discover and extend properties of numbers and geometrical figures based on mathematics students have learned previously
- Type 2: activities to use mathematics in everyday life and in the society
- Type 3: activities to explain and communicate logically and with a clear rationale by using mathematical expressions

The notion of mathematical activity is included even in the *content*. In the section “Approaches to Content Organisation” the contents are categorized in five domains: A. Numbers and Algebraic Expressions, B. Geometrical Figures, C. Functions, D. Making Use of Data. The guidelines (*ibid.*, p. 77) state that to support learning in each of the content areas A to D, as well as to establish connections between them, students should be provided opportunities experience the three types of mathematical activities mentioned above (see Figure 1). To carry out the mathematical activities, the importance of application of problem solving is emphasized: “Of course, as a principle, these mathematical activities are carried out as problem solving...” (*ibid.*, p.32). In fact, several scholars consider that the problem solving—especially, the *structured problem solving* (Stigler & Hiebert, 1999) approach as a most appropriate method to practicing the mathematical activities (e.g. Kunimune, 2016). I will describe the process of the structured problem solving approach as I present detail of the open lesson in next section.

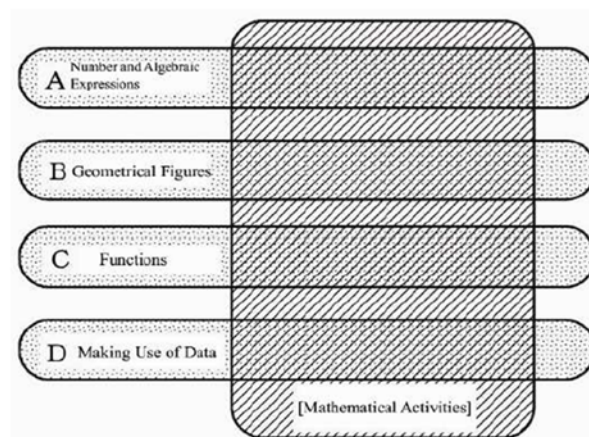


Figure 1. Engaging students in mathematical activities within the five main mathematical contents (MEXT, 2008, p. 78).

In the terminology reference book “Basic knowledge of 300 important terminologies of the teaching mathematics” (Nakahara, 2000), the notion of the mathematical activity is described as “the activities where children create mathematics autonomously” (p. 132) and categorised in three different steps: (1) problem-posing/hypothesis-setting, (2) activities of solving problem, or proving (1), (3) activities of utilization and application of (2). Shimizu (2011) considers that the major part of mathematical activities is coherent with the central property of the structured problem solving itself, and also the implementation of the objectives of mathematics education (p. 5). He states that “mathematical activities should be perceived as the trinity of the objectives of the education, the contents (of the mathematics) and the teaching methods” (*ibid.*, p.5). This expression indicates the normative aspect of the problem solving approach as a method of socialization within mathematics education in Japan.

The Lesson and its didactic praxeology

Yachimoto shows the class a picture of two cones (see Figure 2) and poses the following initial task: *Which of the surface area of the cones is the largest?*

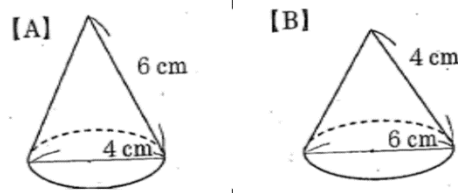


Figure 2. The picture for the task of determination of surface area

The mathematical task is: 1. to notice the unfolded view of a cone is a circular sector, 2. to find out the proportionality between the length of the arc of the circular sector and the whole circle, and 3. to find out the formula for area determination using the generatrix and the diameter [$A = \pi g d / 2$], Yachimoto lets the student raise their hands to vote (teacher's didactic technique τ_1 : to engage the students in the initial task). Five students guess A is the largest, 14 students vote for B, and the rest of the class (about 20) vote for "equally large". The students now consider finding out the solving methods and Yachimoto observes students' work, while circulating in the classroom. This action of the teacher is called *Kikan-shido* (routine θ (technology)), it means "teachers' instruction at students' desk"; teacher's *scanning* of students' individual problem solving process (Simizu, 1999). During this moment, Yachimoto poses a question to the class "what is the problem?" and asks if they know the unfolded view of the cone. When it became clear that the majority of the class cannot imagine the unfolded view of the cone, Yachimoto takes up paper models of the two different cones and lets a student cut and open them. He put the unfolded models on the blackboard (Figure 3). The students notice that they are circular sectors (τ_2 : to let the students realise what information they must know to solve the task).

Yachimoto lets the students to determinate the area of the circular sector A. While he circulates between the desks, he catches a student's murmur "But we do not have the central angle of the sector..." Yachimoto remarks quite loudly (so that all students can hear) "The central angle? Must you have the central angle to determine the area?" Then he asks the class how many of them have a same problem. It shows the majority of them do. He comments: "Ok, you have a trouble not having the central angle. What can we do without the angle?" (τ_3 : to let the students realise that it does not work with the known technique of the mathematical praxeology (MP) and promote to find out a new technique). Yachimoto let a student M to write his solution of the blackboard: $6 \times 6 \times \pi \times 1/3 = 12\pi$. Then he asks the class "Is there anyone who has a problem?" Several students raise the hands and one utters: "How and where the 1/3 comes from?" Yachimoto confirms the other students have the same question (τ_4 : to illuminate the core task of the mathematical praxeology). Then he checks if they know the number 6 comes from the generatrix and asks if there is any other who uses the 1/3. 8 students do. Student N explains: "The length of the arc of A is equally long with the circumference of the bottom (circle). If we compare them, we can find out the central angle of A". Yachimoto then asks the class how the bottom of the cone looks like. It's a circle. He then picks up a circle made by a paper and puts it on the blackboard (Figure 4).

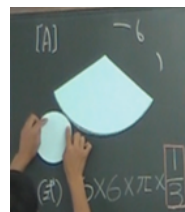
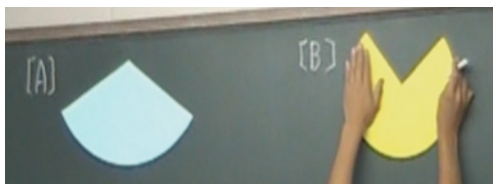


Figure 3. Unfolded views of cones

He repeats what student N said using the model and writes the circumference of the bottom circle and the length of the arc of A is “equally long”. He asks again the class if they now understand where the $1/3$ comes from. They still do not. Student O describes: “If we consider the sector as a big circle, then we can compare to the area of the big circle and the area of the sector” Further, student P explains that one compares the circumference of the big circle (12π) and the length of the arc (which is equally long as the circumference of the bottom circle— 4π), then $4\pi/12\pi = 1/3$ (τ_5 : to give the class several different version of explanations on a certain MP technique by students). Yachimoto writes few key-words on the blackboard: “The whole circumference of the big circle”, “The length of the arc” (τ_6 : to write down key-concept of a technique of the mathematical praxeology on the blackboard to help students’ reasoning) and checks if the class have grasped Student O’s explanation. These whole-class discussions are called *neriage* in Japanese (ibid.) that is the process of polishing students’ ideas and of developing an integrated mathematical idea (routine didactic technology).

After they have found that the surface area of both cones are equally large, and it is possible to determinate the area without the central angle, Yachimoto gives the class a control task: to find the surface area of two cones with the combination of generatrix and diameter 6-8 and 8-6. The students work by pairs and one tries 6-8 combination and the other checks 8-6 (τ_7 : to establish a new technique by explaining it to the classmates). Finally Yachimoto presents student P’s idea to establish a formula for the determination of surface area of a cone: diameter \times generatrix $\times \pi \times 1/2$. He let student P explain how she found out the pattern while she tried to calculate the different combinations of the generatrix and diameter. Yachimoto let the class to look at the textbook where this formula is described (routine didactic technology to *institutionalise* the knowledge) and asks the students what they would associate from this formula. They answer “the formula for area of triangle” (τ_8 : to promote the students explore the technology of the mathematical praxeology). Yachimoto notes that they will work with this concept at the next lesson and with this comment, he closes the lesson.

The reflection session

After the students left, their desks and chairs were arranged so that all 65 participants could be seated in the same classroom during the reflection session (*hansei-kai*). Yachimoto and the chairman, the secretary and a university professor as an advisor sat in front of the rest of the participating teachers.

Reflections regarding a generic didactic praxeology

In this section, I will pinpoint how the participants’ comments relate the generic educational aims to Yachimoto’s actual didactic techniques and technologies. The session starts with Yachimoto’s comments on the mathematics teachers’ work related to the theme of the math department this year—to improve students’ abilities of “to express” their thoughts autonomously and to judge properly:

Since I think it is necessary to improve our students’ ability of mathematical thinking and relate it to the lessons with problem solving, we set this goal. I think the relation between learning mathematics and engaging in mathematical activity is very important.

From the viewpoint of levels of co-determination, posing an educational “theme” as a goal to be realised within the daily work in all disciplines, is derived from the school level. The didactic theory which supports the goal to “improve students’ abilities of to express themselves” is a generic educational conception of the duty of schools. Yachimoto’s comments above address that he deploys the educational theme in a specific didactic and mathematical praxeology by connecting the idea behind the lesson to the problem solving approach. His remark on “problem solving” and “the relation between learning mathematics and engaging in mathematical activity” indicates that the

problem solving is a part of the mathematical activities, which promote the realisation of the educational theme, “to express their thoughts autonomously and to judge properly”. Here, he establishes a clear conceptual link between the generic theory and the teaching practice which has just been observed.

The construction of the conducted lesson and Yachimoto’s perception regarding mathematical activities are partly in line with Nakahara’s (2000) definition of three steps in mathematical activities described in the previous section: (1) problem-posing/hypothesis-setting, (2) activities of solving problem, or proving (1), and (3) activities of utilisation and application of (2). Concerning the third activity, we could not see how all the work of today’s lesson would be applied in the next lessons. However, students’ reactions during the open lesson showed that knowledge gained in previous lessons was invested in today’s lesson. In that way, applying the problem solving also depends on the lower levels of the co-determination like theme and subject, since all these activities (1) to (3) concern about specific didactic technologies. Afterward, Yachimoto began to describe his mathematics group’s concerns about how to design the moment of *kikan-shido*, which in didactic technology refers to the moments of teachers’ observing students’ initial work on a problem. He also explains the background of his didactic techniques (τ)_{1–4} following the flow of his lesson plan. These descriptions of techniques and technologies clearly relate to theme and subject levels.

Participant 1 comments on the goal of today’s lesson, as it figures in the lesson plan: “students will be able to explain how to determine the surface area of a cone”. He asks:

Was the goal realized? How often did the pupils explain during the lesson? To whom did they explain?

Right after participant 1 stated this question, participant 2 criticises Yachimoto’s technique to realise the more generic goal of improving students’ abilities of “to express themselves”:

I think the crucial attitude our students need to achieve is to know the value of mathematics, learning the logic, thinking, communication and so on. You lead the students all the time. Wasn’t there a too small space to let them find out and talk without YOU telling everything? Tell me what kind of activity was used to train their communication skills in today’s lesson. It was great that student P found the formula in the end. Shouldn’t you aim that your students find out things like she did, and let them reason using words and several expressions, rather than to let them follow precisely what you planned?

Then participant 2 begins to talk about the mathematical activity:

There are the text which tell “pupils will learn through mathematical activity” in many different parts in the national curriculum. I think there are three different types in the mathematical activity: 1 finding out (*mitsukedasu*), 2 applying (*riyousuru*: applying the methods one found out), 3 to express and communicate (*tsutaeru*: to tell how one applied it to their classmates). Tell me what kind of activity was done in today’s lesson?

Both participants talk about pedagogy and discipline level issues. The comments such as “goal”, “to whom they explain?” are formed from the generic levels such as school and pedagogy. While the comments regarding “value of mathematics” and “mathematical activity” are formed from meso-level like discipline and domain. Participant 2 gives a direct question how Yachimoto has planned managing of the linking of the logos and praxis part of the DP (“Tell me what kind of *activity* was used to train *their communication skills* in today’s lesson”) and suggestion of a certain didactic technology (“let them reason using words and several expressions rather than following the teacher”).

Yachimoto replies now to the questions. For the planning of today's lesson, he has made a small scale research on his students' knowledge regarding solid bodies. The students could imagine the unfolded view of cubes and cylinders. But only 42 % could correctly determine the unfolded view of a cone:

I wanted to make them realize that they can apply their previous knowledge regarding circular sectors to solid bodies. By cutting the model, they realise that possibility, of using the concept of the plane figure. Then they may see the value of mathematics.

Here, Yachimoto justifies the legitimacy of his DP. First, he describes an element of his paradidactic practice—the pre-research for the construction of the DP of his lesson. Then he explains the didactic technology (*make students realize that they can apply their previous knowledge*), and relates this achievement to a generic goal at discipline level: “*Then they see the value/functionality of mathematics*”. We note that his description is well aligned with the guidelines' description of mathematical activities of type 1 (see the previous section).

Reflections regarding a specific didactic technique

Participant 3 has observed a very specific didactic technique. He firstly mentioned Yachimoto's technique of “questioning” to explain the expression for the area of cone A ($6 \times 6 \times \pi \times 1/3$) to the class:

It's worth noting that the teacher did not simply engage in a dialogue with a particular student. We usually say: ‘discuss among each other’, or ‘discuss with the whole class’. However, I think it is impossible to realize, if the discussion is a ‘free talk’. The teacher must become a manager and connect different persons' remarks. If one masters this technique, one can carry out the problem solving well.

Participant 3 evokes the *professional*, practical knowledge (didactic techniques) for managing a whole-class discussion. Then he asks Yachimoto about how to make the students relate to the idea of proportionality:

My impression is that this idea ($1/3$) is based on the concept: the area of a circular sector is proportional to the length of the arc. Without having this idea, it would never happen that the students find out the $1/3$. How did you do to make them find out that idea?

Yachimoto answers the question:

Actually, when I did a trial lesson in another class, it took 40 minutes to find out the $1/3$. So yesterday, in this class, I asked the students ‘if you know the radius and length of the arc of the sector, can you find out how big the middle angle is?’ Then, they started to talk about the proportionality between the circumference of the whole circle and the arc of a sector. I think they remember what we have done yesterday, and applied that idea.

This dialogue concerns a specific didactic technique (for making the student get the idea for solutions to a specific mathematical task) and generic technology (applying students' previous knowledge), related to the specific mathematical technology (proportional relation between the circumference of the whole circle and the length of the arc). Thus this comment is formed entirely from the lower level such as theme and subject.

Relations among generic theories, specific techniques and technologies

Participant 4 remarks on Yachimoto's technique of organisation of the blackboard:

Every time I see Mr. Yachimoto's lessons, I admire his way to organise the blackboard. Today for example, you used different colors to different matters: orange for the proportionality, blue for the circumference, green for the arc and yellow for the answers. The theme of the year is to improve students' ability of expression. However, the issue of blackboard techniques has not been described in the booklet. If the teacher does not organise the

blackboard properly, the students cannot learn about the proper expression. Can you tell me how we can help students to develop their ability of expression by effective use of blackboard?

Yachimoto replies that he carefully plans the blackboard organisation.

The whole record of the blackboard will remain in the students' notebook. During the last lesson, I was conscious that what I would write on the blackboard would remain on their notebook. They could see it and get some hints from the note. So I plan the use of the blackboard carefully; what topic will be written in which place, recording a student's word verbatim, and so on.

Here, participant 4's question about the specific didactic technique perfectly links to a generic didactic theory. The issue of blackboard organisation for this particular lesson is connected to establishing a shared inventory of ideas about developing students' abilities of expression. Yachimoto's response addresses that blackboard organisation techniques, such as the clear presentation of the problem (and later, solutions from students) by the teacher, are important to students' opportunities to develop their mathematical reasoning and communication, and to record main points developed in the course of the lesson.

Now is the time for the advisor, who is a professor invited from Hokkaido University of Education, giving his concluding comments.

I consider that raising students' ability of expression is about the enrichment of the use of the mathematical language. By seeing all these pictures and expressions on the blackboard (from today's lesson), we can understand exactly how the students thought. I would like you all to emphasise the value of mathematics within your lessons. As we saw in today's lesson, they can discuss and think together in pairs or groups. I observed the students eventually began to understand some issues they did not understand in the beginning, by listening their classmates' comments and writing down classmates' solutions in their notebooks. By doing these activities, their ability to express themselves develops. That is the true training for the ability of expression.

The advisor's comments are related to educational aims and are based on generic didactic theories (derived from pedagogy and school levels). As in participant 4's reflection, these generic pedagogical issues are linked to Yachimoto's didactic techniques as observed; letting the students discuss and think together in pairs or groups, and write down others' solutions in their notebooks. These activities are, according to the advisor, "*the true training for the ability of expression*". Advisor's comments above clearly relate to the mathematical activities type 3: *activities to explain and communicate logically and with a clear rationale by using mathematical expressions* (see the previous section).

DISCUSSION AND CONCLUSIONS

Here I sum up the result and the analysis of the comments to answer my two research questions. For the investigation of RQ1, the components of didactic knowledge can be characterised as follows:

- Category 1: generic DP logos, which discuss how general educational aim such as "to improve students' abilities of expressing themselves" is treated during the lesson.
- Category 2: specific DP praxis, which is the discussion of precise didactic technique, such as how Yachimoto managed letting the students notice the area of a circular sector is proportional to the length of the arc.

- Category 3: the combination of the discussion of generic DP logos and specific didactic technologies and techniques, such as, how Yachimoto organize the blackboard disposition to support the students' development of their ability of expressing themselves.

The analysis shows that the notion of mathematical activities, which has been studied (by teachers) within the Japanese paradidactic practices, promotes having a shared perception of how to capture the dialectic between the generic educational aims and specific didactic technologies. This phenomenon is especially exposed within the category 1 and 3. In the comments of the participants 1 and 2, the issue of training students' abilities of communication, was clearly connected to the mathematical activity of type 3; *activities to explain and communicate logically and with a clear rationale by using mathematical expressions*. The description of this type 3 justifies even the advisor's comment "*By doing these activities-discussing and thinking together in pair or group, and writing down others' solutions in their notebooks-their ability of expressing themselves are developed*".

Also, Yachimoto's remark such as: "*It is necessary to improve our students' ability of mathematical thinking and relate it to the lessons with problem solving, we set this goal (to improve students' abilities of expressing their thoughts autonomously and to judge properly). I think the relation between learning mathematics and engaging in mathematical activity is very important*"; "*I wanted to make them realize that they can apply their previous knowledge (...). Then they may see the value of mathematics*" indicates that he uses the notion of mathematical activities as overall methods for the *learning mathematics*, and the *specific* didactic praxeologies for that are captured by applying the structured problem solving. In fact, the realisation of the notion of mathematical activities in the mathematical and didactic praxeologies of daily lessons is closely related especially to the structured problem solving approach, since this approach is considered as one of the most widespread didactic theories in Japan, and relates to a large set of professional notions (central in the didactic technology) such as problem posing and whole-class discussion (Simizu, 1999). As a teaching approach, the structured problem solving has capacity to carry complex mathematical praxeologies (MPs), also, the structure of its didactic praxeologies—including task construction and whole-class discussion—promotes students' autonomous work on a mathematical task (Author, 2015). As it described in the previous section, the Japanese teachers' and scholars' didactic focus in the use of the structured problem solving is on the cultivation, or socialisation of students, as much as on the didactic techniques to enable development of students' MPs within the daily lessons.

Regarding RQ2, the participants' paradidactic foci—the components of didactic knowledge that were discussed during the post-lesson reflection—were related to different institutional levels of didactic co-determination, such as school, pedagogy, theme and subject. Given that the context was an open lesson in a regional study meeting, where the participants were from different schools, it was nevertheless expected that some comments relate to the levels beyond, or at the level of the discipline, since generic didactic theory is often in focus at such large-scale events. Also, the absence of comments related to discipline, domain and sector, is also expected phenomena, since schoolteachers have limited influence on these higher levels which are fixed by the national curriculum (*thematic confinement*, see Barbé, et al, 2005). Consequently, most of the discussion concerning the MP remains focused on the teacher's theme-specific didactic techniques, as we saw in the participant 3's remarks.

However, in the comments of category 1 (generic DP logos) and 3 (generic DP logos and specific didactic technologies), the both sides of the co-determination (generic levels—school/pedagogy, and specific levels—theme/subject) are related as these categories were extended both to the educational aims and how to actually realise the aims within the DP of the demonstrated lesson, through the notion of mathematical activities as a *mediator*. This notion basically belongs to Japanese teachers' paradidactic practice. As we have seen in the descriptions in the guidelines, the "activity" itself has an aspect of mathematical contents, and serves to develop students' MPs in

different domains. According to the levels of co-determination, mathematical activities as described in the objectives of mathematics education pertain to generic-levels as “school” and “pedagogy”. At the same time, the mathematical activities as mathematical contents pertain to the meso-levels as “discipline”, “domain” and “sector”, since mathematical contents are settled in the national curriculum, which initiates the structure of the praxeologies of mathematics lessons in long span. Thus, this notion is directed from the generic level and meso-level in tandem. Since the main didactic technologies (problem solving, use of students’ previous knowledge, logical communication using mathematical expressions, etc.) mentioned in the post-lesson reflection, were coherent with the characteristic of the notion of mathematical activities (type 1 to 3), we can conclude that this notion functions as a shared *didactic* and *paradidactic theory* for the participants.

As the *hanseikai* exemplifies, Japanese teachers connect these generic theories explicitly to their teaching practice. Depending on the educational goal considered (here, raising students’ ability to express themselves), they design their didactic practice to realise it, and the reflection session is a main moment for evaluating the extent to which it succeeded, and for sharing alternative strategies. The dialectic discourse between generic aims and specific didactic praxeologies, described in this paper, is carried by the notion of mathematical activities, which is an important asset for the Japanese teachers. The analysis shows how this notion reflects both the objectives of mathematics education and the genuine paradidactic practice related to Japanese lessons, as it enables teacher knowledge to be developed and shared beyond a particular theme or domain. For researchers, the dialogue between the teacher and the participants provides detailed knowledge of teachers’ didactic theory blocks and how they seek to manage the co-determination of mathematical and didactic praxeologies. Studying a post-lesson reflection session exposes many aspects of Japanese teachers’ paradidactic practice, including the generic rationales underlying the planning of a lesson, which we cannot see in the lesson itself. The reflection session serves to enhance and share the theoretical block of the teacher knowledge. This and similar elements of Japanese paradidactic practice contribute to develop shared, essential knowledge about teaching, through genuine, professional scholarship.

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Paper III

CONDITIONS AND CONSTRAINTS FOR TRANSFERRING
JAPANESE STRUCTURED PROBLEM SOLVING TO SWEDISH
MATHEMATICS CLASSROOM

Yukiko Asami-Johansson*

CONDITIONS ET CONTRAINTES POUR LE TRANSFERT DE
RÉSOLUTION DE PROBLÈMES STRUCTURÉS EN JAPON À LA
CLASSE SUÉDOISE DE MATHÉMATIQUES

Résumé – Cette étude de cas examine dans quelle mesure une théorie et une pratique spécifique de l'enseignement des mathématiques - la résolution de problèmes structurée japonaise telle que formulée par K. Souma - peuvent être transférées et appliquées dans un nouveau contexte (Suède). L'analyse est basée sur des outils de la théorie anthropologique du didactique. Il s'avère que l'enseignant suédois peut gérer les techniques didactiques proposées par Souma quand celles-ci sont supportées par une technologie didactique qui est familière pour la communauté des enseignants suédois. Pour certaines techniques didactiques de l'approche de résolution de problèmes structurée, ceci n'est pas le cas ; ces techniques étaient en fait difficiles à utiliser pour l'enseignant suédois. Concrètement, il s'agit de techniques liées au *bansho* (organisation du tableau); la pratique consistant à laisser les étudiants formuler le *kadai* (la question essentielle) d'une leçon; et *kikan-shido* (suivi du travail des élèves pour planifier une discussion ultérieure).

Mots clés: transfert de pratiques didactiques, la théorie anthropologique du didactique, résolution structurée de problèmes, modèle de référence praxéologique, infrastructure paradidactique

CONDICIONES Y RESTRICCIONES PARA LA TRANSFERENCIA DE
PROBLEMAS ESTRUCTURADOS JAPONESES PARA LA RESOLUCIÓN
A AULA DE MATEMÁTICAS SUECO

Resumen – Este estudio de caso investiga en qué medida una teoría y práctica específicas de la enseñanza de las matemáticas (la resolución de problemas estructurada japonesa, tal como lo formula K. Souma) se puede transferir y aplicar en un nuevo contexto (Suecia). El análisis se basa en herramientas de la teoría antropológica de la didáctica. Resulta que el profesor sueco puede gestionar esas técnicas didácticas, que están respaldadas por tecnología didáctica compartida dentro de la comunidad de profesores suecos. Existen

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técnicas didácticas comunes dentro del enfoque de resolución de problemas estructurados japoneses para los cuales este no es el caso, y que de hecho eran difíciles de manejar para el profesor sueco, a saber: técnicas relacionadas con bansho (organización de pizarra); la práctica de dejar que los estudiantes formulen un kadai (pregunta central) de la lección; y kikan-shido (monitoreo del trabajo de los estudiantes para planificar una discusión posterior de toda la clase). (by Google translate)

Palabras-claves: Transferencia de práctica pedagógica, la teoría antropológica de lo didáctico, resolución estructurada de problemas, modelo de referencia praxeológico, infraestructura paradidáctica

Abstract – This case study investigates to what extent a specific theory and practice of mathematics teaching – the Japanese structured problem solving, as formulated by K. Souma – can be transferred and applied in a new context (Sweden). The analysis is based on tools from the anthropological theory of the didactic. It turns out that the Swedish teacher can manage those didactic techniques, which are supported by didactic technology shared within the community of Swedish teachers. There are common didactic techniques within the Japanese structured problem solving approach for which this is not the case, and which were in fact difficult to manage for the Swedish teacher, namely: techniques related to bansho (blackboard organisation); the practice of letting the students formulate a kadai (core question) of the lesson; and kikan-shido (monitoring students' work to plan a subsequent whole-class discussion).

Key words: transferring of teaching practice, anthropological theory of the didactics, structured problem solving, reference epistemological model, paradidactic infrastructure

INTRODUCTION

Following up on international surveys on students' knowledge in mathematics, such as TIMSS and PISA, one naturally desires to identify components of efficient teaching, which have led to significant successful performances of students in some countries. Since the early 1990s, educational research has explored efficient teaching approaches from high-performing East Asian countries such as Singapore, China, Taiwan and Japan (see e.g. Stigler and Hiebert, 1999). The next natural step is to try to import these efficient teaching approaches to less successful contexts. This raises several issues: Is it possible for teachers to reproduce the methods, and will their students achieve as well as students in the country where the methods came from? What role does the individual teachers' knowledge and routines play for the successful implementation, and what lies beyond the individual teachers' control? In fact, we do not know much about transfer of teaching approaches from one country in another one, especially long-term transfers that go beyond shorter episodes.

This paper reports on a case study concerning one Swedish teacher, who applied a Japanese problem solving oriented teaching approach as a design tool for her mathematics lessons throughout a full school year. Our research aims to identify exactly what parts of the practices of the Japanese approach were (or were not) successfully implemented, and to identify the conditions and constraints that caused this.

1.1 Related studies

Several international intervention studies have investigated «reconstructions» of teaching approaches of foreign origin (e.g. Ding, Piccolo & Kulm, 2007; Jerrim & Vignoles, 2016). Some of these studies focusing on the Japanese mathematics teachers' practice (e.g. Corey, et al., 2010), and some emphasize that teachers share theoretical assumptions related to the teaching approaches (Jacobs & Morita, 2002). The power and efficiency of Japanese structured problem solving to support students' autonomous learning in mathematics was forcefully reported by Stigler & Hiebert (1999) and is very often related to the practice of lesson study (e.g., Stigler & Hiebert, 1999; Fernandez & Yoshida, 2004; Doig & Groves, 2011; Takahashi, Lewis & Perry, 2013). In fact, the so-called structured problem solving (discussed here in Section 1.3) plays a prominent role in the practice of lesson study (Fujii, 2014).

Mathematics teaching practices are shaped by a number of factors, which often originate from conditions and constraints outside of the school. One of them is didactic theory (i.e. explicit and shared

knowledge and principles related to the teaching of mathematics, cf. Bosch & Gascón, 2014), which differs among societies and among different communities of teachers. This could cause obstacles for applying a specific teaching approach, based on a specific didactic theory, to a new context.

Despite the widespread international interest mentioned above, only few studies have investigated the transferability of Japanese teaching approaches to contexts outside Japan. In particular, only two case studies especially focused on the implementation of the structured problem solving. Groves, Doig, Vale and Widjaja (2016) investigated the possibilities and limitations for the implementation of the Japanese lesson study by applying the structured problem solving in a small-scaled research project in Australia. They found that the Australian teachers realized the value of key-features of the structured problem solving such as making detailed lesson plans with careful consideration of the mathematical goals of the lessons, the use of the initial task which calls on students previous knowledge, and which can lead student to multiple solutions. As a main obstacle to transfer, the authors mention the difficulty in finding suitable tasks to match the Australian curriculum, and teachers' theoretical beliefs. Fujii (2014), in a case study involving two African countries, pointed out teachers' misunderstandings of the structured problem solving approach: «Educational and mathematical values are taught through structured problem solving» is the converse of the misconception «Structured problem solving is just solving a task» (p. 79). In both cases, differing didactic theories are identified as an obstacle to transfer.

1.2. Aim of the study

In this paper, I investigate the transferability of the Japanese structured problem solving approach from an institutional perspective. I first exhibit the institutional conditions in Japan, which favour the development and use of this approach, and compare with similar conditions in Sweden. Secondly, I analyse the outcomes of an intervention in which a Swedish teacher attempted to use the approach throughout a whole year: which parts of the approach were realized, and which were not. Finally, I discuss how the differing institutional conditions could explain why some parts were realized and others were not.

1.3 Kazuo Souma's version of structured problem solving approach

In this study, I focus on a variation of the Japanese structured problem solving approach called *Mondaikaiketsu no jugyou* (problem

solving oriented lessons). This approach was developed by Kazuhiko Souma, who is a professor of mathematics education in Hokkaido, Japan. Souma has written and edited a number of books regarding teaching practice, including textbooks. In these books, he proposes lesson plans and collection of tasks, as well as theoretical ideas. This was a main reason I chose Souma's version: there are sufficient materials to work in several subjects over an entire school year. Souma established his method from long experience as a mathematics teacher in lower secondary school; his research is clearly practice-related. He gives numerous lectures and workshops every year, for mathematics teachers all over Japan. As most other Japanese professors in mathematics education he often participates in lesson study type activities. At the theoretical level, Souma (1995, 1997) refers to Dewey's theory of reflective thinking (Dewey, 1933) and Polya's work on problem solving (Polya, 1957), in particular Polya's emphasis on the use of guessing and conjectures. In Section 3, I will analyse this approach in more detail, based on the theoretical framework introduced in Section 2.

THEORETICAL FRAMEWORK AND RESEARCH QUESTIONS

This paper is based on the anthropological theory of the didactic (ATD). Starting with the didactic transposition theory (Chevallard, 1985, quoted in Bosch & Gascón, 2006), ATD has been developed and offered several different tools to study various problems regarding the dissemination of knowledge in different forms of institutions. In this section, I will describe how I applied which of the tools provided by ATD to investigate didactic phenomena within various didactic systems (Bosch & Gascón, 2014), observed in Japan and Sweden. A didactic system is formed by learners (e.g. students), actors who help the learners (e.g. teachers) and so called didactic stake—a thing to be learned by the learners.

The notion of praxeology contributes to model all kind of human activities, such as learning of mathematics. Since learning of mathematics rarely takes place without activities (such as listening an explanation by a teacher, solving a problem), a praxeology always consists of a practical unit (know-how) and a knowledge unit (know-why). The practical unit is called praxis, and the knowledge unit is called logos. Further, we can zoom in on each unit. The praxis is constituted by types of tasks, and technique, which gives a method to solve the tasks. The logos is constituted by technology, which is a

discourse of the techniques, and a theory, which in turn justifies the technologies. The praxeologies that concern the realisation of mathematical tasks are called mathematical organisations. The teacher's practices that aim to realize the mathematical organisations of his/her lessons are called didactic organisations. Student's mathematical organisations and the teacher's didactic organisations are affected by each other. The dynamic of such didactic system is called didactic co-determination. The use of mathematical and didactic organisation is especially useful to investigate the structure of Souma's design of the lessons, since the notions can technically describe how the didactic stake that students are supposed to learn, and the construction of Souma's didactic organisation are interacted.

To identify the elements of Souma's approach, which the Swedish teacher realized and which were not realized, we need a tool to model the practice: a reference epistemological model (Chevallard, 1985, quoted in Bosch & Gascón, 2006). The reference model (developed in Section 5) provides a theoretical model of the didactic techniques described by Souma's books, and is then used to analyse the Swedish teacher's didactic practice.

Didactic praxeologies depend upon conditions of different kinds and origins. The scale of levels of didactic co-determinacy (Chevallard, 2002 in Bosch & Gascón, 2006) models a hierarchy of such origins, from generic levels like culture and society, to very local ones like the subject taught. The didactic conditions are divided into five sub-levels from discipline, domain, sector, theme and subject. We make use of these notions to study how Souma's didactic technologies and techniques originated from different levels in Japan. It will reveal institutional conditions and constraints that influence the attempt to transfer the Japanese practice into the Swedish context.

Teachers' didactic praxeologies are designed outside the classroom as they make lesson plans, search for appropriate tasks and revise work from lessons done in the past. The notion of paradidactic infrastructure (Winsløw, 2011) describes the totality of conditions for the teachers' work outside of the classroom. A typical example in Japan is *kyozai-kenkyu* (Watanabe, Takahashi, & Yoshida, 2008), which is the study and production of teaching material, in the context of lesson study. Other examples include open lessons (Isoda, 2015) and practice research (Miyakawa & Winsløw, 2019), and common educational conferences for math teachers and researchers. The paradidactic infrastructure in Japan thus provides many frameworks for teachers' collaborative practice (outside of the classroom).

We can now formulate the research questions of this paper:

1. What are the central didactic techniques and technologies in Souma's version of the structured problem solving approach, and what conditions are crucial for its realisation in Japan?
2. To what extent did the Swedish teacher realize Souma's central didactic techniques described in RQ1? How can this be explained by a wider view based on the levels of didactic co-determinacy, and on differences in paradidactic infrastructure?

STRUCTURED PROBLEM SOLVING IN JAPAN

In this section, I present a brief historical account of the structured problem solving approach. Then an overview of the development of this teaching approach in Japan is described. The intention is to give the reader a picture of the paradidactic infrastructure, which enabled the appearance of structured problem solving in the form known today. We also aim to provide a brief account of how this approach is perceived by the community of Japanese educators and mathematicians.

At the end of the nineteenth century, Japanese mathematics education focused on rote acquisition of practical skills and formal knowledge (Nagasaki, 2007a). During the early 20th century, major educational reforms, inspired by Dewey and Thorndike, were introduced in Japan (Nagasaki, 2011). Several leading educators began to criticise the mathematics tasks presented in the Japanese textbooks of that time (Matsumiya, 2007). They found that the tasks lacked links to pupils' everyday-life. Reflecting on such criticisms, the first government-approved textbook in mathematics, the «Green Book» for the first grade, was published in 1936. Matsumiya (ibid.) states that, the Green Book gave Japanese teachers the first opportunity to consider mathematics teaching from the pupils' perspective and to try to develop pupils' ability to use mathematics autonomously.

During the 1950's, the goal of mathematics education transforms into understanding the mathematical concepts (Nagasaki, 2007b). The notion of mathematical way of thinking (*sugakuteki na kangaekata* in Japanese) was introduced to the Japanese national curriculum for upper secondary school in 1955 and for elementary school in 1958 (Nagasaki, 2007a). Nakajima (2015, p. 82, original work published 1982) describes improving students' mathematical way of thinking as something that «trains students so that they independently can generate creative activities, which are mathematical», which means: to explore the problems, to acquire methods for making some hypotheses, finding ideas which lead to solving problems, generalisation/extension of the mathematical ideas, and evaluating ideas/solutions.

This notion of mathematical way of thinking was developed by mathematics educators and mathematicians during the 1960's, and the notion of problem solving became connected as a method to develop students' mathematical way of thinking (Ito, 2010). Akizuki (1968, in Nagasaki, 2007a) noted, «We must provide good problems for students to raise their ability of reasoning. The best way is to give them carefully thought-out problems, if we want to advance their skills for modelling» (ibid., p. 175). Kikuchi (1969, in Nagasaki, 2007a) defined the mathematical way of thinking as «processes in which one finds mathematical facts through understanding of mathematical concepts, constructing mathematical problems and solving them» (ibid., p. 175) and described the process of problem solving as (1) Configuration the initial tasks (2) Observation (to find the core-task) (3) Classifying the facts (4) Making the hypothesis (5) Examination of the methods (6) Proof (the methods) (7) Application (of methods to another tasks) (Nagasaki, 2011, p. 36). This list may have been influenced by Polya's four phases of problem solving. The notions of mathematical way of thinking and problem solving are also connected to the notion of so-called *mathematical activities* (*sugakuteki katsudou* in Japanese) in the national curriculum (see Asami-Johannsson, in review).

Within the structured problem solving, the process of students' problem solving described above, especially steps (3) to (7), are supposed to be realized through students' discussion. One of the representative methods of problem solving developed during 1970's to 1980's, is called open-ended approach. It is based on open-ended problems (Becker & Shimada, 1997). An open-ended problem is a conditional or incomplete problem, which enables multiple answers and the search for different solution methods. The open-ended approach is structured to develop students' skills to relate to other students' discoveries or methods, comparing and examine the different ideas, and modifying and further developing their own ideas (Sawada, 1997, p. 23). Certain classroom activities, such as students explaining their reasoning, and comparing the different solutions, were carried on to the structured problem solving approach.

The basic flow of the Japanese structured problem solving approach is described by Shimizu (1999, pp. 109-111). He explains how the following didactical terms (in Japanese, with English translations) are used by Japanese teachers on a daily basis:

1. Hatsumon: to ask a key question that provokes students' thinking at a particular point of the lesson.
2. Kikan-shido: teachers' instruction at students' desk. Scanning by the teacher of students' individual problem solving process

3. Neriage: whole-class discussion. A metaphor for the process of polishing students' ideas and of developing an integrated mathematical idea through the whole-class discussions.
4. Matome: summing up. The teacher reviews what students have discussed in the whole-class discussion and summarizes what they have learned during the lesson.

The existence of such common didactical terms are indicative that the Japanese educators have an institutionalised perception about the teacher's role in the mathematics classroom.

CONTEXT AND METHODOLOGY

We implemented a longitudinal study involving observations of 40 lessons in grade 7 (September 2010 to May 2011), and 7 lessons in grade 8 (February to April 2011), at a lower secondary school in Sweden. Three months before the lesson observation started, I had the first meeting with Eva (pseudonym), the teacher who taught all lessons of this study. At this meeting, I described the structure of Souma's approach, as the following «script» for a lesson:

1. Present a task, which represents a new concept or solution method for the students, in a way which will incite them to guess answers, or make immediate observations
2. Let all students make such guesses, hypotheses, etc.
3. Discuss the viability of guesses and let students motivate these guesses, then derive a formulation of the core task to be worked on in the lesson
4. Let students work on the core-task, individually or in groups,
5. Let students present their various solutions to the whole class, and discuss the differences among their solution techniques
6. Turn to the textbook (or similar text) for an outline of related theory.

I gave her translations (made by me) of Souma's instructions on conducting whole-class discussion, and for organising blackboard writing (Souma, 1997, section 6-1). I also translated parts of Souma's task collection (Souma, 2000), for her to see the characteristics of his tasks. In August, Eva and I started to fix the details of the lessons for the grade 7 class. We discussed about the goal of the lessons, the initial tasks, the flow of the lesson, and the tasks for homework. She learned how to write up a lesson plan, as described by Kunimune and Souma (2009a, 2009b), which involves three parts: 1. The goal, 2. The

preparation and 3. The flow of the lesson. Part 3 includes three columns, which together form a chronological script for the lesson (examples are given in Section 5):

- a) Teacher's activity (what the teacher will do during the lesson),
 - b) Students' activity (what the students will do during the lesson),
- and
- c) Other things to consider or notice at particular points of the lesson.

In the first column, one describes e.g. the initial task, questions to ask to the students during kikan-shido (teachers' circulation during the individual work) and neriage (whole-class discussion), or in the process of matome (summing up). In the second column, one tries to predict students' guesses and solution techniques, and likely errors are included. In the third column, one notes didactical/pedagogical considerations, such as «make students write down sketches of the figure first» and «if nobody comes up with the solution 1-3, I will mention the subject of the previous lesson».

I observed 40 lessons in grade 7, and 7 lessons in grade 8 during the project period, and video recorded the lessons from January 2011. By that time, it had passed almost a half year since Eva started implementing lessons with Souma's method. For the analysis, the observation notes I made in all lessons was also an important source.

To answer the first part of RQ1, I first studied the overall structure and flow of the approach as presented in two of Souma's books: «Improvement of the lessons of mathematics with guessing» (1995) and «The problem solving approach - the subject of mathematics» (1997). I present the reference model for Souma's didactic techniques and technology by using one particular lesson plan by Souma and Kunimune (2009a), namely, «Introduction to finding the general solution» for grade 7. The analysis of the structure of Souma's didactic praxeologies is used to build the epistemological reference model, to answer the first part of RQ1, and to serve as reference for answering the first part of RQ2. One reason for using the lesson plan «introduction to finding the general solution» to identify and illustrate the key elements of the reference model, is that this lesson plan was also one of those directly implemented in Swedish lessons for grade 7 and 8. Therefore, I could use it also as a case for comparing the outcome of the Swedish lessons and Souma's original lesson plans.

To investigate the second half of RQ1, I needed to identify the didactic theories that are used to justify and explain the praxis part of Souma's approach. For that reason, I studied the commentary sections

of the corresponding collection of lesson plans, written by Souma and other authors of the book. My intention was to identify the theory blocks directly related to the didactic practice described by the lesson plan. The title of the lesson plan collection includes the notion of mathematical activities, and the authors declare that the aim of the book is to propose ideal lesson plan sequences by applying different kind of mathematical activities. The notion of mathematical activities also appears in the guidelines for the Japanese national curricula, and becomes central to our analysis of the lesson plan.

Then I investigated how the texts of the Japanese national curriculum and Swedish national curriculum deal with the notion of problem solving. This contributes to explain how teachers' perception of problem solving differ between the countries. These analyses are used to answer a part of the second half of RQ2, about institutional conditions that explain the degree of transferability of the structured problem solving approach.

The main strategy to answer RQ2 is to apply the reference model to analyse the Swedish teacher's lessons, and the institutional conditions in which the teacher works. Firstly, I made detailed analysis on her implementation of the lesson «introduction to finding the general solution», which was implemented in both grade 7 and 8. I observe the video and checked which of the techniques described in Table 2 (the summary of the didactic techniques from the Japanese lesson plan) she used, did not use, or attempted to use. Then I studied her other 16 lessons in a similar manner. In these lessons, we applied tasks from Souma's task collections (Souma, 2000), lesson plans from the lesson plan collections by Kunimune and Souma (2009a, 2009b), and the tasks that the Swedish teacher has created by herself. I noticed, in particular, how the didactic techniques related to hatsumon, kikan-shido, neriage and matome, since it became obvious that the degree of realisation of didactic techniques had a clear connection to these clusters of techniques. For the analysis of the institutional conditions and constraints, which influenced her lessons, I analysed which didactic techniques are related to the levels in the schema of the didactic co-determinacy, by studying the didactic tasks the each techniques are aimed to solve.

RESULTS

In the first part of this section, I present the general tenets of Souma's structured problem solving approach. I also provide a brief account of a Japanese didactic notion of «mathematical activities» which has strongly influenced this approach. Then, the specific lesson

plan «Introduction to finding the general solution» is analysed. The praxeological analysis of this lesson plan is used to illustrate the reference model, which I then present in general. Finally, I analyse the Swedish lessons and in particular the Swedish teacher's use and non-use of Souma's didactic techniques.

5.1 Souma's problem solving approach

Souma (1997, p. 31) describes the flow of a lesson from the students' perspective as follows:

1. **Understanding the problem:** students grasp the meaning of the problem shown on the blackboard, and try to work with it.
2. **Guessing:** students state a guess for the answer to the problem, or suggest one or more solution methods.
3. **Formulating the *kadai*** (core task) of the problem: within the process of trying out their hypotheses, students formulate the core task (see the description below) that the teacher's initial problem aimed at.
4. **Solving the *kadai*:** within the process of solving the core task, students acquire sufficient knowledge and skills to solve tasks of this type.
5. **Solving the initial task:** By using the knowledge they got during the process above, students solve the initial task.

The teachers' actions follow largely the Japanese structured problem solving: showing the problem, observing students' individual work, then conduct a whole-class discussion. It differs however, in that it initially lets the students state a guess or hypothesis and that it invites them to reformulate the problem and clarify the core task.

Souma stresses some conditions on initial tasks that lead to successful lessons: 1. tasks should trigger students' curiosity so that they immediately start tackling them, 2. tasks should enable students to learn new knowledge, techniques, concepts and ways of thinking (Kunimune & Souma, 2009b, p. 11). He calls the initial tasks open-closed problems in which the students' answers, in the later stage after they make the guesses, should be unique (closed), but still give rise to a variation (openness) of the solution methods. This is in contrast to «*open-ended-problems*» (c.f. Becker and Shimada, 1997). Kunimune and Souma (2009b, p. 11) describe the four different types of initial tasks as follows:

- I. Form of answer is closed: «How many cm is ~?», «What kind of triangle is this? »

- II. Answer is among some given alternatives: «Which of these expressions are actually same? », «Which of these expressions are right /wrong? »
- III. Two options for answer: «Is it correct that ~?» «Is it the same as ~?»
- IV. Open answer: «What can you say about the following expressions? »

To understand what Souma means by *kadai* (a core task), he presents an example case (1995, pp. 102–105): if the initial question is «Show that the difference of the squares of two integers that follow each other is equal to the sum of the two numbers», there will be some students who have no idea where to begin, and possibly some skilled students find the solution easily, and then just wait with nothing to do until the other students find the solution. However, if the teacher silently writes down the expressions $52 - 42 = 9$, $82 - 72 = 15$, $42 - 32 = 7$ on the blackboard, and asks the class, «*What can you say about these expressions?* », students may come up with several initial guesses: «*the differences equal the sum of the integers*», «*the differences equals the first integer times two minus one*», «*the last integer times two plus one*». After the class has verified each of these statements, they start to wonder if those statements always hold and why. They now want to prove them. Then the students have formulated the core task from their guesses. The formulated core tasks should have many possible solutions, so that some students may use the formula for expanding the square of a sum, and others apply the rule of difference of two squares, etc.

The whole-class discussion (*neriage*) is considered as a crucial moment of the structured problem solving approach. It is supposed to be implemented during «solving the core task». Souma explains that there are mainly two types of methods for conducting whole-class discussions (1997, pp. 66-71):

- A. Let students presenting different solution methods one by one
- B. Let students write different solution methods simultaneously, at the blackboard

Further, he describes two additional options for a teacher:

- a. Plan the order of students' presentation beforehand during *kikan-shido* (teacher's observation of students' solutions)
- b. Have students raise their hands and let them present their solutions, without planning the presentation order.

One can combine above patterns A, B and a, b, to yield the combination Aa, Ab, Ba and Bb, with the obvious meaning. According to Souma, teachers should use these different combinations depending on the following four criteria: 1. the goal of the lesson, 2. the characteristics of the initial tasks, 3. levels of the students groups and 4. time issues. He admits that it is difficult to explain the relationship among these 4 conditions and to use them clearly to motivate the choice the method for organising student presentations, since it depends on the teachers' comprehensive judgment of the situation and priorities.

After the moment of *neriage*, Souma (1997, p.72) considers that the teacher should sum up students' various solution methods in one of three different ways:

- P.** When all students explained their solution method, the teacher asks who used which method, and what kind of knowledge from previous lessons they were using.
- Q.** The teacher asks the class which method they prefer and why.
- R.** Teacher asks the class the relevance and differences between the solutions.

The use of these different manners depends on the teacher's focus in the lesson. Souma presents the following possible focuses (1997, p.73):

- P....**To let the students understand different ways to solve the problem
- Q....** To let the students understand efficient solving methods
- R....**To improve the students' «mathematical way of thinking»

Certainly, to enable any of the above, it is important that the students have easy access to all solution methods proposed – in Japan, the blackboard is used to serve this and many other purposes. According to Souma (1997, pp. 74-75), watching the contents written on the blackboard and copying the writings in their notebooks should help students obtain:

1. A sense of necessity to solving the task
2. A sense that they are solving the problem together with the classmates
3. Understanding the process of the problem solving
4. A sense of necessity of thinking about the problem.

In this way, the blackboard should develop a record of the whole lesson, since the teacher usually never erases any of the text written on the blackboard. Souma considers that the issues 1.-4. above are all essential components for the realisation of lessons within a problem solving approach.

5.2 The notion of mathematical activities and problem

The objectives of mathematics education is described in the Japanese national curriculum (MEXT, 2008, translated by CRICED, 2010, p. 15) as:

Through mathematical activities, to help students deepen their understanding of fundamental concepts, principles and rules regarding numbers, quantities, geometrical figures and so forth, to help students acquire the way of mathematical representation and processing, to develop their ability to think and represent phenomena mathematically, to help students enjoy their mathematical activities and appreciate the value of mathematics, and to foster their attitude toward to making use of the acquired mathematical understanding and ability for their thinking and judging.

In the commentary of the lesson plan books for grade 7, Kunimune and Souma (2009a) cite the guidelines' definition of the mathematical activities («various activities related to mathematics where students engage willingly and purposefully», p. 8). They emphasize that mathematical activities are both a method to realize the generic objectives of mathematics education (e.g. «improving students' autonomous thinking») and a method for the teaching of mathematical contents. They exemplify with activities such as «guess the result», «find a pattern», «examine if there is a counterexample», «explain using various solutions», «hypothesize deductively», «think inductively», «apply previously learned knowledge», «explain one's reasoning», «observe and experiment», and so on (ibid., p. 10). Those activities are entirely consistent with the basic ideas of Souma's approach as described in the previous section, as has been extensively discussed among Japanese didacticians and teachers (e.g. Shimizu, 2011; Kunimune, 2016). The guidelines also mention the relationship between mathematical activities and problem solving (MEXT 2008, p. 51; emphases by the author):

In principle, *mathematical activities are carried out as problem solving*. That is, they are a sequence starting with generating wonder and questions, formulating problems by formalizing them, understanding the problems, planning, implementing and reflecting on solution processes, generating new wonder and questions, generating

conjectures, and formalizing problems. Experiencing these series of activities provides opportunities *for students to feel the joy of thinking and learning mathematics as well as the necessity for and the usefulness of mathematics*. In addition, because these activities require students to persist, they *provide opportunities for students to heighten their own self-esteem*.

These descriptions about the process of the problem solving—starting from generating wonder and questions, then formulating genuine problems by formalizing them, generating new wonder and questions, generating conjectures, and formalizing solutions, all have the same feature as Souma’s description of his approach. We can therefore say that the notion of problem solving described in the guidelines mainly refers to the structured problem solving approach. We also note that applying the problem solving approach is strongly connected to generic (normative and pedagogical) purposes of teaching mathematics. In fact, the guidelines also state that (MEXT 2018, p 51. text in the parentheses by the author):

Furthermore, by listening to and adopting different ideas from others, these activities can promote students to understand each other better. Therefore, mathematical activities are not (only) processes of learning mathematics but also particular content (by themselves) ...

To sum up, both the notion of mathematical activities and the structured problem solving approach are essential didactic theories for Japanese mathematics teachers.

5.3 Presentation and analysis of a specific lesson plan

In this section I present a specific lesson plan «Introduction to finding the general solution», proposed by Kunimune & Souma (2009a, pp. 26-29), and an analysis of the corresponding didactical praxeology.

The lesson plan begins with «The idea of this lesson», where the teacher describes his/her pedagogical and didactic considerations and aims (ibid., p.26). Here is an outline: In primary school, students have used some symbols like \square and \square for «numbers to find», as they made calculations with addition/subtraction and multiplication/division. They have also trained how to express a relationship between two quantities, and to interpret expressions. In this lesson, they will learn using variables as a basis of modelling, before starting to learn about solving linear equations. The weight of this first lesson is on letting the students realize the convenience of using formulae, such as in the formulae for the area calculation of a triangle. Students will solve the

task by using several mathematical methods such as induction, analogy and deduction.

Then the initial question is presented:

We will make a square by arranging stones as in the picture below. If one of the sides consists of 5 stones, how many stones are used in total? (See figure 1)

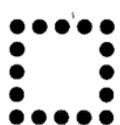


Figure 1. – Illustration of the «Square Problem»; «How many stones are used in total?»


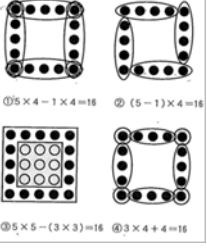
The teacher will let students reason by drawing his or her own figures. During the moment of whole-class discussion, students discuss their various way of thinking. The teacher now gives a second task: to determine the total number of the stones of when there is 20 stones on each side. These first activities will lead to the core task: to find a method to determine the total number of stones in a square with any number of stones on one side. When the students formulate such an expression, they will feel the inconvenience of using the phrase «number of stones on one side» repeatedly. In that way, students will appreciate the convenience of using a letter, rather than a phrase. During matome (summing up) moment, students will explain how the variable represents an unknown number within in some possible domain (here, positive integers).

«The goal of the lesson» is then described as follows (ibid., p. 27):

- To understand the meaning of using variables
- To raise students' ability to explain their reasoning clearly to their classmates, by using figures and mathematical expressions

The next section is «The mathematical activities in this lesson», where the main activities are explained. Finally, the flow of the lesson is described (as in Table 1).

Teaching content and learning activities	To think about	Mathematical activities
1. Initial question T (teacher): Let us discuss about the value of using formulae for area calculation! S (students): Triangles: $\text{base} \times \text{height} \div 2$	They will remember the formulae they have learned in	Through reflecting on their «old» knowledg

<p>Squares: one side \times one side Rectangles: length \times height Parallelogram: base \times height Trapezoid: $(\text{base} + \text{base}) \times \text{height} \div 2$ Rhombus: diagonal A \times diagonal B $\div 2$ Circle: $r \times r \times \pi$ S: If one knows and remember formulae for different figures, one can easily calculate its area.</p>		<p>elementary school, and recognize the meaning of making formulas</p>	<p>e, making students interested in the meaning of making formulas</p>
<p>2. Present the initial task We will make a square by putting stones as in the picture. If one of the sides consists of 5 stones, how many stones are used in total? S: the students will reason in various ways</p>  <p>Figure 2. – predicted solution methods</p>	<p>I will let the students present their various way of thinking. We control that these expressions presents students' way of thinking.</p>	<p>To express students' thought by using several expressions and figures. To communicate and tell their reasoning.</p>	
<p>3. Solve the task by using previous knowledge T: Let us consider if the number of the stones is 20 on one side, S: 1. $(2 \cdot 4) - (1 \cdot 4)$ 2. $(20 - 1) \cdot 4$ 3. $(20 \cdot 20) - (20 - 2) \cdot 2$ 4. $(20 - 2) \cdot 4 + 4$</p>		<p>Analogical argument (applying the argument regarding 5 stones on one side to 20 stones)</p>	
<p>4. Making students conscious about the core task T: «Can we make a formula which we can use for the calculation of the total number of the stones no matter what number of stones on one side, using 1–4 you made? » 1: $(\text{number of stones on one side} \cdot 4) - (1 \cdot 4)$ 2: $(\text{number of stones on one side} - 1) \cdot 4$ 3: $(\text{number of stones on one side} \cdot \text{number of stones on one side}) - (\text{number of stones on one side} - 2)^2$</p>	<p>Letting them reflect the formulae for area calculation, and I present the core-task</p>	<p>Generalize the specific phrase</p>	

4: (number of stones on one side -2)- $4+4$ 5. Recognise the problematique T: Let students present their impression about making the formulae S: It is troublesome using same phrase «number of stones on one side» every time! Can't we make it shorter somehow? S: Can we use a letter instead? 6. Solving the kadai (core task) T: can we replace the phrase to a letter x ? S: 1: $(x-4) - (1 \cdot 4)$ 2: $(x-1) \cdot 4$ 3: $(x \cdot x) - (x-2)^2$ 4: $(x-2) \cdot 4 + 4$	Hopefully students realize that using phrases repeatedly is inconvenient. Let them present the value of using variables	Abstraction: replace a «phrase» to a «letter»
7. Summing up: about the use of letters T: If the value of something is not known, we use a variable. Usually, we use letters.	Sometimes we use Greek letters	

Table 1. – Lesson flow (Kunimune & Souma, 2009a, pp28-29).

The core task was to find a general expression (a formula) to calculate the total number of stones. Students' possible techniques were described as: $(n \cdot 4) - 1 \cdot 4$, $(n - 1) \cdot 4$, $(n \cdot n) - (n - 2)^2$ and $(n - 2) \cdot 4 + 4$ - corresponding to four different ways to model the core task. The teacher does not ask the class to verify that these 4 expressions are equivalent. This work was not included in the mathematical task of this lesson. In table 2, I listed the didactic tasks and techniques proposed by the lesson plan:

Didactic task	Didactic technique
a). Let the students think about why one has formulae and notice the benefits of using formulae for the area calculation	1. Let students present what formulae they have previously learned for the area calculation 2. Letting them explain the advantage of using these formulae
b). Making students curious whether one can find the total number of the stones, without counting	3. Giving a comparatively easy initial task (5 stones), which every students are able to solve (e.g. they can count) 4. Showing a figure
c). Giving the students a clue to help them start reasoning the solution methods for the initial task	5. Showing a figure of the stones (firstly, to understand the task, and secondly, to help the students to reason the solution methods in various ways)

d). Letting the students test and try to find a solution method (of the task with 20 on one side) in various ways	6. Ask all students draw their own figures. 7. Giving them a few minutes to think individually and see which students have gotten which kind of idea (<i>kikan-shido</i>)
e). Letting the class understand the thinking process of making the various solutions on the second task	8. Predicting the possible solution methods beforehand to prepare giving the students a quick response and comments regarding their methods (belong to the paradigmatic praxeology) 9. Showing the students' various solutions (with the figures) on the blackboard, and compare the thinking process of every method (<i>neriage</i> : routine technology)
f). Letting the students be conscious about the core task	10. Asking the class if they can make a formulae, which is able to calculate the total number of the stones in any number on one side
g). Letting the students find out the general formulae using a variable for calculating the total number of the stones	11. Beginning from the specific numerical value: firstly 5 and then 20. Letting them train to find out firstly the common patten between them 12. Let them notice the inconvenience of using the phrase «number of stones on one side» repeatedly during the discussion of finding out the general formulae.
h). Letting the students reflect on the knowledge learned about making a general formulae for the calculation of something, which has a regular pattern	13. Stress value of using variables by giving some other examples 14. Looking at textbook explanations about using a variable

Table 2. – Teacher's didactic task and techniques observed in the lesson plan.

5.4 Reference model for Souma's didactical techniques

I finally present the reference epistemological model of didactic techniques, which are connected to certain type of tasks. Appendix 1 explains how the different techniques are identified from the literature.

Type of didactic tasks	Didactic technique
a) Make students join the lesson willingly, and	1. Giving an initial task for which every student is able to guess the answer by using old knowledge, or by simple guessing

Transferring Japanese problem solving to Swedish classroom 21

making them curious about the target knowledge	2. Write the initial task on the blackboard
b) Letting the students work with the initial task autonomously	3. Using four different question types of initial tasks appropriately depend on the target knowledge and the levels of the students 4. Let students guess in some way (according to the nature of task), and not reply immediately if guesses are correct or not 5. Writing the students' different guesses on the blackboard 6. Comparing students' different guesses recorded on the blackboard and let them consider which of the guesses may be correct
c) Letting the students control if their guesses are correct or not	7. Kikan-shido: routine technology within the structured problem solving. Letting the students work individually for a few minutes and monitor their work
d) Letting the student formulate, or become conscious about kadai (the core task) by referring their guesses	8. Giving students a question, which helps them formulating the core task, such as: «I wonder if it will be the same by any number. What do you think? If so, how can we prove it?»
e) Letting the students think about the mathematical techniques autonomously	9. Never erasing what the teacher and students wrote on the blackboard for the students can look at it and reflect the solution process they have discussed on the initial task. Also, techniques 6, 7 and 8
f) Letting the students express their thinking process and keeping lively discussions	10. Predicting the possible mathematical techniques in advance to prepare students various solutions for giving them a quick response and comments regarding their techniques 11. Under Kikan-shido moment, monitor the students' solutions and plan the Neriage moment, e.g. the order of the presentation 12. Neriage (whole-class discussions): routine technology within the structured problem solving. Letting the students present their mathematical techniques on the blackboard and discuss their viabilities (or letting them raise the hands and present their solutions in any order)
g). Institutionalising the learned knowledge	13. Letting the students reflect the discussion. Compare different techniques, and discuss the different aspects of the solutions 14. Summing up the lesson (Matome: routine technology within the structured problem solving) by:

h) Train the students taking notes i) Give students a record of the lesson, for later use	a) Looking back the issues written on the blackboard, together b) Checking together the explanations in the textbook about the topic learned today 15. Letting the students copy the blackboard in their notebook
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Table 3. – the reference epistemological model of the didactic techniques of Souma’s version of the structured problem solving approach.

5.5 Praxeological analysis of the lessons implemented in Sweden

In this section, I will present a praxeological analysis of the Swedish lessons using the reference model (Table 3), in two steps: first, an analysis of one particular lesson in grade 7, where the teacher used the lesson plan by Souma discussed in Section 5.3. Then, by providing a summary of the teacher’s use of the didactic techniques applied in her 16 lessons.

Observation in the grade 7 class

Eva began the lesson by showing the initial task about squares made by stones. Two students presented their solutions and Eva wrote them on the whiteboard:

Robert’s idea: $2 \cdot 5 + 2 \cdot 3 = 16$
 Ronja’s idea: $4 \cdot 4 = 16$

Eva lets Robert explain what he meant of the expression $2 \cdot 5 + 2 \cdot 3$:

Robert: I do not know...why I said so. Two times 5 like $2+2+2+2+2$, and further three times 2? Ah...I probably meant $2 \cdot 8$.

T (Eva): Two times eight? (To the class) Do you understand how he thinks? However, the question is that if it will be easier to reason by using $2 \cdot 8$. I can write it anyway (writes $2 \cdot 8$ on the whiteboard). Ronja, can you explain yours? (lets Ronja mark the figure)

Ronja: I thought like this way (drawing as in Fig. 3).

T: Is there another idea? Mary?

Mary: $15+1$.

T: Ok, we write it (writes $15 + 1$ on the board)

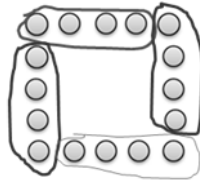


Figure 3. – Ronja’s explanation for the expression $4 \cdot 4 = 16$

Eva used the didactic technique suggested by the Japanese original lesson plan: giving an initial task for every student is able to guess the answer, and can lead to various mathematical techniques. The figure on the whiteboard gave the student an opportunity to find different solutions. Another technique she used was, presenting students’ different guesses on the whiteboard including methods that were not correct, such as $8 \cdot 2$ and $15 + 1$. As will appear below, her intention was to let the students realize that these expressions would not work in the case of different numbers than 16.

Eva now therefore let the students to apply these expressions to bigger digits than 5 and find a certain pattern for the formula. She asks the class if they can express the total number of the stones with one side constructed by 100 stones.

- Ronja: 4.
T: What are you saying?
Ronja: 4 times 99. (she applied her previous expression $4 \cdot 4$)
T: You think it will be 4 times 99. (To the class) What do you think? Which one of these expressions can be used to calculate 100 stones on one side? How about this? (Pointing $2 \cdot 5 + 2 \cdot 3 = 16$) What does this 5 mean?
Ronja: 5 is the number of the stones on one side.
T: (writes «The number of the stones» besides 5) OK, and how about this 3? Anyone?
Samir: 6 divided by 2.
T: But look this (the figure). 5 is these (marking the 5 stones on the both sides), and the rest?

Eva tries to give the student questions to make them conscious about the core task (finding out the formula). However, only Ronja appears to see a pattern clearly. Thus, Eva now shows by herself how the expression of $2 \cdot 5 + 2 \cdot 3$ should be interpreted in the figure. Ronja, points out the expression of $2 \cdot 8$ and $15 + 1$ would not work in the case of 100. Most students still do not find any pattern in the case of 100 stones on

one side. Consequently, Eva changes her strategy, and draws a figure of 3 stones on one side. She asks the class:

- T: How can we write according to this expression (points to Robert's idea $2 \cdot 5 + 2 \cdot 3$) if there are 3 stones on each side?
 Samir: 2 times 3 plus...2 times 3 minus 2, it should be.
 T: Ok, (writes $2 \cdot 3 + 2 \cdot 1$) And it is equal to?
 Samir: 8.

Her didactic technique giving the student a question here is not based on the didactic task letting the students think about the mathematical techniques autonomously, as it was shown the Japanese lesson plan, since the mathematical technique to find the pattern was given by herself. Eva continues asking the class about the cases of 4, 6, 7, 8, 9 and 10 stones on one side for making the students recognise the pattern of the expression. She records numeric chart of the expressions presented by the students, and asks if they can see the pattern in the expressions:

3 stones $2 \cdot 3 + 2 \cdot 1$
 4 stones $2 \cdot 4 + 2 \cdot 2$
 5 stones $2 \cdot 5 + 2 \cdot 3$
 (etc., all the way up to 10).

Several students answer that there are 2 stones between the front term and the rear term. Now Eva asks again the case of 100 stones on one side:

- Tina: 2 times 100, anyway to begin with.
 T: Ok. (writes $2 \cdot 100 +$) How can we express this part (points on the rear term)?
 Kejo: 2 times 70. No, 2 times 80!
 Someone: 2 times 98!
 T: 2 times is agreed. (writes $2 \cdot 100 + 2 \cdot$) What is happening here? (points on 9 and 7 of $2 \cdot 9 + 2 \cdot 7$) 3 and 1, 4 and 2, 5 and 3, 6 and 4, 7 and 5, 8 and 6. Aisha?
 Aisha: 2 in interval. [Someone talked at the same time and could not be heard in the class]
 T: What Aisha said, Kejo?
 Kejo: 2 in interval.
 T: Then in that case, 2 times what?
 Kejo: 80!
 T: Is 2 in interval between 80 and 100?
 Kejo: 98!

The students have found the correct answer. However, the answer was strongly directed by Eva's questioning. Then she asks about 1000 and one million stones on one side. Kejo, who now starts to see the pattern, responds « $2 \cdot 1,000,000 + 2 \cdot 999,998$ ». Eva now steers the direction of the lesson to come to the general solution using a variable n on one side.

- T: In that way, we can calculate whatever, any number. That is called a general solution in mathematics. A general solution is something, which we can apply to any of the cases. (writes n on the whiteboard) How can we express this one (pointing on 4 of the expression $2 \cdot 6 + 2 \cdot 4$) using n ? There are 2 in between to the n . How can we express it?
- Someone: n^2 ?
- T: n^2 ?
- Ronja: No, $n-2$.
- T: (writes n [a space] $(n-2)$) How can we write the whole expression? Compare to those expressions. n is for those (points the number of the stones on the chart) and $(n-2)$ for those (points the rear part of the terms). What is missing?
- Someone: Plus.
- T: Yes, (writes $n+$ [a space] $(n-2)$) and? What else?
- Someone: 2 times and 2 times.
- T: (writes $2 \cdot n + 2 \cdot (n-2)$) Is that correct? Do you [all] understand?

Eva then gives the class a training task to calculate the value of $2 \cdot n + 2(n-2)$ when $n=20$ and $n=40$ to let the students agree that the formula is applicable for any number of stones on one side. At last, she lets the class apply Ronja's initial expression $4 \cdot 4 = 16$, by letting them make the numeric chart again and find the case of n stones on one side. Helped by Eva's leading questions, the class eventually «finds» the formula $4(n-1)$. However there was no time left to compare this expression with the previous expression, $2n+2(n-2)$.

Eva's use of the didactic techniques

During the lesson, Eva used several didactic techniques proposed in the Japanese lesson plan and presented in Table 2. However, there are some techniques she did not use: a)1, a)2, g)12 and h)14. That these techniques were not realised, can be summarised by a change of overall goal of the lesson. In the original lesson plan, it is «letting the students realize the convenience of using formulae (in general)»; in this lesson the goal appear more to be «making the students find the formula (to calculate the sum of the stones) using variable».

Another essential technique she did not use was d)7. For instance, when Eva states the question of the 100 stones on one side, she does not

give them time to think individually, and there is of course then no opportunity to observe such a work. Consequently, the didactic technique e)9 becomes also almost impossible. Since she did not implement kikan-shido, she did not have any data for planning a *neriage*-moment. In addition, she had to develop the numeric chart in very teacher-controlled manner, because at that stage few students could see the pattern in what was essentially ideas put forward by two students (Robert and Ronja). This also means that she did not realise one of Souma's general didactic techniques, d)8 (cf. Table 3). First, she gave the students the task to express the total number of the stones with one side constructed by 100 stones, and then she *suggested herself* to use a variable n for the general case, while according to the lesson plan, both the general problem and the use of a letter should be suggested by the students.

Another technique from Table 3, namely e)9, was not realised in this lesson. What she wrote on the board was: the initial task, the figure, the four ideas for the solution of the initial task, Ronja's figure, a comment on Robert's idea, figures of the squares of the case of 3, 4, and 6 stones on one side, the numeric chart from 3 stones to 10 stones, the formula $2 \cdot n + 2 \cdot (n-2)$, the control task $n=20$ and $n=40$, the numeric chart to find the pattern for making the formula corresponding to Ronja's idea of $4 \cdot 4 = 16$, and the formula $4(n-1)$. No comments were given to explain the intention of these (mainly symbolic) expressions. She did not pay attention to writing different issues in specific placed of the board. She also had to erase the figures to get the necessary space for the final formulae using n , simply to gain the space to write them (see Figure 4).

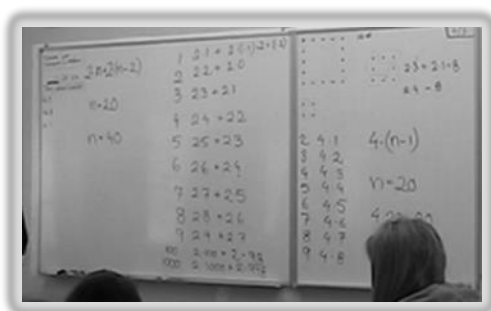


Figure 4. – The whiteboard at the end of the lesson

Eva's use of didactic techniques in 16 other lessons

Based on the reference model in Table 3, I have categorised Eva's use of the didactic techniques, observed in 16 other lessons. The lessons were implemented between January and March 2011 (including the lesson presented in the previous section), and at this point, she had tried to use Souma's approach for almost 6 months. Several initial tasks applied there (lessons 4, 6, 9, 10, 11, 13, 14, 15, 16) were adapted from lesson plans by Kunimune and Souma (2009a, 2009b) and from a task collection by Souma (2000); and other initial tasks were created by Eva herself (lessons 1, 2, 3, 5, 7, 8, 12). Below are the topic of the lessons:

Lessons with grade 7:

- | | |
|-------------------------------------|---|
| 1. Perimeter and diameter | 7. Multiplication with negative numbers |
| 2. Perimeter and area of circle | 8. Priority rules |
| 3. Negative numbers on number line | 9. Introduction to finding the general solution |
| 4. Absolute value | 10. Applying variables in algebraic expression |
| 5. Addition with negative numbers | 11. Addition/subtraction with variables |
| 6. Subtraction with negative number | 12. Application of algebraic expression |

Lesson with grade 8:

- | | |
|--|--|
| 13. Multiplication with negative numbers | 15. Introduction to finding the general solution |
| 14. Division with negative numbers | 16. Addition/subtraction with variables |

The didactic techniques used in the lessons

The didactic techniques 1, 2 and 3 from Table 3 concern the initial tasks. Eva worked diligently planning the flow of the lessons and constructing tasks, especially when she would not (or would not directly) use Souma's lesson plans. Regarding the technique 3 «using four different question types of initial tasks» (the types I-IV are described in Sec. 5.1), Eva used all four types in at least three lessons. The tasks applied in the lessons were mostly open-closed, where students should use their previous knowledge, and could result several solution methods (except lesson 6, 7 and 13, which was about the operations involving negative numbers). In the classroom, she started every lesson by stating the initial tasks on the whiteboard.

The techniques regarding whole-class discussion correspond to 4, 6, 10 and 12 in Table 3. In all lessons, Eva let the students guess the answers to initial task, and recorded how many of them guessed what at the whiteboard. The following discussions often started based on these different guesses: «now we have two different answers. How can we find out, which one is the correct? » Naturally, this technique is connected to the task design, where the task allows for several guesses. Regarding the technique 4, Eva tried not to confirm the students' answers immediately by replying, «Yes» or «It's correct». She pretended she did not know the answer by saying «Hmm», «Ok». However, when student looked uncertain because of a request for explanation, it also happened that Eva confirmed an answer, saying something like: «do not worry, your answer is correct. I am not asking you because your guess was wrong, but I want to know why you thought in that way».

Eva used technique 14b to sum up the learned knowledge for the students. For the preparation, she often made stencils explaining different mathematical laws and rules, which the students have worked with during the lesson, and let the students read it in the end of the lessons. This work had to be done since the textbook, used by the class, did not include such general mathematical explanations.

The didactic techniques not used in the lessons

We now come to the didactic techniques from Table 3 that Eva did not use, or attempted but did not manage to realize in her lessons. These techniques – essentially 7, 8, 9 and 11 in Table 3 – are related to *kadai* (students formulating the core question), to *Kikan-shido* (monitoring the students' work to plan the next *neriage* moment), and to *bansho* (organising whiteboard writing).

We already saw one example of how Eva, in the lesson on squares made by stones, does not even try to make the students formulate the core task. But she sometimes tried, for instance in lesson 16 (with grade 8) which was based on a lesson plan by Kunimune and Souma (2009a). There, the initial task was to compare the expressions $5m + 3m = 8m$, $9x + x = 10x$, $9a - a = 8a$, $5y - 8y = -3y$ and find out what is common to these four equations. The core task was to find a rule to rewrite $ax+bx$ when a and b are given numbers and x is some letter (thus, in essence, a version of the distributive law). At the beginning, Eva followed the Japanese lesson plan closely. However, when the class could not answer the initial question, she wrote an expression (which is her own, not from the Japanese lesson plan) $9a - 3x + 2b - 4x + 3a - 2$, on the whiteboard. She asked the class «Which of the terms are associated with $9a$? » to let the students notice that some terms include the same letters. Then the

class understand they should have said «the equations that have the same letters». On the other hand, the Japanese lesson plan prescribes that the teacher would try letting students find some explanations of why $5m$ and $3m$ can be added. «If one add, say, 5 meters and 3 meters (of something), it will be 8 meters», «If one substitute m for the constant 2, $10+6=16$, and 8 times 2 is also equal to 16», or « $5m$ is $m+m+m+m+m$ and $3m$ is $m+m+m$, thus $5m+3m=8m$ » (Kunimune & Souma, 2009a, p. 33). Of course, these are potential students' solution from the lesson plan. Nevertheless, it is a prominent idea in Souma's approach to try to let students formulate general mathematical rules or patterns by themselves, while this idea is not so common in the Swedish context.

Next, during *kikan-shido*, which Eva tried to use in all lessons, she often gave hints and advice to the students. Compared to what is proposed in Souma's original lesson plans, this phase often took longer time, sometimes several minutes. The most significant difference of Eva's use of *kikan-shido* and the use proposed in Souma's approach was that she hardly ever made use of this moment to plan the next moment, the *neriage*. Only in one lesson did she effectively realise this planning function. Most of the time, she let the students raise their hands and present their solutions in the order they volunteered.

Finally, concerning *bansho*, it was very difficult for Eva to realize the principle of «never erasing the writings on the blackboard», and by consequence also to «look back on the whole lesson by using the contents written on the blackboard». She always had to erase some part of the board during the lesson. According to Souma, the board should function, for students, as a source of inspiration and new perspectives for the solutions, creating an atmosphere of collective work and as a tool for the institutionalisation, by displaying the whole process of the students' mathematical work (Souma, 1997). The didactic technology of *bansho* is recognised and reported by several researchers (Fernandez, & Yoshida, 2004; Takahashi, Lewis & Perry, 2013). However, it is unknown to (and would probably appear obscure) to most Swedish teachers. By contrast, Eva always lets the students copy the contents written on the whiteboard. However, she never succeeded in making the board function as a well-structured record of the whole lesson.

DISCUSSION AND CONCLUSION

This paper, investigates two research questions, formulated in Section 2. We now summarise the answers for these questions and propose further perspectives for research.

6.1 The central didactic techniques and technologies of Souma's approach

The central didactic technologies of Souma's approach follows largely the basic routing technologies (hatsumon, kikan-shido, neriage, matome) of the Japanese structured problem solving, which are shared by most of mathematics teachers in Japan.

Lessons according to Souma's approach begin with the didactic technique «presenting students an initial question», for which students start by stating immediate answers or guesses. The initial questions can be formulated in four different ways, and the guesses produced by students will be discussed and validated collectively by the class. Presenting an initial question and letting students guess or make some kind of hypothesis, is one of the central didactic techniques of Souma's approach. Students' different guesses would raise their curiosity about which answer is the correct one, and supported by the teacher's questions, this situation leads students finding the kadai – the core task. Sometimes the initial questions do not directly ask for a mathematical technique related to the target knowledge. The initial task presented in section 5.1, «What can you say about these expressions $52-42=9$, $82-72=15$, $42-32=7?$ ») led students to different guesses, and these guesses are used to formulate the core task: «Prove that the difference of the squares of two consecutive integers is equal to the sum of the two integers». The core task carries the target knowledge, and should be solvable by several mathematical techniques. Realising such situations requires delicate didactic techniques of the teacher, careful construction of the core task and of the initial task (that would lead to the core task), and use of the questions that help the students formulate the core task, validate solutions, and so on.

The second category of the central techniques concerns the technology of whole-class discussion (neriage). Neriage is connected directly to kikan-shido – the teacher's monitoring of students' work. The teacher has beforehand studied the possible mathematical techniques of the task, and planned in which order these techniques would be presented on blackboard during the neriage moment. Thus, during the kikan-shido moment, the teacher monitors the students' work, and perceives which students use what techniques, in order to make a decision of how to have the students present their solutions. The whole-class discussion can be carried out in several different ways (Aa, Ab, etc., cf. section 5.1). In any case, the crucial didactic technique is to record students' various mathematical techniques on the blackboard, have them explain how they reasoned and found out those techniques, and orchestrate a discussion on their validity, mutual differences etc. At the end, the teacher lets the student reflect on what they have achieved.

This reflection activity is supported by looking at the blackboard together – and this requires the third essential group of didactic techniques, *bansho*.

According to Souma's approach, mastering *bansho* techniques is indispensable for full accomplishment of the *neriage*-moment. The teacher is supposed to conduct the discussion session through e.g. reflecting the transformation of the initial question to the core task and its various mathematical solving techniques – and the whole process should be clearly visible on the blackboard. According to Fernandez and Yoshida, *bansho* is an essential «instructional tool for organizing students' thoughts» (Fernandez & Yoshida, 2004, p. 235). The teacher plans the disposition of the blackboard meticulously, for not need to erase anything written on the board. Consequently, the blackboard functions as a record of the process of the whole lesson, and thereby it can be used as the tool of the institutionalising moment – *matome*. During the *matome* moment, the teacher let the class look back what they have done and confirm the knowledge learned. This routine technology justifies the techniques such as watching the blackboard together and reflecting the new mathematical techniques they have used, and look at the textbook to check the description of mathematical concept they have learned during the lesson.

We note from the above – where we use Japanese terms both to make that visible, and for lack of good translations into English – that there is a rather developed didactic technology in Japanese to describe didactic techniques. This technology is known and used by any Japanese mathematics teacher and is closely connected to the didactic principles or theory which was outlined in Section 3.

6.2 Importance of the notion of mathematical activities

The guidelines for the national curriculum associate the application of the notion of mathematical activities to the use of the structured problem solving. This fact has strongly influenced the central didactic techniques of Souma's approach (cf. section 5.2). Kunimune and Souma (2009a) consider that the implementation of the aims related to mathematical activities from the guidelines (enjoy their mathematical activities and appreciate the value of mathematics) should be realized through students' autonomous thinking activities. Souma's central didactic techniques are motivated by these objectives of the mathematics education.

The notion of mathematical activities has its origin at several different levels of didactic co-determinacy (Asami-Johansson, in review). The characteristics of the objectives connected to this notion (improving students' mathematical way of thinking, their ability of

communicating with others, their recognition of the value of the mathematics etc.) indicate origins from the higher levels of society, school, and pedagogy. At the same time, the notion impacts on the lower levels – such as theme and subject - since mathematical activities also function as methods to realize the educational aims in practice. For instance, the emphasis on the teacher's techniques, regarding helping students formulate the core task by their own, or planning the disposition of the blackboard for giving students inspiration for the learning, are clearly associated to the didactic tasks located at the higher levels. At the same time, these didactic techniques also are connected to didactic tasks directly related to a theme or subject levels. In that way, the central didactic techniques of Souma's approach are related to both the higher levels and lower levels of the co-determinacy. Of course, the notion of the mathematical activities is not the only source that gave massive influence to Souma's approach, but it helps illustrating how it is consistent with constraints and conditions at several levels of co-determinacy.

6.3 In what extent has the Japanese teaching practice reconstructed in the Swedish context?

In the result section, I have reported the didactic techniques that Eva applied and managed to use in her lessons. These techniques relate to *hatsumon*, *neriage* and *matome*. She constructed the initial tasks that allow the students having different guesses, and the core tasks that included various mathematical techniques. Most of the whole-class discussions indeed took the form of the whole-class discussions, directed by her. The students often actively participated in the discussions, as their different guesses made them interested in knowing the correct answers. Eva could pretend not knowing the correct answer, and kept up the students' curiosity by posing questions that encouraged them to explain their way of thinking. However, she never realised the process, whereby the students by themselves formulate the core task based on the initial tasks, which is a central technique in Souma's approach. Instead, she always had to tell the core task directly to the class. She often gave leading questions to help the students find the mathematical pattern or rules aimed at. Also, the whole class sharing and validation of solutions was hardly ever planned based on information collected during *kikan-shido*. She always monitored students' work, but most of the time, this led her to give the students hints and advise. Only in one case, where she used one of Souma's lesson plan with very clear predictions of students' potential mathematical techniques, and where these predictions matched well what she observed among the students' works, did she actually plan the

neriage moment through her kikan-shido. Thus, the proper implementation of these didactic techniques seem to depend in part on how much detail the teacher needs concerning students' possible mathematical techniques.

Regarding the matome moment, she was very keen to produce handouts explaining mathematical laws and concepts, and let the students read it together, in order to establish the knowledge learned. However, she never tried achieving this moment through building up the contents written on the board and then using it as summary for the class. Organising the board writing was something she never agreed to tackle seriously.

Considering the historical, cultural and didactic background that formed the teaching practice of Japanese structured problem solving described in this paper, I believe that these phenomena are not only a matter of the ability or preferences of individual teachers. In Sweden, the problem solving approach does not include the aspect of socialisation, which is so strongly emphasised in the Japanese guidelines for the national curriculum. The concept of problem solving is used to train students' individual competencies, where the focus is on reasoning, modelling, and mathematical literacy (Skoleverket, 2011). The didactic techniques that deal with these foci are located in the lower levels of the co-determinacy, such as providing mathematical tasks and its solving methods, and giving students questions that enable them to find mathematical patterns. Other foci for Swedish teachers are located at purely pedagogical levels, such as planning the work forms of students to train their communication skills, positive attitudes towards mathematics, and so on. However, those foci from two different levels never interact, as we saw in Souma's lesson plans. When Swedish teachers use the didactic techniques in the context of giving tasks and solving the tasks, they usually do not consider the social aspect of educational aims, located at the higher levels and common to all disciplines. On the other hand, regarding the use of the didactic techniques located in school or pedagogy levels (e.g. encourage students listening carefully other students' presentation of the solutions, etc.), the didactic tasks are often disconnected from the mathematical target knowledge. The exception in Eva's case is that she applied the initial questions (that always have some relation to the target knowledge) in order to raise her students' positive attitudes for participating in the problem solving activity.

6.4 Conclusion

The large part of the conditions and constraints that cause the difficulty of «transplantation» of the Japanese didactic practice into the

Swedish teaching context described in this paper can be explained by the essential differences of the didactic theories that justify the teachers' didactic techniques between the countries. We can see the most typical example of this discrepancy in the complexity of the transferring of the bansho technology.

Eva never planned the use of the board in detail, which usually forced her to erase her (and sometimes the students') writings on the board. The didactic theory that justifies the bansho technology differs largely from didactic theory that is commonly known and accepted by Swedish teachers. To begin with, no specific terminology for bansho exist in Swedish, and in addition, board writing is often perceived as somewhat trivial or even old-fashioned. In Eva's lessons, the use of the whiteboard mostly catered to needs coming from the subject level of didactic co-determinacy, e.g. the need to record the initial question and students' answers. That is, it is used just to display questions and answers but does not document the progress of students' reasoning, from the initial task to the core task, and onto the mathematical theorem that justifies the hypotheses they have generated, as the Japanese bansho technology requires. In short, the bansho techniques aim to visualise the process of the mathematical activities on the board (Imazaki, 2017). It caters to needs at both the lower and higher levels of the co-determinacy, and as a result, Japanese teachers' perception of the function of the board relates to both the subject and pedagogy levels simultaneously; recording the whole process of the lessons and helping students' collective learning in mathematics. Thus, Japanese teachers consider erasing the writings on the board almost as a serious crime.

In addition, the discrepancy between the paradidactic infrastructures present in the two countries plays a considerable role for Eva's understanding of the logos parondt of Souma's didactic organisation. Indeed, didactical technology is essential to teachers' collective work in settings such as lesson study (e.g. Fernandez, & Yoshida, 2004, Takahashi, Lewis & Perry, 2013), and there exists a considerable literature in Japanese regarding bansho techniques, written by active teachers (e.g. Tanaka, 2003; Kato, 2007). This literature and other infrastructures such as teachers' practice research (Miyakawa & Winsløw, 2019), Facebook groups on the study of the bansho techniques etc., constitute the components of the paradidactic infrastructure in which both practical and theoretical knowledge of bansho is shared among all teachers.

The present study indicates why and how the transfer of teaching practice to a different context may meet with unexpected obstacles. Teaching is an entrenched practice, since it is based on a cultural script (Stigler & Hiebert, 1999) of each country. In Japan, educational reforms

beginning in the early twentieth century, eventually led to the notion of mathematical activities. Together with the problem solving approach, the notion became a representation of the idea of students' creative and autonomous learning in mathematics education. The Japanese didactic theory that aims for students to formulate core tasks differs in many ways from similarly widespread didactic theory in the West. In the Western countries, showing the object of learning and formulating relevant questions belong to the domain of teachers. Students are supposed to *answer* the teachers' questions. This script is deeply seated also in Swedish teachers' perception of their teaching practice. The Japanese didactic techniques related to task design and whole-class discussions, are relatively easy for Eva to reconstruct. However, the fundamental difference of the teachers' view on the didactic task regarding students' activities (Japan: focus on the students' autonomous learning; Sweden: focus on the students' achievement of the target knowledge), prevented a full realization of other aspects of the structured problem solving approach.

The reference epistemological model of teachers' didactic techniques presented here, and the analysis based on the levels of co-determinacy, highlight the conditions and constraints for transplanting this particular didactic practice, and the same theoretical approach might be applied to study similar attempts. My analysis also indicated that the potential for success of such transfer is beyond what is determined by the individual teachers' abilities or preferences, or their students' backgrounds. Of course, those factors and the compatibility of curricula matter a lot for the realisation of such a transplantation. Still, at a deeper level, one has to consider also the compatibility of the teaching professions' shared technology and theory, which in turned is shaped by several factors at the level of society and culture, and in particular by the paradidactic infrastructure.

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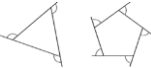
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Appendix 1: Supplementary documentation of the reference model (Table 3)

Type of techniques	The source described in Souma's text
a)1 & 2	«a situation, where students have different guesses on a task, evoke the curiosity with the students and it bring out students' positive attitudes for attending the lesson» (Souma, 1995, p 9) «if the teacher silently writes down the expressions $(52 - 42) = 9$, $(82 - 72) = 15$, $(42 - 32) = 7$ on the blackboard, and asks the class, «What can you say about these expressions?», students may come up with several initial guesses» (ibid., p. 103)
a)1 & 3 a)4, b)5 & 6	«The initial task, which will be given in the beginning of the lesson, is a question, which gives the students an opportunity to start thinking. If the teacher gives a task such as 'reason the solving methods to calculate...!', or 'prove it!' the students do not feel the aim and necessity thinking about the task. (...). Thus we suggest the following 4 types of initial tasks». (Kunimune & Souma, 2009b, p. 11) Also, see section 5.1. «Which of the sums of exterior-angles is the largest? The triangle's, or the pentagon's?» (See Figure 3)
	 <p>Figure 3: Illustration of the problem «Which of the sums of exterior-angles is the largest?» (Kunimune & Souma, 2009b, p54)</p> <p>The goal of the lessons:</p> <ul style="list-style-type: none"> • To understand that the sum of exterior-angles of a polygon is 360° • To demonstrate how to express the sums of exterior angles by using previous knowledge. <p>(ibid., p. 55)</p> <p>«The students should come up with three alternatives: that the sum of the triangle's exterior angles is largest, that the pentagon's is largest or they are equal. The teacher should let the students briefly present the reasoning for each alternative and ask them», «How can we decide which of the guesses is the correct one?» (ibid., p. 54).</p>
b)6 & d)8	(See the task example described in a)1 & 2 above) «The students start to state several initial guesses: 'the differences

c)7 f)10 & f)11	equal the sum of the integers', 'the differences equals the first integer times two minus one', 'the last integer times two plus one'. After the class has verified each of these statements, they start to wonder if those statements always hold and why. They now want to prove it. Then the students have found out the core task» (Souma, 1995, p 103)
f)10	See the lesson plan (process 2) in section 5.3
f)11 12	See section 5
g)13 & 14	«Students will have a goal of 'what they think for what purpose' through watching different things written on the blackboard; the initial task, the core task found by the students, and other classmates' various way of thinking» (Souma, 1997, p. 74). Also, see section 5.1 and the end of the lesson plan (process 6 & 7) in section 5.
h) & i)15	According to Souma (1997, pp. 74–75), watching the contents written on the blackboard and copying the writings in their notebooks should help students obtain: <ol style="list-style-type: none"> 1. A sense of necessity to solving the task 2. A sense that they are solving the problem together with the classmates 3. Understanding the process of the problem solving 4. A sense of necessity of thinking about the problem.