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Educational design in math and science: The collective aspect

Peer-reviewed papers from a doctoral course at the University of Copenhagen

Edited by Marianne Achiam and Carl Winsløw

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Introduction

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This book represents the products of a doctoral course held in Copenhagen, Denmark in November 2015. The course was entitled *Educational design in mathematics and science: The collective aspect* and involved the participants in a series of discussions and reflections about the collective and collaborative phenomena at the heart of mathematics and science didactics[1].

It seems immediately clear that interaction and collaboration is crucial to the outcome of most teaching situations such as lessons, laboratory exercises, or science centre visits. What is perhaps less evident, but increasingly focused on in recent research, is the role of collaboration in the work of teachers, the conception of exhibitions, and the conception and use of textbooks and other educational resources. For example, when a teacher plans a lesson, she draws on resources produced by a professional collective, which she may participate in at various levels (Winsløw, 2010), often though online platforms. Such collectives, understood as groups of actors who work together to achieve common didactic objectives, may be expanded by – and sometimes simply include – researchers in various fields. The influence of these researchers on the didactic phenomena is sometimes deliberate (e.g. in design experiments or action research) but should always be monitored. Larger systems, including institutions and companies, comprise groups that call for a broader didactic understanding with anthropological (Chevallard, 2002) and sociological (Douglas, 1986) perspectives.

A main objective of the course was to emphasise how the collective aspect of didactic phenomena can be often only be understood through studying their epistemic and systemic bases, often in a quite broad sense; this understanding must include a grasp of the tangible and intangible artefacts produced by the systems (professional knowledge, educational materials, resources, etc.).

Studying the collective aspect of education is not new in the didactics of mathematics; indeed, a paradigmatic example is the idea of 'didactic system' (Brousseau, 1997/2002) consisting of persons engaged in the study of mathematical knowledge: Teachers, students, and researchers. In more recent years, new technological developments have caused new didactic phenomena to appear, e.g. online teachers' networks, massive open online courses (MOOCs), and new ways of participating and communicating through digital media; such phenomena constitute variations of didactic systems and corresponding collectives and have been researched as such (Pepin et al., 2013). The collective aspect has also more recently been brought to bear in other contexts, i.e. research on the interplay and negotiations between exhibition developers (Lindauer, 2005; Macdonald, 2002; Roberts, 1997) or on the interactions between researchers and exhibition developers (Stuedahl & Smørdal, 2012).

During the course, we considered the theoretical and methodological issues of addressing collective aspects of didactic phenomena. These issues included the various conceptualizations of a 'collective', the positions that exist within the collective under investigation, the genesis of the collective, and the implications of these issues for didactical research. The course merged research perspectives on collective aspects of education from mathematics and science in both in-school and out-of-school contexts, based on the latest international research and identifying avenues for future investigations.

We initially had fourteen participants from six countries, representing a rich diversity of research fields. The course was arranged over five days of intensive work including lectures, group work, participant presentations, and intensive feedback sessions (see Appendix A). Prior to the course, participants were required to read a number of basic texts (see Appendix B) and on the basis of these readings, to formulate a five-page paper outlining their own research and how they proposed to use the frameworks given in the readings. During the course itself, the participants presented their ideas and actively participated in the discussions and reflections to further develop their proposal under the supervision of the course teachers and

[1] We use didactics in the Continental European meaning to describe phenomena related to the dissemination, teaching and learning of subject-specific content.

guest lecturers.

Three months after the last course day, participants were required to submit a revised and expanded ten-page version of their initial paper based on the course discussions and reflections and the feedback they received. A total of thirteen participants went on to submit this paper and complete the course. Furthermore, the participants had the option of submitting their papers to a peer review process; nine students participated in this peer review process. We are happy to present the resulting nine papers in the following sections of this booklet.

The course benefited from the presence of Professor Marianna Bosch from Ramon Llull University, Spain, and Professor Dagny Stuedahl from the Norwegian University of Life Sciences, Norway. These scholars have worked with educational design and its collective aspects in various ways in their research, and generously contributed their unique insights to the discussions in the course. To them, as to all fourteen course participants, we extend our thanks for having worked so constructively with us.

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How mathematics teacher educators may apply their pedagogical content knowledge to manage the knowledge transposition? A Japanese and Swedish case study

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This paper examines Japanese and Swedish teacher educators' manner of applying the pedagogical content knowledge (PCK). The praxeologies of two specific lessons from Japan and Sweden for prospective teachers are presented. The study shows which kind of institutional conditions and constraints might influence the teacher educators' way of treating the transposition from the subject matter knowledge (SMK) to the taught knowledge. Also, the observations of the lessons points to how the different traditions of teaching strategies in both countries impact the complexities of the praxeologies of their lessons.

Introduction

Teacher knowledge

Various components of the definitions of teacher knowledge are discussed within the research of teacher training (Liljedahl et al., 2009). Schlman (1986) states three areas of teacher knowledge can be seen as the keystones of the emerging work on professional knowledge for teaching: subject matter knowledge (SMK), general pedagogical knowledge and pedagogical content knowledge (PCK). SMK is "the amount and organization of knowledge per se in the mind of the teacher" whereas PCK consists of "the ways representing and formulating the subject that is comprehensible to others… [it] also includes an understanding of what makes the learning of specific topics easy or difficult…" (Shulman 1986, p. 9). PCK can be therefore considered as an essential bridge between the academic subject matter knowledge and the teaching of the subject matter (Brown and Borko, 1992).

In this paper, the notion of PCK is defined in general in line with Brown and Borko (1992) as follow: A crucial bridge to make the transposition from the SMK to the taught knowledge (Chevallard, 1992) clear. Further, I state that one of the most important aspects of the PCK of teacher educator is to illuminate the transposition from the SMK of prospective teachers to the taught knowledge of a classroom. That is, to make the prospective teachers aware how to realize the learning of some specific mathematical contents for their pupils.

The aim

This is a part of an ongoing comparative study project of teacher education in Japan, Finland and Sweden. The aim of the empirical study in this paper is to investigate which kind of institutional conditions and constraints might influence the use of the PCK of the teacher educators and the complexity of the praxeologies (Chevallard, 1999 in Bosch & Gascón, 2006) of their lessons. Among various components of the PCK, it is focused on analysing how the educators handle the process of the knowledge transposition from the SMK to the taught knowledge. To realize these aims, the teacher educators' manners to organize their teaching practices are examined from the perspective of the didactic transposition and the anthropological theory of the didactic (*Ibid.*).

As Winsløw (2012) points out, the teaching practices are constructed by various institutional factors. It requires "that we develop very precise and explicit models of what we want to study and compare" (*Ibid*., p. 291). In line with Winsløw, a methodological attempt is done in this study; to clarify Shulman's notion of the teacher knowledge (SMK and PCK) in terms of the teacher mathematical and didactical praxeologies of different institutions.

Theoretical frameworks and method

The didactic transposition and the anthropological theory of the didactic

Chevallard (1992) called the process of adapting the knowledge content for the purpose of being taught within a given institution, a "didactic transposition" – a transposition of the scholarly knowledge (of e.g. mathematicians) into the knowledge to be taught (e.g. curriculum), the taught knowledge (of e.g. a classroom) and the learned, available knowledge (of e.g. community of students); a transposition of praxeologies that are adapted to different levels within the teaching system. The conceptualisation proposed to study the mathematical knowledge in an institutional context is extended to any other human activity and give rise to the anthropological theory of the didactic (ATD). There, mathematics learning is modelled as the construction of praxeologies (*Ibid.*) within social institutions. A praxeology provides both methods for the solution of a domain of problems (praxis) and a structure (the logos) for the discourse regarding the methods and their relations to a broader setting. A praxeology that describes mathematical activities is also called a mathematical organisation (MO). A didactical organisation (DO) is a praxeology that describes the activities to support achieving the goal of learning or teaching MO, as for instance those used by teachers.

Chevallard classifies the mathematical praxeologies into increasing complexity as: specific, local, regional and regional praxeologies (Garcia, et al, 2006). The form a praxeology takes depends upon a structuring schema in several levels in a "hierarchy of levels of co-determination" (Bosch & Gascón, 2006): civilisation/society (e.g. political, or cultural orientation in education), school (e.g. curriculum), pedagogy (e.g. general teaching principles), discipline (e.g. mathematics, physics,…), domain (e.g. algebra, geometry,…), sector (e.g. equations, similarity,…), theme (e.g. triangles, root,…) and subject (e.g. one simple question). These levels generate the conditions and restrictions that influence the complexity and form of praxeologies.

Analytical methods

As Liljedahl et al. (2009) point out, what is unique with teacher education is *what* educators teach is also *how* educators teach and *what* the prospective teachers learn is also *how* they are learning (p. 29). In that meaning, I categorised the MO and DO of the lessons for prospective teachers in two parallel praxeologies; the mathematical organisation of teacher educators and prospective teachers as the MO_{te} (the MO of teacher education) and the mathematical organisation of, say, a class of grade five, as the MO_{sc} (the MO of school class). In the same way, the didactical organisation of the educator in relation to the prospective teachers, as the DO_{te} and the didactical organisation of a school teacher in relation to her/his school class, as the DO_{sc}.

In this study, the SMK and PCK are interpreted as praxeological equipment. Also, the SMK of prospective teachers is defined as a composite of the scholarly knowledge and the knowledge to be taught. To study the components of the MO_{te} and MO_{sc} would show the teacher educators' intention; what kind of the SMK prospective teachers should be equipped. Correspondingly, to study the components of the DO_{te} and DO_{sc} would show their manner of applying the PCK, i.e. the knowledge which may promote the prospective teachers' acquisition of the SMK and making the transposition from the SMK to the knowledge to be taught clear for them.

Content Representation (CoRe) as an interview template

Kind argues (2009) that it is difficult to investigate PCK in teaching practices. He stresses that using Content Representation (CoRe), developed by Bertram & Loughran (e.g. 2012), as a methodological tool offers a unique insight into teachers' PCK and practices relating to specific topics and areas.

In this study, CoRe template is applied as a tool to investigate how the teacher educators' perception of PCK might look like. Therefore, the treatment of CoRe template differs from Bertram & Loughran's original settings, where CoRe works as a tool for facilitating physics teachers' consciousness using their PCK at lesson planning. Here, the teacher educators were asked to answer to the questions of the CoRe template *after* the conducted lesson, not *before* they plan the lessons. The reason I chose to apply the CoRe as an interview format is that it is a convenient way to orientate how the teacher educators identify the process of the knowledge transposition from the SMK (as Big Idea A, Big Idea B and so on) to the taught knowledge.

Studying the teacher educators' answers to a question such as "What are your teaching procedures and particular reasons for using these to engage your teaching of these ideas?" (see Table 1) gives us a picture of how the educators interpret the transposition between the different phases of the knowledge and designed their lessons based on those interpretations. Also, answers to a question such as "What are the difficulties/limitations connected with this idea?" may tell us from which level of the codetermination, the praxeologies of their lessons are generated and formed.

Table 1 A CoRe Template

Year level for which this CoRe is designed:____________	Important Science ideas/concepts					
Content Area:____________	Big Idea A	Big Idea B	Big Idea C	Big Idea D	Big Idea E	Big Idea F
What do you intend the students to learn about this idea?						
Why is it important for students to know this?						
What else do you know about this idea (that you do not intend students to know yet)?						
What are the difficulties/limitations connected with teaching this idea?						
What is your knowledge about students' thinking that influences your teaching of these ideas?						
Are there any other factors that influence your teaching of these ideas?						
What are your teaching procedures (and particular reasons for using these to engage with this idea)?						
Specific ways of ascertaining students' understanding or confusion around this idea (include a likely range of responses).						

Table 1. CoRe Template (Bertram & Loughran, 2012, p. 1029).

Beside the interviews with CoRe temperate, classroom observations of the lessons "Quantity and Measurement" (Japan, with 55 prospective teachers) and "Area and Perimeter" (Sweden, with 20 prospective teachers) have conducted with video recordings. Also, an analysis on two countries' curricula and textbooks, concerning the concept of measurement was made.

Result and analysis

The treatment of the concept of measurements in the curricula and textbooks

In the Japanese *The Elementary School Teaching Guide* (MEXT, 2008) for the curriculum for grades one to six, the determination of length, area and volume is described in domain Arithmetic. There, the contents for the each grade are described in the detail. As guidelines for teaching methods, it is stressed to apply pupils' previously learned knowledge and their various way of solving problems.

The introduction to "Quantities and Measurements" usually consists of four phases in Japanese elementary schools (Miyakawa, 2010): 1. Direct comparison of two objects. 2. Indirect comparison of two objects with a third object, having the same kind of quantity. 3. Comparison of two objects with arbitrary object as a unit (e.g. a pencil). 4. Comparison using standard units (e.g. meters). This order is clearly followed by Japanese textbooks (see e.g. Seki & Hashimoto, 2008).

Unlike the Japanese curriculum, the Swedish curriculum places this content in the domain of Geometry. The content regarding quantities, units and measurement is shortly described in the curriculum for grades 1-3 and 4-6. The Commentary Material to the Swedish curriculum (Skolverket, 2011) for primary school only retells the curriculum and does not give any practical guidelines for teachers.

Comparing these two educational contexts, the Japanese curriculum makes the transition from arbitrary to standard units understandable in the lower grades. Moreover, locating the section "Quantities and Measurements" in the domain of arithmetic makes a natural connection between area calculations and the basic arithmetical operations. The broad description of the concept of quantities, units and measurements in the Swedish curriculum for grades 1-3 makes the progression of the concept unclear. The concept of measurements is randomly presented in several Swedish textbooks for grade one; Prima matematik 1A (Brorsson, 2013) introduce arbitrary units, direct comparison and standard unit (cm) simultaneously. The direct comparison is sometimes presented after the introducing of comparison with standard units. Also, these presentations in the textbooks for grades 1-3 are often placed in sections covering Arithmetic, although the Swedish curriculum for grades 1-3 states that this concept should be introduced in Geometry.

It indicates that the Swedish curriculum does not give sufficiently clear guidelines for the teaching of this content and therefore teachers and authors of textbooks interpret the intentions of the noosphere (Chevallard, 1992) in different ways. The transposition between the knowledge to be taught and the taught knowledge in the Swedish context is therefore not as predetermined as in the Japanese context.

The lesson "Quantities and Measurements" in Japan

The course Arithmetic Education for grades 1-6 teachers focuses the content of primary school mathematics and the teaching methods for those contents. The requirement for attending this course is "Math II"-level from upper secondary school, which covers integral, logarithm and trigonometric functions. The lecturer of this course Mr. Matsui explains the four phases in the process of pupils learning about quantities by referring the Teaching Guide and clarifies those different comparison methods for the class. Thereafter, he discusses how the above mentioned four phases are treated in digital textbooks for grades one to five.

In the textbook (Seki & Hashimoto, 2008), the determination of the area of rectangles is given as an initial task where some techniques are justified by algebraic reasoning. The theory, which justifies this technology, is both algebra and geometry. It may be stated that the praxeology of the section Quantities and Measurements in the textbook is at least local, since it features multiple methods for solutions and even regional, when it constructs technological discourse of arithmetic and algebra to describe the process of area determination. The textbook clearly follows the specified praxeology in the curriculum in this section.

The lesson moves now to experience the structured problem solving approach. The approach emphasises to create learner's active participation to mathematical activities by challenging problems and collective reflections (Stigler & Hiebert, 1999). Mr. Matsui lets the prospective teachers find out several different methods for the determination of the area of parallelograms aiming to teach pupils of grade five. Four chosen students draw pictures and explain their different solutions on the blackboard. Mr. Matsui points out that the different kinds of "shifts" used by three students; parallel translation and rotation. Mr. Matsui gives final problem to find out methods for determining the area of trapezoids, using same didactical approach. This time, he chooses seven students to demonstrate their different solutions. One student doubles the trapezoid so as to transform it into a big parallelogram. Mr. Matsui uses this solution and establishes the formula for the area of trapezoid; (a + b) h/2.

The didactical task of this lesson is to make the prospective teachers *explore* how pupils would reason about such problems concerning area determination. At the same time, the prospective teachers *experience*, how the taught knowledge in the section Quantities and Measurements may look like. Here, I describe the praxeology of the last demonstrated lesson above:

The mathematical organisation of the teacher educator and the prospective teachers (MO_{te}): Types of tasks (**T**): to derive a formula for the area of parallelograms/trapezoids. Techniques ($\boldsymbol{\tau}$): transformation of shapes, using formulas. Technology ($\boldsymbol{\theta}$): comparison, figures, translation, rotation, formulas. Theory (**Θ**): Euclidean geometry, figures and area.

The didactical organisation of the educator in relation to the prospective teachers (DO_{te}): **T**: determine how pupils in grade five would reason area determination of polygons during a lesson, by considering the pupils' previous knowledge. $\boldsymbol{\tau}$: make the student participate in an example lesson using the structured problem solving approach, and follow it up with whole-class discussions. $\boldsymbol{\theta}$: statement of previous knowledge, mathematical textbook and curriculum used as reference. **Θ**: structured problem solving.

The mathematical organisation of a school class (MO_{sc}): **T**: to calculate the area of a parallelogram/trapezoid and to derive a formula for these geometrical figures. **τ**: transformation of shapes, using formulas for rectangles. ***θ***: figures, parallel shift, rotation. **Θ**: Euclidean geometry, figures and area.

The didactical organisation of a school teacher in relation to her school class (DO_{sc}): **T**: making the pupils participate in the lessons and to reason the determination of area of parallelogram and trapezoids. **τ**: questioning, giving the task, whole-class discussion. ***θ***: statement of previous knowledge, mathematical textbook. **Θ**: Structured problem solving.

The complexity of the mathematical organisation becomes large, since the praxeology is generated by connecting several local praxeologies.

The lesson “Area and perimeters” in Sweden

The course Mathematics and Learning for Primary School, Grades 4-6 Teachers II, Geometry, treats the knowledge in mathematics and mathematical education in relation to the current Swedish curriculum. The requirement for attending this course is “Math B”-level from upper secondary school, which covers quadratic equations and similarity of triangles.

The lecturer Mrs. Nilsson begins the lesson by asking the prospective teachers to reflect on their own perception of area and perimeter. Then she gives five group-exercises concerning concepts of area and perimeter. The sixth exercise consists of determining the area of different geometrical figures by using Geo-board. Mrs. Nilsson demonstrates a method for area-determination of an isosceles triangle by using a rubber band around the triangle. She divides the rectangle into two squares which are in turn divided into two halves. The half of the area of the squares is subtracted from the each side. Now the prospective teachers ponder the method for area-determination of another isosceles triangle in groups. Mrs. Nilsson then demonstrates student A’s solution where an identical method is applied as the one she explained. (See Figure 1a &1b). Thus, 4 – 1 – 1 – ½ = 1½ area units.

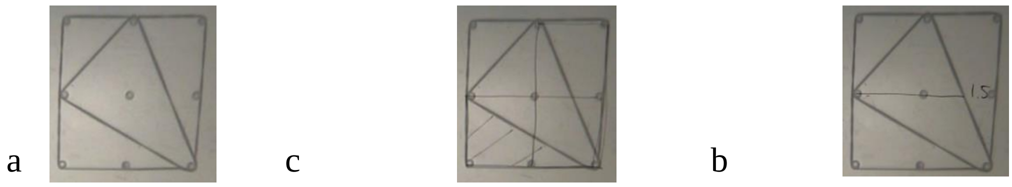

Figure 1a: an isosceles triangle. 1b: with an auxiliary line. 1c: Student B’s figure

Then student B asks if he can apply the formula of the area determination for a triangle; first, dividing the original triangle into two triangles with the base of 1.5 length units (see Figure 1c), and then adding the area of the two triangles. Thus, (1.5 · 1)/2 + (1.5 · 1)/2 = 0.75 + 0.75 = 1.5 area units. Some of the students express that they do not grasp directly how it works. Mrs. Nilsson comments “One can understand (this method) if one has more mathematical skills”.

The common didactical goal of the six exercises is to give the prospective teachers opportunities to explore some teaching methods for area and perimeter determination. Here, I sketch the praxeology of the last exercise, the determination of the area of an isosceles triangle:

MO_{te}: **T**: to determine the area of an isosceles triangle. **τ**: division of figures and subtraction of area. **θ:** additivity of area and similarity, formula for area determination of rectangles. **Θ**: Euclidean geometry, figures and area.

DO_{te}: **T**: to explore teaching methods for area and perimeter determination. **τ**: group discussions about using manipulatives (Geo-board) **θ:** rules and terminology regarding the use of manipulatives in lessons. **Θ**: rules and terminology regarding the use of manipulatives in lessons.

MO_{sc}: same as MO_{te}.

DO_{sc}: **T**: to give the pupils opportunities to explore some methods for area and perimeter determination. **τ**: questioning, giving the task, using manipulatives (Geo-board), group discussion (not stressed as much as DO_{te}). **θ:** absent. **Θ**: lessons with manipulatives.

The boundary between the MO_{te} and MO_{sc}, also DO_{te} and DO_{sc} is not clear in the last and all other exercises. Mrs. Nilsson's intention is to train students' computing skills and establish their own perceptions of area and perimeter. In the last exercise, Mrs. Nilsson let the student B explain his alternative method, however she did not validate it by, say, verifying that the base is 1.5 length units as stated. Her intention was not to discuss the viability of different mathematical organisations for the grade five class, but to establish a certain technique which is possible for all prospective teachers to manage.

Interview with CoRe template

Mr. Matsui mentioned that his intention for students learning about Big ideas – the concept of area of geometrical figures and formulas to generalize the calculation of the area – is that area of polygons can be determined in *various ways* by using pupil's *previously learned knowledge*. He stresses also that the prospective teachers should be able to use some mathematical terms; the terms describe the various ways for determination and help them in understanding the pattern of the different methods. Mr. Matsui aims that prospective teachers learn some teaching methods which let pupils find out the formulas for area determination of geometrical figures, rather than memorising the formulas. These answers indicate that the conditions which influence the scale of the praxeologies of his lessons are generated from disciplinary levels in the hierarchy of didactic co-determination; at least from *theme*-level, because *theme* unifies different but related techniques. Even a condition from *sector* level, which is defined by a *theory*, generates the scale of the praxeologies, since it unifies several local praxeologies which are already constructed and experienced by pupils and a teacher. Thus Mr. Matsui designs the MO_{te}/DO_{te} to illuminates the prospective teachers that the MO_{sc} as connected local MOs.

Mrs. Nilsson highlights the importance of prospective teachers' understanding mathematical concepts and their definitions. As examples of concepts that the students do not yet need to know within this area, she names the concept of alternative angles and vertical angles. However, these items are included in the curriculum of secondary school and all prospective teachers have already learned these concepts. These answers indicate that her focus of the lesson is to establish a specific praxeology of the MO_{sc}. Regarding the prospective teachers' difficulties and limitations in this content area, she mentions that some of them have learnt formulas for area determination by heart without a deeper understanding and sometimes incorrectly. Also their perception that "Geometry is a difficult subject" results in a blockage of their learning process. Furthermore, the students have not developed mathematical terms that allow them to explain their solutions. The lack of mathematical knowledge of the prospective teachers and their anxiety for learning and applying mathematics form Mrs. Nilsson's teaching strategies: to use manipulatives, working in small groups and finding one confident technique. It would be stated that this condition is generated from pedagogy level in the co-determination and not from the disciplinary level.

Discussion and conclusion

The detailed Japanese curriculum which gives a lot of specifications about the mathematical organisations to be taught enables the textbook authors to give uniform proposals for praxeologies to aim for in classrooms. Accordingly, the transposition between the SMK and the taught knowledge becomes explicit. One hypnotise is that these facts might facilitate the development of the teacher educator's PCK, since many materials are already averrable to apply. This in turn might promote the prospective teachers' PCK, that is, in this case, the ability to transpose the SMK to large and complex local or regional praxeologies of pupils/teachers. The Swedish curriculum does not give the same kind of impact to the textbook authors and thereafter, to teacher educators. Thus every teacher educator must interpret the transposition in his/her own way and construct the praxeology on her/his own.

The structured problem solving approach, which the Mr. Matsui encourages prospective teachers to apply, illuminates the double construction of the praxeology (MO_{te}/DO_{te} and MO_{sc}/DO_{sc}) clearly. Also, it gives them possibilities to develop their ability to use some professional terms in the context of the doubled praxeologies. In Sweden, lessons with manipulatives are institutionally established (see e.g. Rystedt & Trygg, 2010). The aim is to support pupils' conceptual understanding of mathematical operations by using hands-on materials. This approach helps the understanding of the concept of the area/perimeter and illuminates some different components of MO_{sc}/DO_{sc}. However, my conjecture is that it does not facilitate the prospective teachers to work with large and complex praxeologies. Here, the MOs are more isolated and disconnected. The observation and the interview show that Mrs. Nilsson's intension is not to let her students to learn how to construct an epistemologically well connected lesson sequence but provide them an established teaching method which they can feel confident of.

My attempt in this paper is to identify and analyse the conditions and constraints which generate the complexity of the praxeologies constructed by teacher educators using their PCK. Although, the scale of this case study is limited, I consider that some facts like the Japanese curriculum encourages applying pupils' previously learned knowledge and various solving methods point to that Mr. Matsui's lessons might not differ too much from other Japanese teacher educators' lessons. Likewise, the low prerequisite of mathematics credits to enter the Swedish teacher education may indicate that even other Swedish educators have similar situation regarding the lack of prospective teachers' mathematical knowledge and their low self-confidence learning/applying mathematics.

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A model for analysing teachers' work in lesson studies

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Based on Winsløw's (2012) model of Didactic Systems and Paradidactic Systems it is the aim of this paper to propose a framework for analysing teachers' collective work as unfolding in lesson studies. The original model doesn't take into account that lesson studies are often conducted in cycles. A second aim is to extend the model by adopting variations of elements of the Theory of Didactical Situations (TDS) in order to advance the analytical usage of the model.

Introduction

In my research project, teachers experiment with inquiry based mathematics teaching through the use of lesson study, outlined in Winsløw (2012) and referred to in Gueudet & Trouche (2012).

One of the research questions says: "Under which conditions can lesson studies contribute to the development of teachers' qualifications to design and conduct [inquiry based] lessons?".

In this paper I try to develop a model that can help me in analysing the lesson studies and hence to answer the above research question.

Lesson studies

In lesson studies teachers collaborate about designing, testing and analysing lesson plans deployed in didactical situations. This involves work outside of the didactical situation, i.e. preparation, observation, analysis and reflection. Hence to answer the above question I need tools to analyse the didactical situation itself and situations outside the didactical situation. To differentiate between these, Winsløw (2012) proposes the notions and models of *Didactic Systems* and *Paradidactic Systems* (Ibid., pp. 292-3).

My research project: Open-ended Approach and Lesson Studies (OEALS)

As suggested in the above research question, one aim of OEALS is to examine how working with lesson studies can help teachers develop their didactical knowledge and skills.

Through the school year 2015-2016 teams of 3-4 teachers of mathematics on three different public schools conduct three lesson studies. The teachers all hold a major in mathematics, have more than five years of experience and they all teach in grade three, four or five. The lesson studies focus on teaching Open-ended Approach lessons in these grades.

The structure of lesson studies conducted in OEALS is depicted in figure 7.

Open-ended Approach

Open-ended Approach is a Japanese model for teaching inquiry based mathematics. Open-ended approach is based on certain principles that are introduced to the teachers of the research project. One essential feature of Open-ended Approach lessons is that during the lesson the pupils must work autonomously. The notion of *autonomous work* in Open-ended Approach resembles key features of *adidactical situations* in the Theory of Didactical Situations, which will be describes later. Since Open-ended Approach itself is not a focus of this paper, it will not be elaborated further.

Theory of Didactical Situations

To analyse didactical situations within the didactic system, the Theory of Didactical Situations (TDS – e.g.

Brousseau 1997) is embraced. The analytical work consists of an a priori analysis of the knowledge at stake and the adidactical potential of the milieu proposed. Based on observations within the didactical situation and with reference to the a priori analysis an a posteriori analysis is conducted. The aim is to expound vital information about the pupils' acquisition of the knowledge at stake through their interplay with the didactical milieu under the constraints of temporal moments (i.e. didactical phases) and didactic contract.

In particular interest for this paper is the notion of *adidactical situation*, which is used to describe situations in which pupils work detached from the teachers direct didactical influence. In such situations pupils' actual knowledge and skills can be observed as they interplay with a mathematical problem and not the mutual expectancies of the teacher and the pupils them selves.

As is the case with pupils during adidactical situations, teachers' actual knowledge and skills come about and are observable when they produce a work of their profession, i.e. plan a lesson. Usually this process is not easily observed but in lesson studies teachers are compelled to explicitly utter their thoughts and through designing the lesson plan we, as researchers, are able to see, what they actually 'can do'.

Thus TDS provides a powerful tool of analysis when it comes to didactic situations but I have no theoretical framework to model and analyse at a paradidactical level. Winsløw's (2012) model of didactic and paradidactic systems doesn't constitute an analytical tool as such but it does offer a theoretical framework for analysing lesson studies. Based on the above considerations related to adidactical situations I propose to apply variations of elements of TDS to develop Winsløw's (2012) model into a tool to analyse paradidactical situations.

Paradidactical Systems

Besides offering substantial and advanced framework for analysing didactical situations, TDS can also be used to design (experimental) lessons and hence it offers to a certain extend tools to analyse pre-didactic situations, i.e. paradidactical situations in which the lesson plan is being developed and designed.

Pre-didactic situations (and hence systems) are by definition isolated from didactical situations (and systems) and they can – obviously – only be subject to a priori analyses. Didactical situations on the other hand can be subject to a posteriori analyses (related to an a priori analysis). This requires systems for observation of the didactical situation and of a post-didactical activity such as analysis, reflection and evaluation. Winsløw (2012) depicts the didactic system, the paradidactic system end their internal relationship like this: Such para-didactical activities are similar to what researcher do in action research. One of the most interesting aspects of lesson studies is that teachers themselves – often assisted by a facilitator – do such research on their own teaching, hence referred to as teacher led action research (Krainer 2006).

In 'normal' teaching emphasis is usually only on activities in the pre-didactic and the didactic systems. To a certain extend teachers also do 'observe' and analyse their own teaching but I use the notion of paradidactic system for paradidactical situations that are distinctly there, and are clearly divorced from the didactical situation.

Quasi paradidactical systems

As a remark to the above it's interesting to consider the situation where a given teacher is rather skilled in activities employed throughout the para-didactical system. From informal discussions with Japanese teachers and researcher, I've come to be aware that some Japanese teachers over time build up skills in observing and analysing their own teaching. In situation like this the paradidactical system is less clearly divorced from the didactical situation, yet serves the same purpose as that of the para-didactical system.

Paradidactical Situations

In this section, variations of the TDS model of didactical situations are adopted to the model of didactical and paradidactical systems. It's out of the scope of this paper to go deeper into this and to give examples. The models are simple and some need further improvement.

As mentioned above I apply variations of elements of TDS to analyse the paradidactical situations. Didactic situations and paradidactic situations are not directly comparable but there are some similarities. In TDS, the didactical situation is usually depicted like this: Paradidactical situations all relate to this.

Pre-didactic situations - PrS

In pre-didactic situations, teachers design a lesson plan for an intended teaching based on a thorough a priori analysis of the target knowledge and the pupils' anticipated responses (which is again based on knowledge about and analysis of the pupils' present knowledge, working methods, interest, individual and social properties and functions and so forth). During this process teachers gain new knowledge due to a priori analysis and their reflections. This new knowledge is used to validate the didactical ideas in question and their possible use in the lesson plan.

Figure 1: Didactical and para-didactical systems Winsløw (2012, p 293).

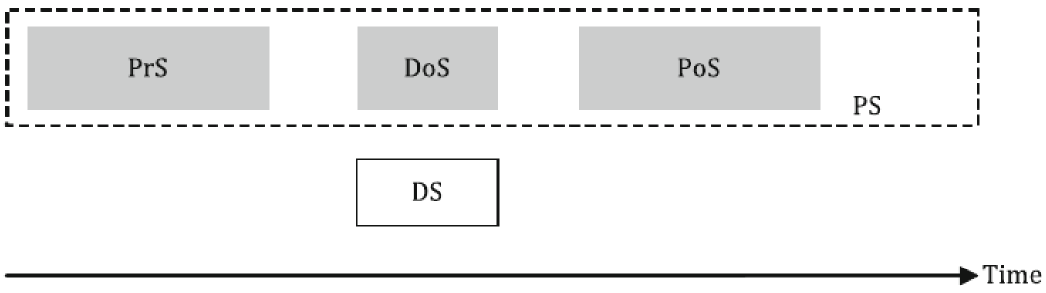


Fig. 15.1 Graphic representation of the main components of our theoretical framework, showing the distribution in time of the systems in which teachers (may) work: pre-didactic (PrS), observation (DoS), didactic (DS) and post-didactic (PoS)

Figure 1: A didactical situation: T, the teacher, interplays with the interplay between S, the student(s), and M, the milieu.

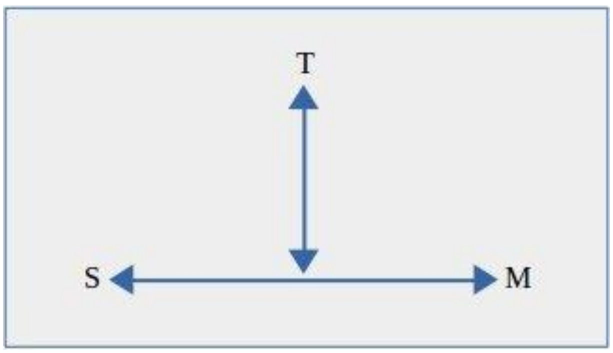
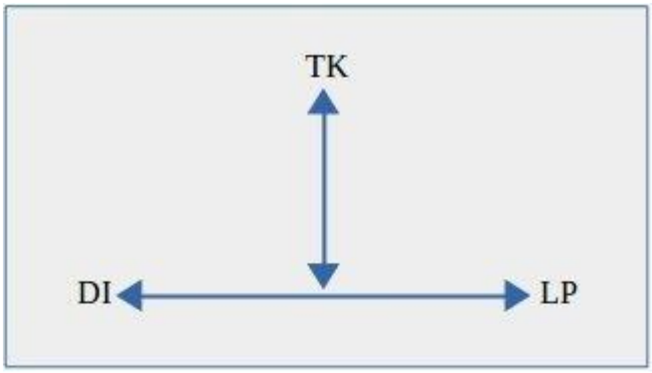


Figure 2: A predidactical situation: Teacher knowledge (TK) interplays with with the interplay between didactical ideas (DI) and the development of the lesson plan (LP)



Didactical observation Situations - DoS

During – but detached from - didactical situations a didactical observations are conducted (DO). These observations collect data from the interplay between the didactical situation (DS) it self and the lesson plan (LP). Note that there should be no vertical interplay here, as the observations are not supposed to influence on the didactical situation.

Post-didactical situations - PoS

In Post-didactic situations, the teachers reflect on what was observed in relation to what was planned and hypothesised. Hence there is an interplay between the teacher knowledge (TK) and the interplay between the analysis of data from the observation of the didactical situation (A_{DO}) and the lesson plan (LP)

Finalising a lesson study

In lesson studies the analysis of observation data from DoS is used to propose revisions to the lesson plan. This is true even if the lesson will not be taught again. Hence, in lesson studies the post didactic situation turns into a predidactic situation as suggestions for a revised lesson plan (LP_R) are proposed and reflected upon. Thus a full cycle of a lesson study can be depicted as in figure 6, in which the initial PrS and the final PrS are differentiated as PrS_I and PrS_F:

Figure 3: A didactic observation situations

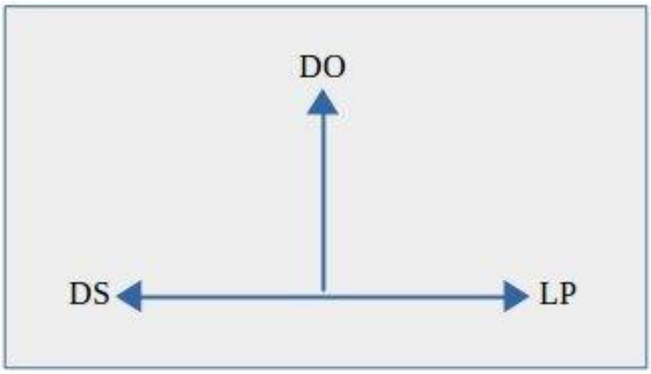
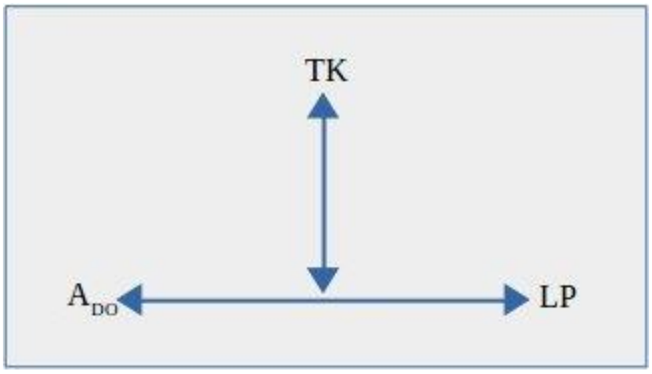


Figure 4: A postdidactic situation



Teacher knowledge

As the aim of lesson studies is to develop teachers' knowledge and skills, the outcome of lesson studies can be understood as the as the 'growth' of teacher knowledge (TK) from PrS_I to PrS_F.

More advanced models can be advocated for as the situations are more complex than described here. Nevertheless these illustrations may give a simple overview of situations within each system and the interrelation between these.

An Analytical Model for Cyclic Lesson Studies

Like other research, lessons studies consist of designing a research script, data collection and analysing the data collected. In lesson studies this often takes place in a cyclic manner where the results of the first data analysis modifies the script for further data collection and so forth. Hence the model of didactic and paradidactic systems is 'repeated' for each cycle (of paradidactic situations, including an appertaining didactical situation).

The lesson study used here to exemplify the use of the extended model of didactic and paradidactic systems is referred to as HTS1[2].

Like the rest of the lesson studies in my research project HTS1 was conducted using this pattern:

Modelling this structure with our extended model, one would have to repeat and merge the model three times:

2 The 1st lesson study at Hummeltofteskolen

Figure 5: Cyclic structure of HTS1

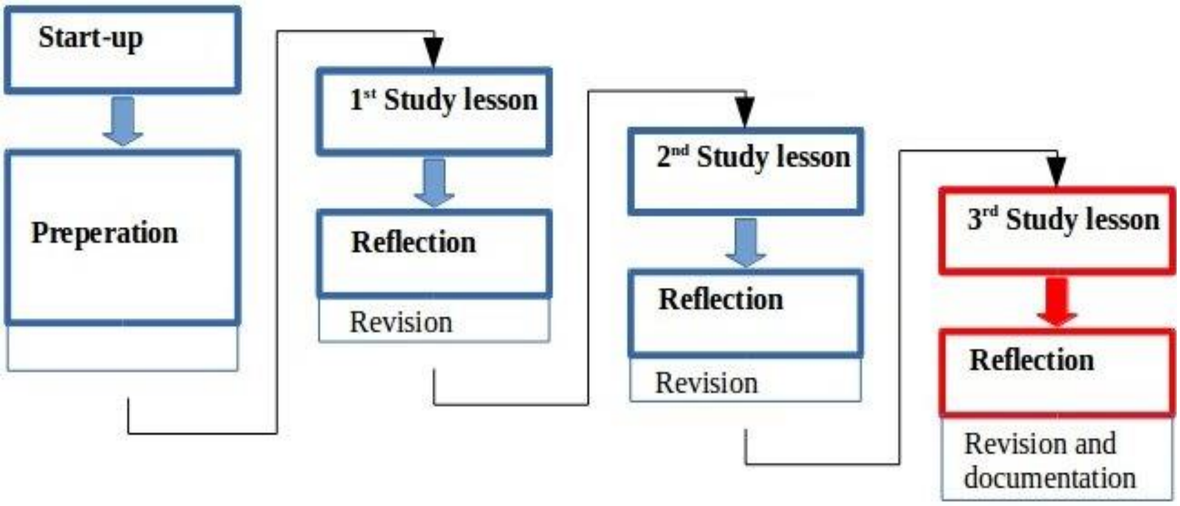


Figure 7: Winsløw's (2012) model of Didactic systems, paradidactic systems and their interrelations, extended with examples of situations, based on TDS.

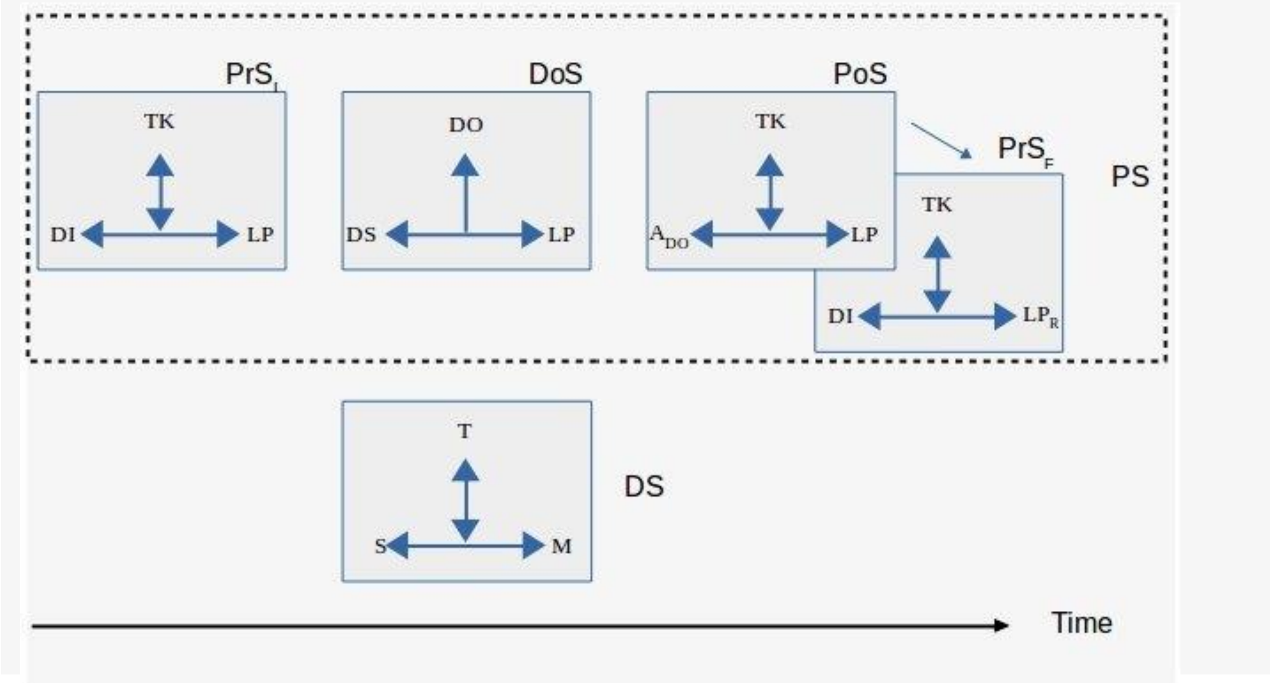
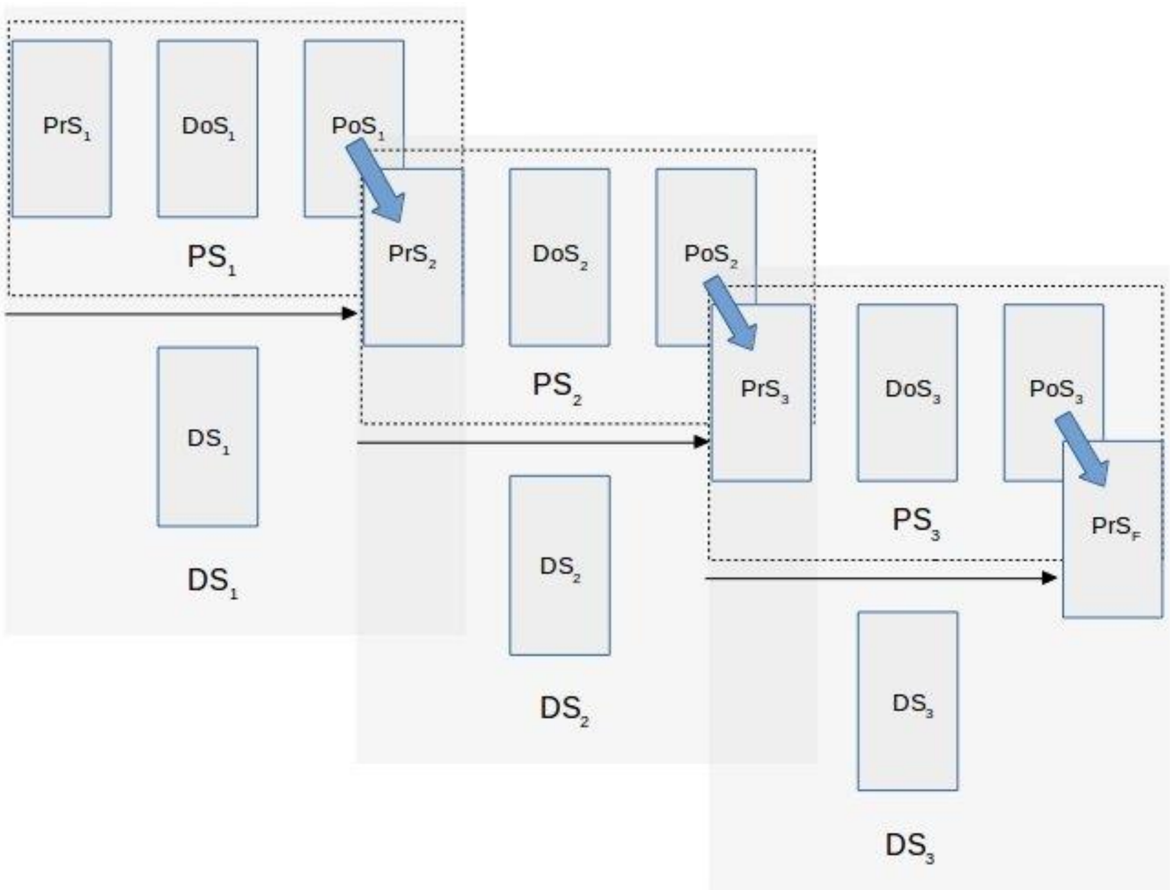


Figure 6: Didactical and paradidactical structures of HTS1.



Teacher Knowledge

For the overview a lowered number is added to each situation. The total span can be described as from PrS1 to PrSF (remembering that the final PrS is aiming at an imaginary didactic situation).

The model says nothing about the length of situations. Though it isn't specified in figure 7, PrS_1 is by far the lengthiest situation in this lesson study. That's a general feature of lesson studies even the actual time span may differ. In HTS1 PrS_1 lasts approximately 8 hours, where as all other situations last about one hour each.

The use of the model can be exemplified by the case of HTS1. A key finding of that lesson study is the significance of the use of – or restraint from using - verbal and written key expressions. During the three cycles of the study, the teachers realised that sometimes being explicit about the mathematical topic can be a hindrances for the pupils to achieve the target knowledge.

Using the Model

The target knowledge of HTS1 was the three strategies to subtract by counting. That is knowing the existence of them and the properties of each:

a) Count down from the minuend the subtrahend number of steps:

- starting at the minuend and counting down the number of steps equivalent to the subtrahend, the *number of the last step* being the difference.

b) Count up from the subtrahend to the minuend:

- starting at the subtrahend and counting the number of steps to the minuend, the *number of steps* being the difference.

c) Count down from the minuend to the subtrahend :

- starting at the minuend and counting the number of steps to the subtrahend, the *number of steps* being the difference.

As the lesson takes place in a 3rd grade class, numbers are implicitly understood as positive numbers including 0, i.e. N0 (though 0 isn't used or mentioned in the lesson).

The lesson is basically build up of three tasks, T1, T2 and T3:

T1: The teacher introduces six sentences, presenting formulations related difference, e.g. 'how much is 18 larger than 15?", "How far is there from 12 down to 2?"[3] etc. The pupils are to discuss the meanings and solve the problems.

T2: Pupils are to make maths stories that include one of the formulations from T1.

T3: The pupils are to investigate and explain how they 'find' the difference when using a number line.

Initially the teachers wanted to design a lesson in which pupils would realise that there are different strategies to find the difference between two numbers when counting, and that these strategies have different properties. It was an underlying wish for the pupils to understand that each strategy is more or less

3 A direct translation from Danish: "Hvor langt er der fra 12 ned til 1?". The exact formulation represents a more daily language and was believed to be important.

appropriate for different problems of subtraction.

PrS_1

During the a priori analysis of subtraction the teachers realised that there are the above mentioned three strategies. This was unknown to them before. During discussions about the pupils old knowledge and preferences, the teachers also came to understand that the pupils must discover for them selves which strategy is appropriate for them in solving which problem of subtraction. Originally the teachers wished to 'tell' the pupils in which case which strategy would be appropriate (from a/the teacher perspective). It was also during discussions of the a priori analysis that the teachers realised the difference between strategies (i.e. the above mentioned strategy a, b and c) and tools (i.e. number lines or number tables).

In interplay with this development of teacher knowledge didactical ideas and the lesson plan was developed in interplay with each other. That is, based on the teacher knowledge a didactical idea emerged and was tentatively put into the lesson plan. When designing and discussing the lesson plan the didactical ideas were reconsidered based on and/or adding to the teacher knowledge.

From this process the teachers developed mature didactical ideas that was hypothesised to lead the pupils to realise that by counting (using a number line) there are the three ways of finding the difference between two numbers. On the way to this the pupils were expected to consider what 'the difference' is and how it can be found. An implicit hope would was for the pupils to be aware of possible appropriatenesses of each strategy.

DoS_1

In DS_1 the focus on finding the difference by subtraction is explicated by the teaching teacher during task 1. In the course of DoS_1 the observers notice

1. In T1 almost all pupils don't consider other ways to find the difference than by applying a minuend minus subtrahend strategy (which resembles strategy a)
2. In T3 only strategy a) is used.

PoS_1

During PoS_1 the analysis of data (A_{DO}) from DoS_1 led the teachers to hypothesise that 1) and 2) were the results of orally explicating the focus on subtraction. The underlying reasoning was that once 'subtraction' is explicated pupils will expect that the teachers expect that the pupils use standard procedures for subtraction (due to the didactic contract).

PrS_2

Turning PoS_1 into PrS_2 the teachers decided to not mention subtraction during DS_2 in accordance with their new hypothesis. Hence the lesson plan was modified with the changes proposed during PoS_1 so that subtraction wasn't to be explicated by the teacher in DS_2.

DoS_2

In DoS_2 it was obvious that the change had to a certain extend had the intended effect. During T1, pupils were at the same time more open and more insecure of what was correct. Pupils uttered several ways of considering what difference is and how to find it. All three strategies were resembled. Still, when it came to solving problems of subtraction in T3, the absolute majority of pupils stuck to strategy a)

PoS_2

Though issue 1) of DoS_1 was seemingly solved issue 2) was still apparent. Through analyses and reflections in $PoS_{2,}$ it came about that observing teachers had noticed that 2) seemed to be related to the written expression of subtraction problems as for instance 7-5. Reflecting further on this, the teachers suggested that 1) and 2) were different instances of the same problem, i.e. the explications led the pupils to act in certain ways (again due to the didactic contract).

PrS_3

Transforming PoS_2 into PrS_3, LP was modified so as in DS_3 problems of subtractions would be written as finding the difference between e.g. 7 and 5 or 4 and 9 (the larger and smaller number changing position), omitting the subtraction sign. '-'.

DoS_3

In the course of DoS_3 the observing teachers noticed that pupils in T3 were more insecure about which were correct answers but worked more freely with finding the difference. It was also observed that all three possible ways of finding the difference by counting in T3 were used with a relatively good balance among them.

PoS_3

In PoS_3 the above observations were considered key findings which the teachers adopted for their further work.

PrS_F

Teachers agreed that for a future (imaginary) DS, they would suggest not to explicate the the focus on subtraction as it seemed to lead the pupils to think uniformly of strategy a).

Conclusion

The purpose of developing further Winsløw's (2012) model and to use it in the above analysis was to be able to follow the development of teacher knowledge during the course of a cyclic lesson study.

Though the model still needs refinement it helped to describe the flow of the lesson study in an accurate manner, allowing for clearly identifying when and how teachers knowledge developed under way.

In this example the teachers clearly developed new knowledge about

- the mathematics itself and about the didactics of mathematics.
    - e.g. the three strategies
- the didactics of mathematics
    - e.g. the significance of explicating the focus of the lesson

This conclusion could be reached without the model, but the model helps giving an overview of the progression whilst providing the author and the reader with a common notation of the different instances of the lesson study.

As mentioned the model need refinement for future work. For instance the double interplays of the paradidactical situations should be scrutinised for further development.

It seems clear to me that a refined version of the model may play a vital role in future analyses in my project. And hopefully of others' too.

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A lesson study approach to develop instrumental orchestration for student teachers

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With our research we are interested in developing a course for mathematics student teachers to develop, evolve, enhance and sustain their practises and justification of actions when teaching using a technology as a tool for the students, where both the pedagogical and mathematical potential of the technology is exploited. We use the notions of praxeology from the Antropological Theory of the Didactics and instrumental orchestration.

Introduction

With the presence of technology in the school institution, the pedagogy, the content of the disciplines taught and therefor the didactics are changed (Trouche, 2004). In our study we will consider computer algebra systems (CAS), a type of mathematical program that can handle symbolic expression, equations and inequalities. The strength of CAS is to automate algebraic manipulation tasks. Thus a CAS can be used as a tool by the students to skip otherwise time consuming routine work and instead leaves the possibility to focus on identification of patterns and structures, representation, generalisation and modelling but also more sophisticated algebra (Chick, Stacey, Vincent, & Vincent, 2001).

Though the use of CAS in mathematics educations sounds promising and we have proof of existence of "good" CAS based teaching (Drijvers, 2012). The transition of implementation of CAS into the daily classroom is a challenge. In some countries such as France (Guin & Trouche, 1998) or Australia (Ball, 2014) it is part of the national curriculum to include mathematical programs as a tool for the students to solve exercises in mathematics. However as noted by Guin and Trouche (Guin & Trouche, 1998):

the use of (graphic) calculators had […] become an explicit aim in these curricula and the Ministry of Education and Technology has supported many experiments to promote the integration of new technologies into teaching. Nevertheless, no more than 15% of the teachers include graphic calculators in their teaching, in spite the fact that all students have a graphic calculator in scientific classroom

Studying the teacher and the teachers' orchestration of technology both in the setup, the planed faces of the teaching, but also the spontaneous occurring situations research began around 2000 (Chick et al., 2001; Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010; Trouche, 2004). The research shows that teachers orchestrate technology in the classroom in various ways without a common perception of what constitutes "good" orchestration of technology (Doerr & Zangor, 2000; Lagrange & Erdogan, 2009; Monaghan, 2004; Sensevy, Schubauer-Leoni, Mercier, Ligozat, & Perrot, 2005).

Hence we stand with the challenge of how to integrate CAS into the daily teaching of mathematics such that the potential of CAS both pedagogically and mathematically is exploited?

In this article we will describe a course for mathematics student teachers inspired by Japanese lesson study, a program for teachers' and student teachers' professional development, to develop, evolve and sustain the student teachers orchestration of technology as a tool for the students in the classroom with the terms of praxeology from the Anthropological Theory of the Didactics (ATD). The Anthropological Theory of the Didactics is assumes to be known by the reader.

Background

In Japan, lesson study is a format integrated both in the teacher education (Elipane, 2012) and the profession of the teachers (Isoda, 2007; Isozaki & Isozaki, 2011). Lesson study has for many years been an object of interest also for many western researchers in the didactics, from the US to Norway, and has been given its own thematic working group at the 13TH International Congress of Mathematical Education to be held summer 2016.

Lesson study exists in many variations, but consists primarily of three stages: a planning face, a research lesson and a reflection meeting. The planning face is also referred to as a pre-didactical system (PrS) (Winsløw, 2011), where a group of teachers or a group of student teachers prepare a didactical system. The product of the PrS is a lesson plan stating the learning goals of the students, the teacher activities, predicted behavior of different students and the teacher's reactions to the students' behavior, classroom set-up, research objective, and observable items during the research lesson to support the research objective. The research lesson is also referred to as the didactical observation system (DoS) (Winsløw, 2011). The DoS is the performance of the lesson plan carried out by one of the teachers participating in the pre-didactical system. During the research lesson the other members of the PrS observes items in the didactical system in accordance with the lesson plan, invited teachers or student teachers can also observe during the DoS. The reflection meeting is also referred to as the post-didactical system (PoS) (Winsløw, 2011). The PoS is a meeting attended by the group of teachers or students teachers from the PrS, invited teachers or student teachers and a knowledgeable other. The object of the PoS is to evaluate and reflect on the DoS, the research objective and if the DoS was to be carried out again what would be changed and why. A lesson study cycle is defined as consisting of the coherent PrS, DoS and PoS.

Based on the ATD Miyakawa and Winsløw present a theoretical approach for analyzing mathematics teacher praxeologies engaged at a lesson study cycle (Miyakawa & Winsløw, 2013). The study describes largely the type of knowledge of the mathematic teachers detectable in the lesson plan, the DoS and the PoS. Using the same theoretical approach focusing on the PoS Rasmussen presents a study of the lesson study format adapted to a mathematic teacher education program in Denmark (Rasmussen, 2015). The study focuses on the level and type of praxeology of the mathematic student teachers present at the PoS and shows that even though the parties involved in the lesson study were new to the format, it was still a rich environment for the mathematic student teachers to engage, evolve and develop praxeologies related to the teaching of mathematics.

Theory

The notion of technology from praxeology will be named praxeological technology to avoid confusion with the word technology implicating information technology. We will now introduce the reader to the notions of *instrumental orchestration* (Drijvers et al., 2010; Trouche, 2004). Instrumental orchestration sees the teacher as the conductor in the classroom of the artefacts involved and the students, with the purpose of guiding students' instrumental genesis. The arrangement by the teacher of the objects involved, such as the task for the students or the technology involved, is denoted the *didactical configuration* (Trouche, 2004). In relation to the cycle of lesson study the didactical configuration could be explicitly stated in the product of the PrS, the lesson plan. *The exploitation mode* of the didactical configuration is the way that the teacher introduces the task(s), the performance of the artefacts, the approaches used to solve the task chosen by the teacher, etc. (Trouche, 2004). Converted to the notions of praxeology the elements of the exploitation mode will be the parts of the pedagogical techniques, the didactical techniques, the mathematical techniques, the technological techniques but also the instrumental techniques. These elements of the instrumental orchestration will be explicitly stated in the lesson plan of the PrS. *The didactical performance* (Drijvers et al., 2010) is the teacher's impromptu decisions on how to play the elements of the didactical configuration during the exploitation mode such as bringing a students work forward in the class by projecting on to the smart board. Expressed in the notion of praxeology this will be the part of the didactical and pedagogical techniques that the teacher cannot be certain will occur during the exploitation mode of the didactical configuration and is dependent of the students work. The didactical performance of the exploitation mode will optimistically be part of the lesson plan of the PrS, but will be reliant on the student teachers and their ability to predict the students' behaviour.

Several examples of different types of instrumental orchestration have been identified (Drijvers et al., 2010). The categorization focuses on the teacher, the students whether individual or as a collective group, the technology and the type of knowledge whether mathematical or technological at stake. We use the term *technical-demo* defined to concern the demonstration by the teacher of a technological technique to the whole class. We have added a variation *technical-demo-student* defined to be the demonstration of a technological technique by a student to the whole class. *Sherpa-at-work* concerns a student, the Sherpa, presenting her work with either a mathematical or technological point for the whole class.

The lesson study inspired course for mathematic student teachers (LS-TE)

In this section we will give a brief description of the lesson study inspired course for mathematics students teachers to be held autumn 2016 at the Institute of Education at the University of the Faroe Islands. The purpose of the course, related to our research, is to develop, evolve and sustain instrumental orchestration for the student teachers, where the technology is the CAS function of the program GeoGebra.

The course is the first in a series of three courses, over one school year of fulltime studying, constituting the part of the teacher education for mathematic teacher students. The course is titled "Numbers, Arithmetic and Algebra" and last for 12 weeks of full time studying. The course has in comparison with previous year been extended three weeks to accommodate focus technology as a tool for the students and the lesson study aspect. Approximately 25 teacher students will attend the course; the student teachers will be divided into study groups of 3 to 4 teacher students. As part of the course each study group will be assigned a class in lower secondary school, grade 7 to 10, to be involved in the lesson study program.

The first three weeks the students will amongst other things be introduced to the format of lesson study, its uses in Japan and potential for developing praxeologies. The student teachers will also apply the notions of praxeology as a tool for analysing potential mathematical praxeologies in an exercise or didactical praxeologies in the classroom. In relation to the notions of praxeologies instrumental orchestration will be introduced as way of considering and grouping different levels of different praxeologies. and the theory of didactical situations to analyse and use as tools for designing exercises and lessons. Furthermore the teacher students will be introduced to the research project and in particular the research questions. The student teachers will be given time to digest the research project and format of lesson study, with the possibility of asking questions ,suggestions and adding of variables or elements in the project.

The first study cycle takes place three weeks into the course. In the first lesson study cycle each study group will be given a sketch for a lesson plan. The student teachers will then have to personalise the lesson plan before the research lesson and formulate the learning goal of the lesson. The lesson plan is meant to give the teacher students an example of CAS based teaching with a rich milieu and varying types of orchestration (Drijvers, 2011; Drijvers et al., 2010). For each DoS another study group is invited; they will also participate in the reflection meeting.

The following table is a draft of the part of the lesson plan describing the actions of the classroom. The setup in the classroom is not described since the classes to participate in the project have not yet been visited. It is expected that the student teachers, during the PrS, make minor changes, add more students and teacher responses, describe the set-up in the classroom and as part of the course writes about the mathematics involved.

Time	Teacher	Students
10 minutes	Introduction: Get the students to turn on electronic device and open the program GeoGebra and the CAS window. (Technical-demo) Presents todays exercise sheets and program.	
10 minutes 1.1)	Ask if any of the students have an idea on how to use GeoGebra to answer the first part of the first task.	If no respond, the teacher should ask if anybody could type in the first equation in GeoGebra? Or what do we need to know in order to type in the first equation? (Discuss-the-screen) If receiving respond, the teacher should ask the student to explain their idea. Then the teacher should ask if the other students understood. Find a student if possible that didn't understand the idea and ask a third student who understands the procedure to guide the second student through the first part of the first task. (Technical –demo student) The work of the second student should be projected to the smart board such that the whole class can follow the proceedings.
20 minutes 1.2) – 4)	The teacher observes the students' progress and if necessary ask them questions to help them overcome problems.	The students work individually, if they have troubles with syntax they can ask their math partner for help. If a general syntax problem occurs the teacher will make a general inquirer to the students on how to solve said syntax problem. Presentation of problem and solution (Technical-demo). If a shot-cut is found by one of the students, it is to be shared with the class. (Sherpa-at-work)
15 minutes 5)-6)	The teacher observes and take notes on students progress and ideas to prepare the order of the presentations	If students get stuck on task 5) the teacher can give the hint to look at 2) again. Or if they think adding an equation with solution different from x=2 will give a new equation with solution x=2, why or why not?

15 minutes	The teacher asks the students to present their ideas. (down-up). With the more advanced ideas the teacher will ask if other students can explain the idea as well and have them do it as well. The teacher will furthermore draw connections between the ideas of the students.	Some of the students will present their ideas. Other students will repeat the ideas others will ask for clarification.
5 minutes	The teacher will repeat today's hypothesis and proof.	

TABLE 1 FIRST LESSON PLAN

In the following three lesson study cycles the teacher students will not be given lesson plans, but only the exercise for which the students must work on. This is done to ensure that the milieu and media are rich but not too rich and suited for the level of the students participating. In the lesson plans the teacher students will have to, not just describe their intended lesson by predicting the students work and how to respond to the students work as a teacher, but also to explain and justify their instrumental orchestration. If deemed necessary the student teachers will be asked to add a section on the lesson plans on the mathematical content involved and the technological techniques.

The PoS, of the lesson study cycles, will focus on the different types of instrumental orchestration conducted during the DoS in relation to the mathematical stake. The focus of and the questions to be answered during the PoS will be given before hand by such that during the DoS the members of the study group can observe particular behaviour of interest. In particular the different types of instrumental orchestration will be of interest related to the mathematical and pedagogical praxeology.

Collection and analysis of Data

As data for our research we will be collecting the lesson plans and be recording the PoS.

We will use the lesson plans to study the techniques and the praxeological technology of the praxeologies involved. We will use the PoS to examine the praxeological technology and the theory of the praxeologies involved. We will also be considering the evolvement of the student teachers' instrumental orchestration and type of orchestration over time and follow the progression of each of the study groups.

The didactical performance of the instrumental orchestration we can access through the lesson plans, that describes the set-up of the objects involved in the classroom. The task, for the students to work on during the research lessons, will be given and thus the student teachers have only a change to do minor changes. Though the students are free to use any technology, they will only in class have time to get familiar with one or two specific CAS.

The exploitation mode, we hope, will change over time and that the teacher students will be able to compare and evaluate the difference in the performance of the techniques. We hope to access this development in the student teachers' knowledge in the reflection meeting following the research lesson, and the progression of the reflection meetings. Using the notions of praxeology to look at instrumental orchestration will enable us to divide the didactical configuration, the exploitation mode and the didactical performance into more refined elements and thus more easily study the changes and in what type of praxeology the changes appear, and ask the question why this praxeology and not other praxeologies?

We hope to evaluate the format of lesson study with respect of implementation of technology in the classroom, teacher students' development of instrumental praxeologies, the mathematical praxeology and the didactical praxeology.

Discussion and conclusion

The format of lesson study seems as a great tool for researchers to access and study praxeologies of the parties participating (Miyakawa & Winsløw, 2013; Rasmussen, 2015; Winsløw, 2011), but also as a tool for student teachers and teachers to support professional development.

The PoS will be crucial for our research in studying the development of instrumental orchestration for student teachers. However in settings where participants have not before hand had much training in or are unfamiliar with the concepts related to lesson study, the PoS have been reported not to live-up to the usual forum for development of professional knowledge (Bengtsson, 2015). Thus the setting for the PoS and the reflections and discussion will be given extra consideration beforehand and questions will be formulated with the purpose of developing student teacher knowledge, accessing the technology and theory of the different praxeologies of the instrumental orchestration and the relation between the praxeological technologies and techniques.

One of the many challenges of research projects involving teaching with technology is, that the practices the teachers develop and evolve during the research project is often not implemented in lessons after the finish of research projects. This we will not be able to check, since it can take many years from the student teachers take the course to the student teachers get a job as a teachers. However we still have the possibility to follow the students during practice later in the school year and that might give us an indicator of their future implementation of technology in their teaching.

One still unsolved challenge of combining teaching with technology and lesson study is how to observe the students work in front of a screen without breathing down students' necks during the DoS. There are several technological solutions for this such as having a mother computer, that lets you look at the screen of the students' computers. However you will still not be able to see the screen of all computers in the class in a sufficiently representation to study the students work. Other programs have logs such that you can monitor the students' work on each screen. However the logs can very quickly become unmanageably long. Nonetheless they are good for a more in depth reflection of the students' development of or diagnosing the students' technological techniques. Another approach to the challenge of observing students' work while in front of a screen could be to but more emphasis on linking screen and paper-and-pen. This approach is apparent in the lesson plans and the exercises for the students.

The conclusion of this article is that though lesson study seems a fruitful format for teaching experiments and professional development of the teachers or student teachers involved, it does not solve all challenges when teaching with technology and it is not given that the format of lesson study resolve in developing professional knowledge for the student teachers or the teachers involved.

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Contribution of the epistemological analysis to the design, experimentation and analysis of Study and Research Paths

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This paper presents the research work developed by the author and its preliminary results. Two main empirical works have been carried out in order to analyse the epistemological needs of teachers, researchers and students when implementing an SRP.

Firstly, a course for in-service teachers of a Master in Mathematics Education centred in the epistemological analysis of the mathematical activity. The course took as a starting point the Study and Research Path lived by the teachers in a previous course.

Secondly, an SRP in a third year General Elasticity course of a Mechanical Engineering Degree is presented. Results of both courses are still in the analysis phase in order to stablish a preliminary answer to the research questions.

Introduction to the research problem

The implementation of SRP at school institutions plays a crucial role in the new paradigm of "questioning the world" proposed by Chevallard (Chevallard, 2006). The previous works developed in the Anthropological Theory of the Didactic (ATD) revealed important difficulties caused by the *ecological constraints* that appear when designing and implementing Study and Research Paths (SRP) as didactic devices at both secondary and tertiary levels (Barquero, 2009) (Ruiz-Munzón, 2010) (Winsløw, Matheron, & Mercier, 2013). In fact, the design and implementation of these devices usually shows up aspects traditionally banned from school such as collective work or the search of relevant literature in real inquiry processes.

In the ATD, the notion of "ecology" is used as a metaphor in the following sense:

> The *ecology* deals with the scientific study of the interrelations between organisms and their environments. In fact, *ecology* studies the physical and biological factors that influence these relations and that are influenced by them. By a deeper analysis of the etymology of the word *ecology* (oikos logos) it can be observed that in Greek oikos means "place to live", so literally "ecology" is the study of living organisms (in our case knowledges) "at their home", in their environment (in our case it would be the specific institution). (p .21 Ruiz-Munzón, 2010)

The epistemological component and, more specifically, the *prevailing epistemological model* in the considered institutions appear to be in the root of many of these institutional constraints.

In order to better explore these constraints, researchers need to assume an external position from the institutions involved in the didactic transposition process. The external position is needed to be able to question knowledge conception at the institution. The strategy is to elaborate an alternative epistemological model known as the *Reference Epistemological Model* (REM). This model is supposed to be in a continuous evolution: it is not a static model but an initial scientific hypothesis, which is supposed to be modified as long as it is used in every experimentation. However, it is not usually described as such in the aforementioned research works. In these works the REM is always presented in its final and finite form.

When considering the construction and role of REM in the design, implementation and analysis of SRP, new problematical questions emerge at the crossroads between epistemology and didactics. These questions are taken as the starting point of the research work:

- The first question approaches the relationship between REM and SRP: How to describe an SRP in order to implement it in a school institution? How to transform a given REM into possible didactic organisations that could live in current school institutions? This problem also includes the need to make this process available to the school institutions, especially to the profession of teachers.
- Another important and difficult question is the degree of explicitness that should be adopted with the new epistemological models and tools necessary to design, implement and evaluate new teaching and learning processes depending on the participants of the study communities addressed (students, teachers, mathematicians, etc.). Do teachers need an explicit definition of REM? Is a question-answer map a rich enough material form for the REM?
- Finally, we also wish to study which new notions or tools are needed to describe and manage the dynamic nature of the mathematical activity that will take place in study processes. In fact, how the SRP and REM descriptions can take into account its dynamic nature? How to describe these tools depending on the role addressed (didactic researcher, teacher and student) and how to make them available in the teaching institution and to the participants of the didactic process.

These questions arise from a study of previous ATD works and the need to transpose research results to the teacher profession. A more detailed justification of the questions was presented in the 9th Congress of European Research in Mathematics Education (CERME9) (Florensa, Bosch, & Gascón, 2015).Actually, these questions are taken as the first step of the research work presented here.

First research phase: a teachers' professional development course

In order to establish a preliminary answer to the research questions, the first work of the research phase was centred in designing an on-line teachers’ professional development course. Prior to the design of the course implementations of SRPs at secondary and university levels (Ruiz-Munzón, 2010) (Barquero, 2009) (Winsløw, Matheron, & Mercier, 2013) were analysed as well as teacher training works in the ATD framework. Teacher education in the ATD is based on the use of the so called *SRP in teacher education* (SRP-TE). This device has been described in the works of Ruiz Olarría (Ruiz-Olarría, 2015) as well as in the works of Barquero and colleagues (Barquero, Bosch, & Romo, 2015). These two works establish a formal methodology in teacher education structured in five modules. Other theoretical approaches about teachers’ knowledge have also been analysed: especially in what concerns the so-called “Pedagogical Content Knowledge” (PCK) (Shulman, 1986) as well as the notion of “Mathematical Knowledge for Teaching” (MKT) (Ball, Thames, & Phelps, 2008). Other devices such as the *Lesson Study* or the *Multidisplinary Teaching* (Winsløw, 2012) should be analysed in order to incorporate some aspects to further editions of the course.

Assuming these contributions, a teacher's training course on “The nature of mathematical thinking” has been designed and implemented on March 2015. Actually, the students had already lived a SRP-TE in the sense of Ruiz-Olarría in a previous course in November 2014. Because of this previous course, the designed course is centred on the epistemological analysis of the mathematical activity developed during this previous course. Both courses are part of an online Master in Mathematics Education coordinated by the CICATA (Mexico) and were coursed by the same students, all of them teaching mathematics on the secondary level at different Latin-America countries. The initial course (SRP-TE) is presented in detail in (Barquero, Bosch, & Romo, 2015). It includes an adaptation of the five modules of a SRP-TE presented in the work of Ruiz-Olarría (Ruiz-Olarría, 2015). Specifically, the SRP-TE took as generating question: “How to teach mathematical modelling and regression in secondary level?”

The aim of the course is to provide the teacher-students with ATD tools to carry out an analysis of the mathematical activity that enables them to emancipate from the school dominant viewpoint on mathematics. For that, the notion of reference praxeological model (RPM) and Herbartian schema become crucial for us as educators and designers, even though they were not explicitly defined in the course. The main mobilized tool during the course are the question-answer maps: they play an important role as a partial representation of the RPM.

Four activities were presented to the teachers during the course. In the first activity, students were asked to work in five teams and generate a tree-map of all the questions and answers that emerged when they lived an SRP as students. This initial map will be used as an analysing tool during the subsequent activities. The second activity was centred on the use of the question-answer map to analyse curriculum-related documents. During the first phase of the activity, students were asked to collect official documents (such as textbooks, curricula extracts…) related to “functions” and “regression”. These domains appeared in the first course when teachers designed and experimented a sketch of the SRP with their students. The second phase of this activity was the analysis of the collected documents and its comparison with the lived and experimented SRP. This analysis was proposed in terms of praxeologies and its articulation and rationale. The third activity asked the students to extend the map from the first activity or to propose new generating questions and its associated questions-answers maps. The main goal was to include new questions and answers in order to include all curriculum requirements in the fields of functions and regressions that may had not been considered on the first version of the map. In fact, students were asked to evaluate the (new) proposed generating questions to possibly cover all notions from curriculum. The last activity intended to highlight the importance of having an explicit conception of the “nature of mathematical thinking” and more precisely of the concrete school mathematics activities and domains that is a RPM, when tackling with didactic research questions. The activity included the analysis of a textbook based on the principles of the socioepistemology (Cantoral & Montiel, 2001) and its comparison with the ATD principles used in the course.

The use of the question-answer maps appear to be a powerful tool for teachers to analyse knowledge mobilized during and inquiry process. In fact, the construction of the maps was quite natural to them to describe the followed path during the inquiry. Moreover, by using maps students are capable to describe mathematical aspects usually ignored in the school mathematics such as “real” inquiry processes (in the sense that mobilized knowledge is not pre-defined) and collective work. In fact, the collective aspect appear crucial in teachers work when designing the SRP. In fact, the analysis of the followed path includes the analysis and the work with a "lesson plan". This guide was generated by teachers during the first phase as a way to transmit the knowledge acquired to their colleagues. In fact, the geneses of this document could be deeper studied by the approach of Gueudet and Trouche (2012). In fact, we could assume that teachers participating in the course form a Community of Practice, especially when they generate a common material to transmit new didactic tools.

For researchers, maps also appear as a good tool to connect different blocks of contents. However, despite of the real experience where regression and functions were co-used, teachers showed big difficulties in accepting alternative paths compared to the official ones: at the end of the course, only one team really connected the work with families of functions and regression. We can attribute this confinement in disconnected themes to the strength of the didactic phenomenon labelled by Chevallard as “thematic autism” (Barbé, Bosch, Gascón, & Espinoza, 2005).

The materials generated by the teachers as well as the analysis of the final survey filled by the participants are being analysed. In fact, the analysis of their answers shows up interesting preliminary results. For example, 72 % of the teachers accepted that “describing and understanding mathematical activity as a question-answer sequence allowed them to incorporate new knowledge and pedagogical tools to their practices”. In conclusion, the empirical work during this first phase enables us to generate a partial answer to the first and second research questions presented previously. In fact, the use of these maps as a partial and material representation of the REM empower teachers to make explicit mobilised knowledge and enable them to carry out an epistemological analysis of the mathematical activity. Moreover, teachers use these maps to describe the activity in the lesson plan both in a priori and a posteriori analysis.

Second research phase: An SRP in general elasticity

Mathematical modelling

In order to continue to generate partial answers to the research questions previously stated, we have designed, experimented, and analysed an SRP on General Elasticity. The main goal of this work was to analyse the dynamics of SRP, to develop tools in order collect this kind of data and to study the role played by mathematical modelling in an engineering course. We consider engineering as an interesting field to develop an SRP because of the important presence of mathematical modelling. The designed SRP is developed in a third year General Elasticity course of a Mechanical Engineering Degree.

The integration of mathematical modelling into current educational systems has been tackled by numerous investigations but still remains a major challenge. Many examples of mathematical modelling in various domains of engineering education exist: modelling acoustic properties of materials (Hernandez, Couso, & Pintó, 2014) or the works of engineering teaching in US high schools (English & Mousoulides, 2011). Numerous theoretical approaches agree on the need to incorporate mathematical modelling in mathematics and engineering teaching in consequence. As a result, some new curricular approaches try to introduce mathematical modelling in some university degrees (Gould, Murray, & Sanfratello, 2012) (Dangelmayr & Kirby, 2003) (Dangelmayr & Kirby, 2003). Some studies consider that mathematics in engineering play such an important role that engineering could not exist without them. Because of this strong interdepence between mathematics and engineering, the classical modelling cycle approach cannot be applied in this case (Bihler, Kortemeyer, & Schaper, 2015).

However, many institutional constraints and limitations appear when designing and implementing modelling devices in university teaching institutions (Barquero, Bosch, & Gascón, 2010). The institutional ecology plays a crucial role in the study of these conditions and constraints. The Anthropological Theory of the Didactic (ATD) framework enables us to describe these conditions and constraints affecting the implementation of mathematical modelling in scholar institutions, especially at university level. The necessary conditions for mathematical modelling at the undergraduate level have been studied in the case of first-year students of a business administration degree (Barquero, Serrano, & Serrano, 2013).

Course design and *a priori* analysis

Beyond the mathematical role played by mathematics as a service subject and the importance of mathematical modelling, a second motivation justifies the adoption of a SRP for the General Elasticity course. Until the last academic year this course was structured in mixed theory and problem sessions, and practical sessions. The latter included six 2-hour sessions on the following topics:

- Tensile test in three different metals (AISI 304 Stainless Steel, SR 275 Structural Steel and T6061 Aluminium).
- Charpy test in three different metals (AISI 304 Stainless Steel, SR 275 Structural Steel and T6061 Aluminium).
- Finite Element Method (FEM) simulation of a tensile test (using SolidWorks™ simulation as software).
- Oral presentation about failure criteria in different family materials.

During the practical sessions in the past two academic years three didactic facts were observed. First, a *thematic autism* in the sense of Barbé et al (2005) explicitly appeared. This means that all four activities were 'lived' as independent by the students even if the activities were closely connected. For example: FEM simulation (3rd session) simulated the real test carried out in the 1st session. The second didactic fact is related to the role played by the computer during the FEM simulation. Students introduced geometrical data, loads and meshing conditions to obtain the required results. Important difficulties appeared when they tried to understand "how the computer solved the problem" and "validating the results obtained". The students tended to validate all the results without any validating process. Both factors can be understood as a "black box" phenomenon: computer simulation is not understood by students and thus hinders them when judging the adequacy of the results obtained. And thirdly, we detected a clear absence of rationale in the four practical sessions. Both for students and for lecturers the presence of these sessions was more due to its "classical" character in elasticity than to a well-founded and justified didactic choice.

It seems that the adoption of a SRP based on a substantial enough generating question may partially overcome these limitations. The choice of the generating question emerges from the question "Why is General Elasticity taught in engineering?" which necessarily leads to the missing rationale. Once this question is posed, it is clear that the main reason to teach the subject is to provide engineers with tools enabling them to design any part of a machine working under an elastic regime. The connection between themes comes up immediately. To begin with a specific issue, the two lecturers teaching the subject agreed to start the SRP with the generating question: "How to choose one material (with unknown mechanical properties) from a set of three and design a part for a bike given in advance (brake lever, crank, gear, and bike lock key)?"

As an *a priori* analysis of the SRP, we have studied what kind of knowledge is expected to emerge when the students work on the design process. As a partial representation of this mobilised knowledge a question-answer map has been used (Figure 1). This tool was already used when modelling knowledge geneses from a generating question (Winsløw, Matheron, & Mercier, 2013) (Jessen, 2014).

Experimentation, data collection and analysis

The SRP has been experimented in December 2015 and January 2016 with two groups of 25 students. One of the groups was taught by a teacher without any didactic training and the other group was taught by the author of the paper.

The students have work in the mechanical laboratory during eight 2-hour sessions. The laboratory is equipped with a universal tensile test machine, a Charpy test machine, computers with simulation software and two 3D-printers. Each large group of students have been divided in groups of 4 or 3 students: each small group will have one specific part to be designed.

Each group is asked to design a specific part of a bike. At the end of the eight sessions they were asked to write a final report addressed to a fictional "bike design company". The report included:

- Specific dimensions of the part including its dimensional plans,
- Estimated loads
- Justification of the choice of the material
- Estimated strains that it will suffer while being used
- The adopted safety factor for stresses and strains
- Justification of the results regarding the computer simulation and the mathematical model used
- Final cost of the whole design process calculated by using given prices.

The requirement of explicitness of these aspects are expected to partially "enlighten" the existing "black boxes" such as computer simulation and mathematical models.

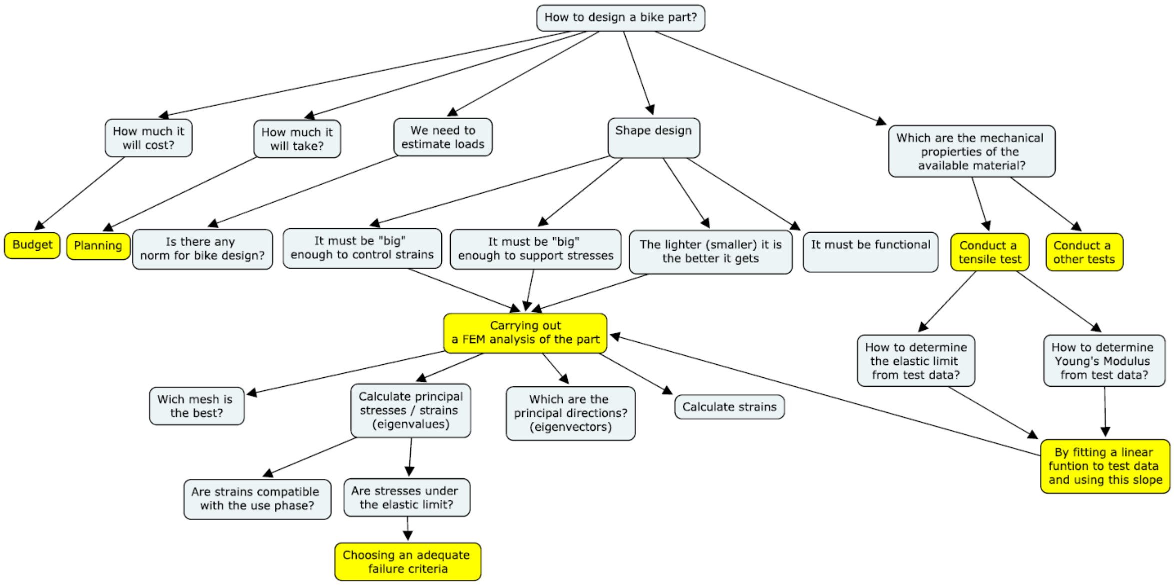

FIGURE 1 A PRIORI QUESTION ANSWER MAPS

During the first session each small group of students received three samples of different metallic materials, whose mechanical properties are totally unknown to the students. Then students were asked to write a first partial report that had to be delivered after the first week. It included:

- Time planning for the whole design phase
- Initial budget
- First questions that have emerged and that are planned to be solved during the following week.

After this first report, a weekly report was generated by the students. The content of the weekly reports is intended to collect data from the dynamics of the activity. In order to collect this kind of data the proposed content was:

- An updated time planning
- The questions that the team planned to ask during the week
- A description of the tasks carried out even if obtaining wrong results
- The obtained and validated answers that they have obtained (and how) from the questions of the week and derived questions.
- New questions for the next week

To evaluate the SRP the students had filled in a survey at the end of the course and now seven semi structured interviews are planned: one to the teacher without didactical training, two to retaking students, two with good marks at the reports and two to students that have not passed the subject.

Preliminary results of the survey show that most of the students (80%) consider that weekly reports have helped them to follow the SRP and that the “lived” SRP has helped them to change their initial idea of the rationale of General Elasticity. The 75% of the students considered very positive their participation to the project. A deeper analysis of the results of the collected data is now carried out.

Conclusions

Both teacher training course and the engineering course allow us to establish preliminar answers to the research questions. On the one hand, working with question-answer maps help teachers to develop an epistemological analysis and to question mobilised knowledge during teaching-learning processes. On the other hand, asking students to make explicit their study process in weekly reports enabled students to make explicit aspects that are usually absent from traditional scholar settings. Both tools are easily adopted by both communities making explicit in some degree the epistemological model sustaining the lived SRP.

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The collective aspect of implementing study and research paths – the Danish case

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For more than 10 years papers have been published showing the potentials of designing teaching based on Study and Research Paths (SRP) – a notion developed as part of the Anthropological Theory of the Didactics (ATD). SRP relates to the proposed teaching paradigm of Yves Chevallard: "Questioning the world". In this paper it is being argued that teachers collaboration in implementing reforms or changes of a dominant teaching paradigms is an important element in making these initiatives real alternatives to existing situation. Elements of current collaboration are analysed and the analysis points towards teachers need of a common language to discuss and articulate design and evaluation of or reflection upon teaching.

The Anthropological point of view

It has been proposed by Yves Chevallard that it is time for change of teaching paradigm. He characterises the current teaching paradigm as "visiting monuments" and proposes a new one called "questioning the world" (Chevallard, 2015). The paradigm of visiting monuments refers to teaching where a theorem or other pieces of mathematical knowledge is presented, it is shown how to use it for solving exercises and maybe proved. Students are expected to learn and appreciate the piece of knowledge. However the piece of knowledge does not necessarily answer a question being of any interest of the student who is supposed to learn it. This might end up with mathematics being perceived as a list of monuments to visit, a number of techniques to solve exercises – but without a raison d'être and without relations between the monuments (Chevallard, 2015, pp. 175).

On the contrary, Chevallard proposes, that teaching should be based on living questions, which leads students into a study and research process developing answers. Study and research path (SRP) is a design tool where teaching is based on an open question, which is supposed to initiate a study and research process for one student or among several students. The idea is that teachers pose a generating question, which the students understand but cannot answer immediately. The students have to study different media (textbook, video, internet, teacher etc.) decompose and reconstruct the new knowledge in a research process in order to formulate an answer for the generating question. The research process takes place in a milieu consisting of old knowledge, knowledge offered by media, pen and paper, calculator etc. A strong generating question Q_0 will lead to derived questions Q', Q'' and so forth (Jessen, 2014). The answers to the derived questions add up to an answer for the generating question Q_0.

The process and dynamics of SRP can be described using the herbartian schema:

$[S(X;Y;Q_0) \to M] \hookrightarrow A^{\blacktriangledown}$

Where the system S consist of a group of students X interested in studying the question Q_0 assisted by a group Y. In doing so the system interacts with a milieu M developing their personal answer, $A^{\blacktriangledown}$. The milieu $M = \{A_1^{\diamond}, \dots, A_n^{\diamond}, O_{n+1}, \dots, O_m, Q_{m+1}, \dots, Q_p\}$ consists of p elements where $A_i^{\diamond}$ represents existing answers within the X, O_j are the works of others which are being used in the study process and Q_k represents the questions raised during the study an research process. The processes described by the herbartian schema further the development of the raison d'être of the developed knowledge, which can be characterised as praxeologies or mathematical organisations (Kidron et al., 2014 p. 157). Chevallard formulates the aim of the new teaching paradigm as: "the new didactic paradigm wants the future as well as the full-blown citizen to become Herbartian" (Chevallard, 2015, p. 178), meaning a person who questions the world and develops answers based on existing knowledge, the works of others and who deconstruct and reconstruct all this into knew answers. Regardless how interesting this sounds, it is evident that the change of teaching paradigm will not happen easily. Therefor the research question of this paper is:

What constraints and conditions exist in the implementation of SRP based teaching and the paradigm of questioning the world from the collective perspective on the teaching of mathematics?

Moreover it will be discussed what infrastructures exist in the teaching system of upper secondary mathematics in Denmark today? What elements support or hinder the diffusion of the paradigm of questioning the world and SRP based teaching? It is evident that a first hindrance is that almost no Danish high school teachers are familiar with ATD or SRP. But even if they were, the analysis of this paper points towards challenges.

In the analysis the herbartian schema is applied, partly to teachers own descriptions of their professional development (electronic survey with more than 1000 respondents across the country), as it is described in a recent report giving a status on mathematics teaching at upper secondary level in Denmark conducted by Jessen, Holm and Winsløw (2015) and partly on the ministerial regulation for bidisciplinary work at this level. Hence, the basis of the following discussions are the previous work of myself and colleagues on the teaching of mathematics and other disciplines at the high school level, i.e. ((Jessen, Holm & Winsløw, 2015), (Jessen, 2015), (Jessen, 2014) and ministerial guidelines). Further the analysis engages the notion of paradidactic infrastructures by Carl Winsløw (2012). The paradidactic infrastructures are "everything which conditions and constraints the PS [Paradidactic System] in its different phases and in the interplay between phases" (Winsløw, 2012, p. 293). The phases of the PS is the pre-didactic system (PrS), the didactic observation system (DoS) and the post-didactic system (PoS), which relates respectively to the planning, the observation and the evaluation of the didactic system (DS). The didactic system is defined as a group of people studying some objects or organisations using some artefacts in doing so (Winsløw, 2012, p. 292). The PS runs parallel with the DS and in some institutional settings not much attention is paid to the Prs and the PoS. This is not the case in Japan where lesson study is a formalised structure of the elements of the paradidactic system. Lesson study in Japan functions as means for professional development (Winsløw, 2012, p. 295). Initiatives have been made with respect to implementing lesson study in the Danish school system, however these efforts do not cover the upper secondary level.

The Danish context

In Denmark a major reform of upper secondary education took place in 2005. The education was divided into a large number study lines (more than 200 see (Ministry of Education, 2013c)). The students must choose one before entering upper secondary education. A study line consists of three disciplines at a certain level (there exist three levels, C to A, A highest). An example of a study line could be biology A, mathematics B and social science B or mathematics A, Physics B and chemistry B. Students have the freedom of choosing extra disciplines (psychology or philosophy etc.) or they can use this liberty to raise the level of one of the disciplines in the study line. Further there are a number of compulsory disciplines as native language, English and History among others. Moreover students follow general study preparation which is evaluated in an oral, high stake exam based on a bidisciplinary synopsis treating a case linked to the topic of the year determined by the ministry of education (Winsløw, 2012, p. 299 & Ministry of Education (2013d)). On top of this the students attend another high stake exam, the study line project, which is a bidisciplinary written report of 15-20 pages students hand in a half year prior graduation (see further in (Jessen, 2014) and (Hansen & Winsløw, 2011)).

Regulations of the teaching of mathematics at each of the levels A, B and C is stated in curriculum and elaborated in documents called ministerial guidelines. In addition, the written exam represents a strong constraint on the teaching as it has been pointed out in (Jessen et al., 2015, pp. 13 & Jessen, 2016). During the first half a year at high school students attend two crossdisciplinary subjects: general introduction to natural sciences and general introduction to language structures (our translations). The first subject must be taught by at least two teachers representing minimum two disciplines within the natural sciences introducing students to different methods across natural sciences – mainly focusing on different ways to work in laboratories and inquiry based. The other discipline introduces students to commonalities with respect to language, an equivalent to the former focusing on grammar and methods from humanities.

Previous experienced study and research

In 2012 Jessen designed study line projects based on SRP and handed out generating questions instead of the usual problem formulations (a list of questions students should answer). The result of this study is to be found in Jessen (2014). It is concluded that SRP is a suitable tool for designing questions for the study line projects, but Jessen also points out some of the challenges in designing these collaboratively (Jessen, 2014). It is here worth noticing that in Denmark most teachers have a minor in one discipline and a major in another (often linked as mathematic and physics) and teach both disciplines in high school. However it is still reported to be a challenge to go across disciplines and collaborate with teachers of a third discipline and connect these (EVA, 2015) – teachers are not trained in this.

In 2014 Jessen explored the potentials of SRP as design format. In this study every day teaching at level C and B was designed around generating questions in mathematics (Jessen, 2015). In this context the questions were designed so it lead to development of praxeologies and mathematical organisations given in curriculum among the students. This means in terms of herbartian schema that students develop answers closer the teachers answer: $A^{\heartsuit} \approx A_{y}{}^{\diamond}$. These designs are called study and research activities (SRA). This teaching did not explicitly require collaboration with other teachers however an examiner at oral exam expressed serious concerns regarding the teaching. The teacher did not find the format of using open questions suitable for students not being fond of or gifted in mathematics. The exam was a success and the examiner would like to know more about the teaching. However this scepticism is a constraint with respect to implementation of a new paradigm for teaching.

Collaboration in bidisciplinary activities

The above mentioned bidisciplinary exams naturally requires collaboration of some kind between teachers representing different disciplines – and sometimes different faculties as the humanities and natural sciences. The two introduction courses to natural sciences and humanities also require a certain amount of collaboration or at least coordination of shared topic for each class to work with. The reform further require from teachers to “tone” their disciplines, meaning that it should be visible for students how elements of the content of each discipline can support the study of main disciplines or the other way around. Hence in mathematics classes in a language study line it is suggested to let students study topics of history of mathematics in original language or draw anthropological studies as examples when discussing descriptive statistics. Even though the reform heavily relies on successful collaboration between teachers there seem to lack knowledge on how to do this productively. As described by Winsløw (2012) most schools have committees arranging general study preparation (and the other bidisciplinary elements as well). Their work, focus on delegation of teaching tasks and responsibilities instead of actual collaboration. There is not much focus on content knowledge from the involved disciplines – or at least it is expected that the respective teachers plan and design the teaching individually (Winsløw, 2012, p 299). Likewise the bidisciplinary written reports are often planned individually in parallel, which is reflected in the questions handed out to the students and similar in the written reports handed in by the students (Hansen & Winsløw, 2011, p. 687).

Collaboration within mathematics

In general the collaboration among teachers of mathematics at upper secondary level in Denmark happens in informal contexts. In an evaluation of upper secondary mathematics in Denmark Jessen, Holm and Winsløw found that for 88% of the teachers the main forum for discussing the teaching of mathematics is the group of mathematics teachers at the school where they are employed – during lunch. 36 % read the magazine produced and distributed by the mathematics teacher association and 29% points to a closed Facebook group for teachers of mathematics at upper secondary level (Jessen et al. 2015, p. 63). Hence, situation is much like the one described in (Winsløw, 2012, p. 302), where a teacher is arguing that he did not apply for a job of collaboration but for teaching individually. In light of the quote it is positive that a relatively high number of teachers seek professional development and inspiration from colleagues. Nevertheless the report by Jessen, Holm and Winsløw states by quoting a teacher that the quality of what is being shared differs a lot. A teacher formulates it in an interview as: “it is free of charge to discuss Maple commands over lunch compared to discuss how to improve teaching” (A high school teacher in an interview asked about current situation for professional development in (Jessen et al, 2015, p. 63)). To some extend the classroom is perceived as private property – and no one but the teacher is to discuss or comment on what is going on in there. However two teachers in the report by Jessen, Holm and Winsløw mention joint preparation as a source for development and another teacher gets his inspiration from “two teacher arrangements” (in total the system employ more than 2000 mathematics teachers at upper secondary level). The latter is a lesson where one teacher plans the lesson and the other one participate in order to assist students while solving exercises or to guide them in project work. The second teacher does not take part in the planning of the lesson. Usually, no pre-didactic nor post-didactic system (in the sense of (Winsløw, 2012, p. 292)) are related to this kind of activity.

This means, that to some extend teachers are given “external infrastructures” by school management meaning time to meet, suggestions from committee to teachers on topic, materials and so forth. However the “internal infrastructures” are missing real collaboration in the sense that teachers still plan bidisciplinary didactic systems in parallel ending up with situations as described in (Winsløw, 2012, p. 301, figure 15.2). In this paper we will discuss this situation further applying the herbartian schema in order to be more explicit about the challenges in collaboration in terms of ATD.

In Denmark many schools facilitates professional development of different teacher groups. Close to half of the teachers answering the electronic survey in the report by Jessen, Holm and Winsløw have “mathematics teachers group meetings” for all mathematics teachers at their school. The meetings can be organised around workshops with topics as how to lower the rate of students failing mathematics B or how to improve students’ ability to write mathematical text? The workshops are often a sharing of best practices, sometimes combined with an invited speaker giving an introduction to the topic. Afterwards it is the individual teachers responsibility to implement the new knowledge in his classroom. This is another example of existence of some external infrastructures for professional development however it does not seem to affect teaching much, when the internal structures are missing.

Herbartian analysis of Danish collaboration

In this section we describe the collective dimension of teachers work in terms of herbartian schema meaning identifying what questions are raised, what answers are consulted and who brought them in and what works are shared?

As mentioned above, most teachers get inspiration for improving their teaching over lunches with colleagues. This means teacher y_1 raises an open question e.g. Q_0: How do you introduce differential calculus in your second year high school classes? The study of this question is supported by the group of mathematics teachers $Y = \{y_2, \ldots, y_n\}$. The teacher y_1 plays the role of student x, but could also take part of the answer development, bringing in his or her own existing answer $A_1^\diamond$. But often the sharing will be the sharing of materials $\{O_1^\diamond, \ldots, O_m^\diamond\}$ combined with some answers in terms of didactic praxeologies $\{A_1^\diamond, \ldots, A_n^\diamond\}$. In this case the development of the didactic praxeology of introducing differential calculus can be described as this schema:

$$[S(y_1;Y;Q_0) \rightarrow \{A_1^\diamond, \ldots, A_n^\diamond; O_1^\diamond, \ldots, O_m^\diamond\}] \hookrightarrow A^\heartsuit$$

Teacher y_1’s answer to Q_0 presumably will be closely related to a preferred answer given by one of the teachers e.g.: $A^\heartsuit \approx A_i^\diamond{}_{y3}$. Teachers take over materials from others revise it a little according to their initial praxeologies but it is not decomposed and reconstructed in the sense of a rich study and research process even if the teacher experiment the new $A^\heartsuit$ in the classroom. This could be characterized as cooperation in the sense that teachers are not engaging in a study and research process improving and developing a new answer to a generating question they share as part of their professional work. If teachers joined together in such a process bringing in existing answers and works of others the schema would look like this:

$$[S(y_1, \ldots, y_n; \emptyset; Q_0) \rightarrow \{A_1^\diamond, \ldots, A_n^\diamond; O_1^\diamond, \ldots, O_m^\diamond, Q_{m+1}, \ldots Q_p\}] \hookrightarrow A^\heartsuit$$

In this the teachers will share the same answer to Q_0. But further a research process in this context would to some extend require the test of ideas meaning classroom interventions and observations or what was characterised as didactic observations systems (DoS). It is worth noticing that this is not part of the infrastructures offered by management. In the report by Jessen, Holm and Winsløw teachers mentions the magazine distributed by the mathematics teacher association as a mean to professional development. It adds to the milieu M of the study of Q_0 but it is up to each teacher whether it acts as a monument to visit or it is studied and incorporated in the teachers practice and development of new $A^♥$. The mentioned Facebook group mainly functions as a sharing of teaching material why this collective element is characterised as the first cooperation schema and equals lunch talks.

Looking at the planning of bidisciplinary works as study line projects, general study preparation or the general introduction to methods of natural sciences there is not much shared development of an answer to a Q_0. Take the example of general study preparation: teachers of two or three disciplines are told by school management to carry out one thematic week where the teachers find a case to study (e.g. global warming) and then they must cover some of the learning objectives for general study preparation. Examples of these objectives are "to write a synopsis", "to find and study suitable media at the library" or "be able to discuss what contributions each discipline can bring relative to the methods of the discipline" (Ministry of Education, 2013d). In these situation y_1, y_2 and y_3 often finds a common field to built a case for the students to study based on their $\{A_1^♥, \ldots, A_n^♥\}$. These answers, which the teachers have developed, are partly "darlings" related to their own study of the discipline as well as their "professional darlings", meaning their teaching praxeologies with respect certain disciplinary organisations. Each teacher offers to cover a certain part or angle to approach the case. From the perspective of the other teachers this means that, what is brought in is existing praxeologies, which they never decompose or reconstruct, hence they play the role of $A_i^{\diamond}$'s. Further, they present media for the students to study, meaning they offer some monuments for the other teachers to visit, $O_j^{\diamond}$'s. Teachers do not often find time to engage in a study and research process trying to do a thorough a priori analysis of the student activity, hence they agree upon a topic and do not cross disciplinary boundaries but hope students will be able to do this.

This means that modelling this teacher work a vague pre-didactic system can be described with the schema below:

$$[S(y_1,y_2,y_3;?;Q_0) \rightarrow \{A_1^{\diamond}, \ldots, A_n^{\diamond}; O_1^{\diamond}, \ldots, O_m^{\diamond}\}] \hookrightarrow A^♥$$

However the answer to Q_0, which the teachers share is more an equivalent to the milieu they are planning to offer the students rather than a newly developed teaching praxeology. Even though management form committees for planning the general study preparation they do not offer the setting of a real paradidactic system, meaning the current situation have some vague pre-didactic system, but no didactic observation system and no post-didactic system. This might not be a problem however students report on parallel structured teaching where there is no relation between what they are taught (EVA, 2015, pp. 42), but the critique does not affect the professional development of each teacher much or how the work is planned. This is mainly, because the teachers cannot change this situation themselves.

To sum up there do exist elements of teachers planning teaching or assisting each other in the planning of teaching at upper secondary level in Denmark. However the quality of these activities lack in richness of the media and milieu of the teachers study process of their didactic question and the presence of the raison d'être of the activities is not clear. Teachers are eager to share notes, experiences and teaching materials. And they do plan bidisciplinary teaching together but it is not clear if they actually collaborate, cooperate or simply coordinate and delegate the different lessons between them. And it seems that the activities do not offer the possibility of teachers discussing teacher practices and scopes of different approaches.

The report by Jessen, Holm and Winsløw shows that when mathematics teachers at upper secondary level are asked what kind of in-service teachers courses they would like to have 51% points to "stofdidaktik" or content didactics. This is course activities where teachers are taught how scholarly theory can become "teachable" theory. Courses offering this have been characterised as "capstone courses" and a presentation of such a course and the need of those are given by Winsløw and Grønbæk (2014). The report by Jessen, Holm and Winsløw further shows that 48 % of the teachers wish for courses in applied mathematics and 39 % answer courses in didactics of mathematics (Jessen et al., 2015, pp. 59). It is argued in the report that it is reasonable to assume that this means teachers actually request courses enabling them to reflect upon teaching and improve it. But it also shows that teachers feel a lack in their competences with respect to engage in a full blown paradidactic system as the one described in the Japanese case in (Winsløw, 2012).

Challenges in Introducing SRP in Danish context

Barquero, Bosch and Romo (2015) illustrates how SRP can be introduced to in-service teachers by letting them carry out a SRP-TE designed in order for them to discuss the raison d'être of the mathematical content to be taught but also to teach them useful notions from didactics of mathematics. Emphasis was put on how these notions could solve problems in relation to teachers practice rather than being notions a teacher ought to know (Barquero et al, 2015). However the teachers has a tendency of falling back on old habits planning the SRP for their own classes. Nevertheless the collective aspect seemed to affect some teachers to design activities with a more open character as intended (Barquero et al., pp. 6). From the recent report by Jessen, Holm and Winsløw there are reasons to believe that it will be equally difficult to change the teachers practice into the paradigm of questioning the world by engaging them in a single course activities. In the course activity presented by Barquero, Bosch and Romo there are paradidactic structures supporting the in-service teachers professional development. In the SRP-TE the stages 1-4 constitute the PrS (Q_0: How to teach…?, live a SRP, analysis of lived SRP and design of an SRP). Stage 5 implementing and posteriori analysis of a designed SRP covers the DoS, letting the living of the SRP be the DS. The posteriori analysis points in the direction of PoS, including revised lesson plans (Barquero et al., 2015, p.7). But it is unclear to what extend this is done as a collective work in the sense of the Japanese case presented in (Winsløw, 2012). What is observed by colleagues who in details know the lesson plan, compared to what the teacher registers during the lesson might be slightly different and lead to different reflections about the lesson afterwards.

It could be interesting to design activities for mathematics teachers at upper secondary level in Denmark drawing on the experiences with SRP-TE incorporating the notion of communities of practice to let teachers develop SRP's in collaboration (Gueudet & Trouche, 2012, p. 307), implement them and reflect upon them in groups in the same school as well as in minor scale across schools. Moreover to offer in-service teachers the full paradidactic system in order to develop sustainable professional development opposed to courses where ideas are never truly implemented. Materials, designs, ideas and experiences could afterwards be shared with the community of mathematics teachers at involved schools and further with the association of mathematics teachers at upper secondary level – to some extend in the sense of the of Sésamath (Gueedet & Trouche, 2012, p. 310). It seems crucial in this context still to emphasise the development of a shared "didactic of mathematics language" in order for the teachers to formulate challenges in their practice in precise terms and further to discuss and solve these challenges. Moreover it would be needed in terms of dissemination of designs and their teaching potentials or simply the idea of SRP and questioning the world.

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Inquiry – When, what & Why? Examining how Teachers decide to implement Inquiry-Based Instruction in Science Lessons

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Inquiry-based instruction continues to remain at the forefront of science education, in both formal and informal settings. With the introduction of the Next Generation Science Standards (NGSS), U.S. science teachers now have a practical set of inquiry standards to guide their instruction. As more and more states adopt NGSS, science teachers in both formal and informal settings are now tasked with identifying when to utilize inquiry and with what content. This decision making process requires teachers to determine what content they wish to teach through inquiry, when in their curriculum they want to teach it, and why inquiry is the 'best' approach. Many factors can shape these decisions, including influences from collectives in which the teachers are involved. Didactic transposition is a process in which knowledge transforms as various 'users' interact with it in different contexts. In this essay, I propose that didactic transposition may serve as a theoretical lens through which to examine this process and develop a research epistemological model that may support this area of research.

Introduction

There is currently not a national curriculum in the United States. Individual states have decision making on what content and practices should be included within their learning standards. In recent years, the Common Core State Standards (CCSS), an initiative put forward by the National Governors Association (NGA) & Council of Chief State School Officers (CCSSO) and promoted by President Obama's Race to the Top education initiative, have been adopted by most states to address math and English skills (CCSS, 2010). While science is not featured within the CCSS, the Next Generation Science Standards have been developed by a consortium of states and non-profit science education organizations to act as complimentary standards for addressing science skills and content across the nation (NGSS, 2013). While individual states still utilize their own science content standards, the NGSS focuses on science skills and practices, with particular attention paid to inquiry-based instruction in science classrooms.

Inquiry is certainly not a new concept; it has roots back to Dewey's work in the early 20th century (Supovitz, Mayer, & Kahle, 2000). However, with resurgence of emphasis on inquiry, some teachers struggle with implementing inquiry in their classrooms effectively. Several possible explanations have been put forward to explain this problem. Perhaps it the seemingly conflicting nature of 'testing culture', in which students are assessed through standardized exams, and inquiry-based learning, in which students engage deeply in problem solving (Sacks, 2000). Perhaps it is the bureaucratic institutional structures set up, seemingly to assist teacher planning, but often causing a reduction in work (Winslow, 2012). Perhaps it is confusion that occurs when teachers lack the self-efficacy to write and implement content that they have mastered (Loughran, Mulhall, & Berry, 2004). Or, more likely, it is combination of all of these explanations and more.

One potentially influencing factor would be the effects of collaborative, or "collective", work amongst teachers. For nearly two decades, collaborative practice has been a heavily researched component of teacher research (Ball & Cohen, 1999). Increasingly, it has become apparent that collaborative practice must be carefully planned and supported, such as with supported collaborative inquiry, in order for teachers to maximize their effectiveness in these working environments (Capobianco, Lincoln, Canuel-Browne, & Trimarchi, 2006). As teachers continue to be required to work collaboratively, the influence of the collaborative context likely plays a role in shaping the individual teacher decision-making.

Regardless of the influencing factors, there is a disconnect between science teacher beliefs and practices with regard to inquiry (Windschitl, Thompson, & Braaten, 2007). As teachers continue to be asked to include inquiry based practices into their teaching, more and more they will find themselves in positions where they need to make conscious decisions about when and where and with what content they utilize inquiry practices.

Given these conditions, science teacher decision-making with regard to inquiry-based practices can be seen as a critical construct to unpack.

Framing Literature

Inquiry-Based Instruction

Hmelo-Silver, Duncan, & Chinn (2007) define inquiry as a process in which "students learn content as well as discipline-specific reasoning skills and practices (often in scientific disciplines) by collaboratively engaging in investigations" (pg. 100). Supovitz, Mayer, & Kahle (2000) provide their own definition of inquiry-based instruction as "a student-centered pedagogy that uses purposeful extended investigations set in the context of real-life problems as both a means for increasing student capacities and as a feedback loop for increasing teachers' insights into student thought processes" (pg. 332). For the purposes of this paper, I draw from these definitions as I define "inquiry based instruction/practices" as those in which the teacher engages his/her students in first-hand learning through investigations that allow the learners to extrapolate meaning about science content through the process.

Specific to science education, inquiry is often associated with "good scientific practice" (Abd-El-Khalick, et al., 2004). Indeed, inquiry has been viewed as a way forward in both teacher practice and teacher research (Cochran-Smith & Lytle, 2009). Inquiry in the sciences has been viewed as an approach that allows interdisciplinary perspectives to be recognized by learners through an inductive learning process (Nargund-Joshi, Liu, Chwdhary, Grant, & Smith, 2013). However, there are many challenges associated with inquiry in science education, particularly with regard to standards-based teaching. One common complaint is that inquiry, while leading to a meaningful learning experience for students, is a time-consuming process that can't be afforded given the large scope of content indicated to be covered in the standards.

These points lead me to pondering – What factors influence when and where inquiry is implemented in science classrooms? Based on the aforementioned points, my inclination is that there is a litany of influencing factors that shape the decisions that teachers make with regard to when and where they utilize inquiry-based practice in their classroom and with what content they are inclined to do so. Therefore, I feel that it is necessary to unpack the decision-making process in order to identify these influencing factors. To do so, I propose didactic transposition as an appropriate theoretical lens for this task.

Didactic Transposition

Chevallard (1985) originated the concept of didactic transposition, which he describes as one in which knowledge is significantly transformed as various users interact with it in different contexts, thorough his research on the Anthropological Theory of Didactics (ATD). When applied to education, it refers to the ways in which content changes as it shifts through a continuum beginning with scholarly knowledge and eventually ending as knowledge learned by the student. This learned knowledge may then be applied to inform the scholarly knowledge of the content as the process begins anew (Bosche, 2014).

Bosch's process of didactic transposition (Figure 1) demonstrates several stages through which knowledge is transformed. Of particular interest to me is the transpositive process that occurs between the "noosphere", or "knowledge to be taught", and the "taught knowledge" occurring within an individual teacher's context. The noosphere, in this case, represents the inquiry-based NGSS standards. Teachers must then transform the acontextual practices described in the standards to their own setting, i.e. their classroom. They must not only decide with which content to teach using inquiry, but they must decide how and why this practice is the 'best one' for their purposes. This knowledge is then manifested in their classroom practice, or "taught knowledge".

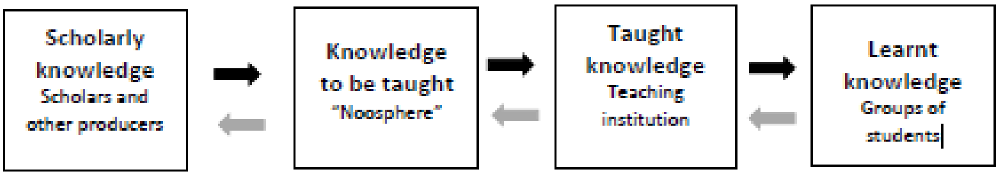

Figure 1: Process of didactic transposition (Bosch, 2014)

The context in which a teacher is located, the "cultural, historical, and social reality" of their teaching institution, plays a role in the ways in which teaching occurs (Gueudet & Trouche, 2012). Therefore, that same context, informed by the ways in which that teacher views the implementation of new practices, will shape the transpositive process and, ultimately, have an effect on the inquiry taking place in their classroom. Given that we know that context often acts a barrier to knowledge acquisition from learners (Mortensen, 2010), I feel it is critical to pay specific mind to this factor when addressing teacher decision making with regard to inquiry.

Recommendations for further study

Reframing the Problem

To reframe the problem with reference to the literature, science teachers are increasingly asked to utilize inquiry practice. However, due to a wide variety of influencing factors, many science teachers are reluctant to implement inquiry in many, if any, of their lessons. Given the current teaching climate where teachers must transform knowledge from the NGSS about inquiry learning (the noosphere) into inquiry-based practices in their classroom, how do teachers decide what content should be taught through inquiry? Also, what external factors influence this decision making process? Finally, are there patterns with how individual teachers make meaning of the knowledge related to both their content and inquiry teaching?

Didactic Transposition as a Theoretical Lens

Given the previously stated concerns, I propose the ATD as an appropriate theoretical lens through which to explore the stated problem. The decision-making process through which teachers select or deselect inquiry-based instruction for specific components of their curricula is influenced by several factors: the teacher's philosophical stance on education, their experience as a learner, their interpretation of the content knowledge, the collective teaching experience at their school, and the school community and culture. These factors all are contributing components in the process between content knowledge and didactical practice. The combination of cognitive and cultural aspects suggests that ATD is an appropriate theoretical lens, as it will allow attention to be paid to both internal and external driving factors.

A tool that is utilized in ATD is that of Study and Research Paths (SRPs). SRPs are tools used to question the world (Chevallard, 2006) by focusing on research questions that challenge traditionally held views on systemic functioning (Gazzola, Llanos, & Otero, 2013). The corresponding research question (Q_0) in this instance is: What factors influence a science teacher's decision to implement content utilizing inquiry-based practices in their classroom? The phrasing of this question is broad enough to allow for both cognitive and cultural influences to be accounted for, yet narrow enough to concretely link between content and inquiry-practice.

As mentioned earlier, didactic transposition is a process that describes the ways in which knowledge is transformed from scholarly knowledge through knowledge learned. For this Q_0, didactic transposition seems particularly well suited as a theoretical lens through which to examine teacher practice. Specifically, the transposition occurring between the second and third boxes in Bosch's process of didactic transposition. In this case, the "noosphere" would represent two types of 'content' knowledge: both content knowledge (CK) related to scientific concepts and pedagogical content knowledge (PCK) (Shulman, 1987) related to the inquiry-based didactics being promoted through current science education reform efforts. My assertion is that teachers must navigate the didactic transpositive process for both types of content knowledge. That is, teachers must identify which science CK is required to teach as well as the science PCK (inquiry-based practices) that they are expected to utilize in their classroom.

Referring back to Bosch's process of didactic transposition, the third box is "taught knowledge". For this Q_0, this box would refer to the science CK that is taught in the classroom. Of particular interest is the transpositive process that occurs where teachers evaluate the various science CK that is required to be taught and identify which are best taught through the inquiry-based PCK that is also being required in their classrooms. It is through this process that I believe a meaningful didactic transpositive process is occurring. Teachers identify various science CK and then select or deselect specific content to be taught through inquiry-based practice. I suspect that the decisions that they make as to which science CK is best taught through inquiry-based practices are influenced by both external factors and internal factors. The external influencing factors can include school context (such as norms accepted within a specific school, limiting resources, and collective planning/teaching experiences) and education policy (such as national standards, mandatory laboratory activities, and school district policies on inquiry). The internal influencing factors can include teacher cognitive processes (such as the teacher's self-efficacy for specific science CK and the ways in which past experiences as science learners have shaped schema related to specific science CK).

A useful tool within ADT research is the Reference Epistemological Model (or REM). An REM is model created by a researcher that elaborates on the didactic process, as imagined by the researcher, and is used to demonstrate how the DT process might occur with regard to a specific research question (Bosch & Gascon, 2006). For this particular Q_0, I have developed an REM that I suspect demonstrates a superficial representation of the influencing factors for the didactic process in quest (Figure 2).

As this REM suggests, the noosphere represents the standards-based content and didactics that are required to be taught. Specifically, it represents the science CK that is present in state and national standards, as well as the inquiry-based practices that are being promoted in national standards. The noosphere is influenced by scholarly knowledge – both science content and science didactics. For the purposes of this Q_0, the didactic transpositive process occurring between scholarly knowledge and noosphere is not of concern, although I feel it worth mentioning that there is influence happening.

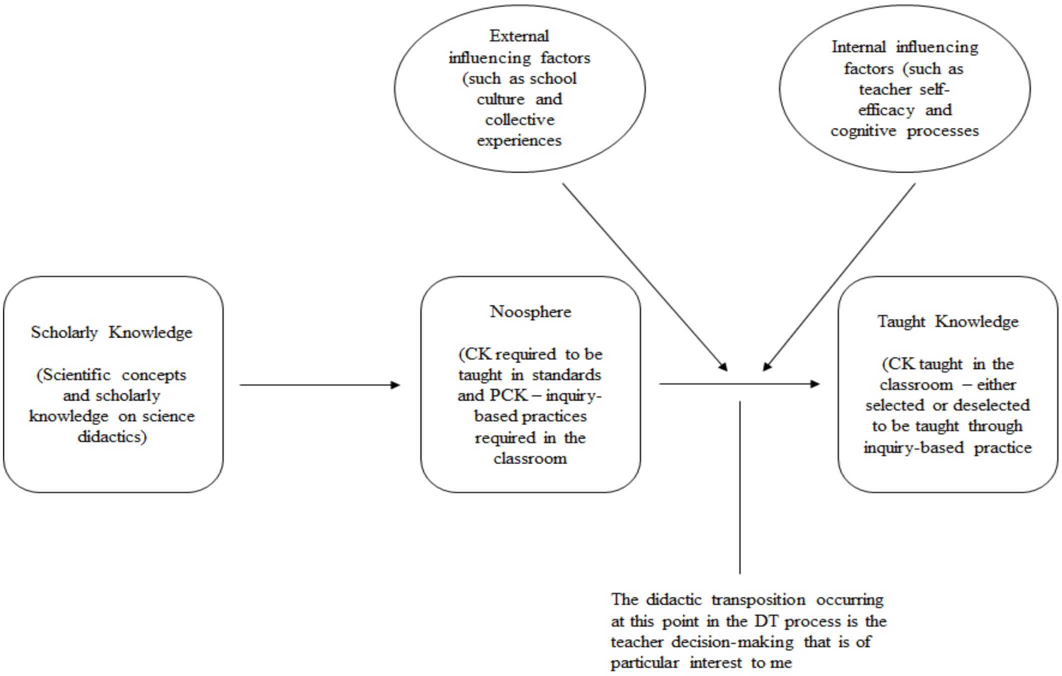

Figure 2: Johnston's Reference Epistemological Model for Science Teacher Decision-Making for Inquiry-Based Practices

Individual teachers are, then, required to take the science CK in the noosphere and teach that to their students. Additionally, they must also demonstrate inquiry-based practice in their teaching. Therefore, they must decide which science CK is best taught through inquiry. This decision-making process will vary based on individual teachers, given the external and internal influencing factors that are highly specific to individual teaching context and teacher experiences. I maintain that this process is one of didactic transposition. The outcomes of this process are the science CK that is taught in the teachers' classroom, some of which will have been selected to be taught through inquiry-based practices and some of which will have been deselected.

Implications for Research

I feel that this line of research could be effective for addressing several 'holes' in the current research literature. This study would allow the groundwork to be laid to explore the similarities and differences regarding inquiry practice that can be influenced by these contexts. Specifically, this study would illuminate the role that collaborative experiences play in influencing individual teacher decision-making.

Additionally, while there is a developing corpus of literature around the use of didactic transposition as a theoretical framework in science education, there is little work being done in this area within the U.S. Finally, the proposed study would include the intersection of two currently unrelated frameworks to looking at teacher practice: didactic transposition and teacher typology. It is my hope that, throughout the course of my dissertation study, I will contribute to the emerging body of research around these concepts, shed light on the ways in which science teachers navigate the implementation of inquiry practice in their classrooms, and identify areas for further study in my academic and professional careers.

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Collective aspects of pre-service LOWER secondary teachers' knowledge on density of rational numbers

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This study is about the design of hypothetical teacher tasks (HTTs) on density of rational numbers, developed based on the anthropological theory of the didactic (ATD), and used to investigate pre-service lower secondary teachers (PLSTs)' mathematical and didactical knowledge. The PLSTs' knowledge considered in this paper concerns mathematical and didactical techniques to solve the specific tasks on the HTTs. The collective aspects, mutual engagement, joint enterprise, and shared repertoire, are considered during their discussion. The findings show that there is a link between mathematical and didactical techniques, and some didactical techniques purposed by PLSTs are too general.

Introduction

Research on pre-service teachers' knowledge on rational numbers has been studied by various researches during the last decades. Some studies focus on testing their competences related to content knowledge, such as fraction arithmetic (Bradshaw et al, 2014). Other studies give more attention to their competences on pedagogical content knowledge related to problem posing (Toluk-Uçar, 2009). There are also studies looking for relationship and differences between these areas, as known by pre-service teachers (Depaepe et al, 2015). These studies use a similar approach to access teachers' knowledge through diagnostic tests.

I consider a different approach to investigate pre-service teachers' knowledge on rational numbers. The idea is designing hypothetical teacher tasks (HTTs) (Durand-Guerrier et al, 2010) that is used to investigate pre-service mathematical and didactical knowledge, here specifically pre-service teachers' knowledge on density of rational numbers.

In this study, I do not only consider mathematical and didactical techniques used by pre-service teachers but also study the collective nature of pre-service teachers' knowledge on rational numbers. Teachers as a part of a community have a chance to work and act collectively for instance to develop common teaching resources (Gueudet & Trouche, 2012), and the collective organisation of teacher work turns out to hold important potential for improving the learning of students, as shown in comparative studies of East Asian education (Ma, 1999; Winsløw, 2012). Hence, I try to answer two questions for this study in this paper. The first question is how can HTTs be used to investigate pre-service lower secondary teachers (PLSTs)' mathematical and didactical techniques related to the density of rational numbers? and the second one, what shared praxeologies can be observed during collaborative works of PLSTs to solve an HTT about density of rational numbers?

The anthropological theory of the didactic and the collective aspects

There are two main frameworks used for this case study. The first one is the *anthropological theory of the didactic* (ATD) that is used to design the HTT about density of rational numbers and to analyse the result. The second one is the collective aspects finding during the implementation of the HTT.

The ATD is known as a general epistemological model of mathematical knowledge that can be used to observe human mathematical activities (Chevallard, 1992). The object of knowledge that will be learnt by a human related to mathematics can be identified into two aspects, a practical block and a knowledge block, which are main components of praxeological reference models. The practical block is formed by a type of task (T) and a technique (τ). A type of task (T) is a specific class of problems such as finding a number between two rational numbers. The students need a technique (τ) to solve this problem such as finding the middle numbers by adding two rational numbers and then dividing by two. Then, the knowledge block consists of a technology (θ) used to explain the practical block and a theory (Θ) to justify and reason about the technology (θ). The technology (θ) for the case is that between two different rational numbers, someone can find at least a number. While the theory of the ordered field of rational numbers, especially denseness-in-itself of rational numbers is uses to justify the technology. Those four elements (T, τ, θ, Θ) are interdependent.

When I look at back to the process of transposition of mathematical knowledge, I consider that knowledge is collectively produced in communities. In the case of learning mathematics by PLSTs in a teacher college, some mathematical knowledge does not only transpose from scholars to PLSTs, but is also shared among them. They tend to share information and experiences within the group and learning from the activity itself (Engeström, 1987).

Lave & Wenger (2012) introduce the concept of *communities of practice* (CoP) to describe a group of people sharing an interest, a craft or a profession. There are three essential conditions of CoP (Lave & Wenger, 2012): 1) *mutual engagement* (members establishing norms and building collaborative relationships), 2) *joint enterprise* (members creating a shared understanding of what are the common objectives), and 3) *shared repertoire* (members producing resources – material or symbolic – which are recognized as their own by the group and its members). I interpret the *mutual engagement* based on the theory of social norms (Yackel and Cobb, 1996) as an interaction among PLSTs during a discussion such as questioning each other's thinking, explaining their ways of thinking, working together to solve problems, solving problems using a variety of approaches, and so on. Then, the *joint enterprise* that I also interpret based on the theory of sociomathematical norm (Yackel and Cobb, 1996) is an acceptable mathematical explanation and justification by PLSTs during the discussion. Finally, the *share repertoire* is interpreted as results, mathematical and didactical techniques, for the tasks argued by PLSTs during their discussion.

The concept of density of rational numbers

The concept of density of rational numbers is closely related to the concept of infinity (Vamvakoussi, & Vosniadou, 2004). More specifically, between any two different rational numbers there are infinitely many rational numbers. The numbers that students know as concrete objects change into continuities, and some numbers are really difficult to explain to pupils.

Some pupils probably get difficulty to deal with the concept of density. It is because they just look at a finite number of different numbers between two given rational numbers. They come to the idea of discreteness as a fundamental presupposition, which constrains pupils' understanding of density (Vamvakoussi, & Vosniadou, 2004). Specifically, pupils in the discreteness–density knowledge give seemingly inconsistent answers when a teacher asks how many numbers between two rational numbers when those numbers representing in two different forms, for instance, decimals and fractions.

Arithmetic mean is one way prompting pupils to find the concept of infinity. For example, between two rational numbers a and b, there is m_0 as the arithmetic mean, repeat the procedure for a and m_0 to find m_1 and so on. It may lead the pupils to infer that there are infinitely many numbers between a and b. This is a way of approaching the notion of actual infinity in a potential manner (Vamvakoussi, & Vosniadou, 2004). Then, Brousseau (1997, p.166) explained the properties of rational numbers in order to make measurements are mostly topological properties that are related to the idea of arithmetic mean. He said between two rational numbers, we can always put a number in between, and we can measure all the intervals so obtains. Moreover, when pupils work with the operations of rational numbers tends to choose decimals instead of fractions because they allow pupils rapid calculations and a convenient representation of rational measurement. This situation probably leads pupils to the inconsistent answers that I explain in the previous paragraph.

Design of hypothetical teacher tasks

The present study is part of a pilot study, in which five HTTs about rational numbers have been tested. In this paper, I just focus on the third HTT that is about density of rational numbers. The task was chosen based on pupils' difficulties to figure out how many numbers between two rational numbers (Vamvakoussi et al, 2011). The task given to the students is originally written in Danish that was translated from English as follows:

> You ask fifth grade students how many numbers there are between $\frac{2}{5}$ and $\frac{4}{5}$, and how many numbers between 0.4 and 0.8.
>
> *Your students say that there is only one number between* $\frac{2}{5}$ *and* $\frac{4}{5}$ *namely* $\frac{3}{5}$ *; they also say 3 numbers between 0.4 and 0.8.*
>
> How do you interpret this claims? (*solve individually in 4 minutes*)
>
> Explain your ideas to teach this students? (*discuss in pairs in 5 minutes, use the space below if necessary, and write your ideas to support the discussion*)

In this study, 11 first year PLSTs from Metropolitan University College (MUC), Denmark, volunteered to work in a group of two, but a group consists of 3 students. Each group worked and discussed for 9 minutes in different schedules. All students wrote their answer on the paper for the individual task, but only few students wrote their answers for the discussion task. I also recorded their activities using video recording for the discussion task. The data was collected at Wednesday, January 6th, 2016.

A-priori analysis

The task given to PLSTs can be described into praxeological reference models. There are three possible tasks can be interpreted as follows:

T_1 = given two different rational numbers, $\frac{a}{b} = m, c_1c_2 \cdots$ and $\frac{c}{d} = n, d_1d_2 \cdots$, find how many numbers between $\frac{a}{b}$ and $\frac{c}{d}$, and $m, c_1c_2 \cdots$ and $n, d_1d_2 \cdots$.

T_2 = given two different student answers about denseness of rational numbers between $\frac{a}{b} = m, c_1c_2 \cdots$ and $\frac{c}{d} = n, d_1d_2 \cdots$, interpret these answers.

T* = given problems and student responses to the type of task T, determine what ideas as a teacher to teach students.

The first two type of tasks, **T_1** and T_2, are used to assess pre-service lower secondary teachers' mathematical knowledge about density of rational numbers. Meanwhile, the last type of task, **T***, is used to evaluate their didactical knowledge related to teach density of rational numbers.

I consider that the type of task **T_2** is interrelated to the type of task **T_1** because when someone interprets that students' claim is true, one probably will show a mathematical technique to solve the type of task **T_1**. Instead of describe each mathematical technique for both of them, I rather concern to describe the mathematical techniques to the type of task **T_1** as follows:

τ_{11} = change fractions into decimals or vice versa. e.g. 2/5 = 0.4 and 4/5 = 0.8, so there are same numbers between two decimals and two fractions.

τ_{12} = first show that there is one number between x and y (x, y representing the general terms for rational numbers such as $\frac{a}{b}$, and $m, c_1c_2 \cdots$). There exists z, so $x < z < y$, then use this to find z_1 so that $x < z_1 < z$, continue to z_2 so that $x < z_2 < z_1$, and etc.

There are some possible techniques to find z such as:

$\boldsymbol{\tau_{12a}}$ = find *z* between $\frac{a}{b}$ and $\frac{c}{d}$ using a formula: $\frac{a+c}{b+d}$.

$\boldsymbol{\tau_{12b}}$ = find z between $\frac{a}{b}$ and $\frac{c}{d}$ using a formula: $\frac{ad+bc}{2bd}$.

$\boldsymbol{\tau_{12c}}$ = if $b = d$, take a number *m* between *a* and *c*, then $\frac{a}{b} < \frac{m}{d} < \frac{c}{b}$.

$\boldsymbol{\tau_{12d}}$ = find *z* between two decimals $m, c_1c_2 \cdots$ and $n, d_1d_2 \cdots$ by considering a number between two numbers after comma. e.g. between 0.4 and 0.8, there exist for instance 0.6.

τ_{12e} = find *z* between $m, c_1c_2 \cdots$ and $n, d_1d_2 \cdots$ using a formula: $\frac{n,c_1c_2\cdots+n,d_1d_2\cdots}{2}$.

$\boldsymbol{\tau_{13}}$ = represent those numbers in a number line, find other numbers between two numbers using one mathematical technique from $\boldsymbol{\tau_{12}}$.

There is also a specific mathematical technique that can only be applied by decimals ($\boldsymbol{\tau_{14}}$) or by fractions ($\boldsymbol{\tau_{15}}$).

$\boldsymbol{\tau_{14}}$ = put 0s after decimals and show that the numbers between two decimals can be written as many as by adding 0s.

$\boldsymbol{\tau_{15}}$ = find equal/equivalent fractions for $\frac{a}{b}$ and $\frac{c}{b}$, and show that the bigger denominators, the more fractions with the same denominator to be found.

There are also possibilities that someone gives correct mathematical techniques for fractions but not for decimals or vice versa. The incorrect mathematical technique for fractions as follows:

$\boldsymbol{\tau_{16}^{-}}$ = change both fractions into the same denominator (in case they have different denominators), find some natural numbers between two numerators. e.g. between $\frac{2}{5}$ and $\frac{4}{5}$ is $\frac{3}{5}$, because 3 is a number between 2 and 4.

Meanwhile, the incorrect mathematical technique for decimals can be written as:

$\boldsymbol{\tau_{17}^{-}}$ = consider the numbers as natural numbers by omitting commas, and find natural numbers between them. e.g. there are 3 numbers between 0.4 and 0.8 because there are 3 numbers between 4 and 8.

Actually, those mathematical techniques described above are just partial techniques that can be more varied when I implement my research into a big scale research. Meanwhile, the possible technology (**θ**) to justify is that between two rational numbers, there exists at least a rational number, and I consider that it is based on the theory (**Θ**) of the order field of rational numbers.

Theorem. Whenever $q < s$ are rational numbers, there is a rational number *r* such that $q < r < s$.

Proof. $r = ½ (q + s)$ is rational and satisfies the inequality.

Corollary. We can construct a sequence of rational numbers $r_1, r_2, \ldots$ such that $q < r_k < s$ for all *k* (and, in fact, $r_1 < r_2 < \ldots$).

Those mathematical techniques lead me to describe some of didactical techniques that could be applied to solve the type of task **T***. To make it simple to recognize those tasks, I put * on the type of didactical techniques correspond to mathematical techniques such as $\boldsymbol{\tau_{11}}$*****. This didactical technique means that PLSTs explain to pupils using the mathematical technique of τ_{11}. From those, I get 7 different didactical techniques, but I also consider other didactical techniques that can be coded as $\boldsymbol{\tau_{18}}$***** and so on.

$\boldsymbol{\tau_{18}}$***** = shows to pupils that 0.*a* and/or 0.*ab* lie in between, and ask them to consider about other numbers in between two rational numbers.

τ_{19}* = explain to pupils to change decimals into percentage.

τ_{20}* = ask pupils to compare decimals/percentages and fractions through pizza experiment.

τ_{21}* = ask students to reduce fractions. e.g. $\frac{8}{10}$ can be reduced into $\frac{4}{5}$.

τ_{22}* = explain to pupils through a simple example such as how many numbers between 0 and 1.

τ_{23}* = use visual representations such as ruler and relate to measurement.

τ_{24}* = show pupils a contextual activity through dividing pizza into more slices.

τ_{25}* = introduce other contextual activities related to everyday life.

Result

A-posteriori analysis

The mathematical techniques described by PLSTs were not only taken from their answers on the worksheets but also elaborated from their discussion. The reason to do this because of the mathematical task given on the worksheet was not explicitly stated (question a), so not all of them wrote their answers to the type of task $\mathbf{T}_1$. Instead of describing the mathematical and didactical techniques separately, I consider to describe them together and show links between the mathematical and didactical techniques.

Starting from group 1 consisting two female students, student A and student B, graduating from B level. None of them wrote a mathematical technique in their papers explicitly, but student B shared a wrong mathematical technique, τ_{16}^{-} , during the discussion. She said "*it is true that* $\frac{3}{5}$ *is the only number that is not present. It is the number that we find in the middle when we say 2, 3, 4".* In the discussion, they also tried to link between teaching fractions and decimals altogether. Student A, for instance, said *"and we could take the decimals in percentage and get 40% and 80%. Then we could ask them to remove 40 % of the pizza and* $\frac{2}{5}$ *of the pizza"*. Here, I categorized those didactical techniques into τ_{19}* and τ_{20}*. Student A also wrote in her worksheet and suggested during the discussion to teach pupils how to rewrite fractions into decimals and vice versa (τ_{11}*). Both of them also argued that pupils have to find reduce fraction such as asking pupils whether they could reduced $\frac{8}{10}$ (categorize as τ_{21}*). However, even they knew that both decimals and fractions were same value and only different representations, but they still shared an agreement that there was only a number between two fractions in the end of the discussion.

Group 2 consists of 2 male students, student C and student D, who graduated from A level. There was only student D wrote explicitly two mathematical techniques to solve the type of task $\mathbf{T}_1$ on his worksheet. The first mathematical technique is clearly about finding equal/equivalent fractions (τ_{15}), and the second one I interpret as τ_{12} because he wrote $\frac{2.5}{5}$ between $\frac{2}{5}$ and $\frac{3}{5}$. Then, the first didactical technique to solve the task was τ_{11}* suggested by student C. This technique was supported by student D that he said *"We can make them try to write the other numbers as fractions, such that 0.5, 0.6, 0.7 are written as* $\frac{5}{10}$, $\frac{6}{10}$, *and* $\frac{7}{10}$, *and they also get* $\frac{4}{10}$ *and* $\frac{8}{10}$. *We get* $\frac{2}{5}$ *and* $\frac{4}{5}$ *when we reduced those and in the middle we have that 0.6 become* $\frac{4}{5}$. *We could also write (writes down on paper:)* $\frac{2.5}{5}$ *between* $\frac{2}{5}$ *and* $\frac{3}{5}$*".* The other didactical techniques could be drawn from this argumentation were τ_{12}* (related to τ_{12c}*), τ_{15}* and τ_{21}*. During the discussion, student D stated precisely that there ware infinitely many numbers, and the numbers, for instance, complex numbers.

Group 3 also consist of two male students, student E graduated from B level and student F from A level. They also did not write mathematical techniques in their worksheets, but I can imply what techniques did they use during the discussion. Student F started the discussion with the didactical technique of τ_{18}* that was showing to pupils 0.7 and 0.79 lie between 0.4 and 0.8. Then, student E suggested to give a simple example and student F supported by suggesting to show pupils numbers between 0 and 1. He said *"this is a $\frac{1}{2}$, they know that, and we can write it here. We can continue and write an interval, 0.25, they can do that. Then we can introduce those numbers now (points on the numbers 0.4 and 0.8). Then they can see that there must be numbers in between them, the same way that they saw the numbers in between 1 and 2".* From this answers, I interpret that student F had the mathematical idea of $\boldsymbol{\tau_{12}}$ that probably links to the didactical idea of $\boldsymbol{\tau_{12}}$***** and also $\boldsymbol{\tau_{22}}$*****. On the other hands, this group got difficulties for the first time to realize that there are many numbers between two fractions. They realized after student F suggested pupils to rewrite fractions into decimals. Here, they applied the didactical technique of $\boldsymbol{\tau_{11}}$*****, and finally realized that both questions are same, so there were also many numbers in between. Then, they suggested to used the didactical technique of $\boldsymbol{\tau_{15}}$***** to teach fractions as well.

Group 4 consists of 3 male students, student G, student H, and student I, who graduated from A level. Student G and student I justified pupils' mistake based on the mathematical techniques of $\boldsymbol{\tau_{16}}^-$ and $\boldsymbol{\tau_{17}}^-$. Student G also wrote in his worksheet that $\frac{4}{6}$ lies between two fractions and 0.41 lies between two decimals. I interpret that he used the mathematical technique of $\boldsymbol{\tau_{12}}$ as well. Then, the discussion started with student G argumentation about 0.41 lies between 0.4 and 0.8. To teach pupils about denseness of decimals, student H suggested to use visual representation through ruler, and this idea was supported by others. I categorize this didactical technique as $\boldsymbol{\tau_{24}}$*****. Then, student G said *"we need to be concrete about the pizza. We should divide in $\frac{2}{5}$ and then $\frac{4}{5}$ and also $\frac{3}{5}$. Then we can show them that there also a slice in between these divisions of the pizza."* This didactical technique ($\boldsymbol{\tau_{24}}$*****) was supported by other students. The last didactical technique was $\boldsymbol{\tau_{12}}$***** that I found from student H argumentation *"they have to understand the relation between decimals and fractions such that they can transform the numbers. If they struggle with one of the notations, then they can transform it to the other notation".* However, I could not see explicitly whether all of them agree that there are infinity many numbers in between two fractions and two decimals.

The last group was group 5 consisted a male student, student J, graduated from A level with 3 years teaching experiences, and a female student, student K, graduated from B level with 1 year teaching experience. During the discussion, they tended to use various contextual problems to teach pupils. The first one was to use pizza and division to give pupils an idea about numbers in between. It seems for me this didactical technique in line with $\boldsymbol{\tau_{24}}$*****. Other ideas were pouring milks from a jar to cups, and sharing chocolate bars, but they did not give clear explanation how to use it in teaching denseness of rational numbers. All of them I categorize as $\boldsymbol{\tau_{25}}$***** as part of mathematics found in everyday life. It was also stated by student J in his worksheet that students should be introduced to everyday mathematics. The other didactical idea was $\boldsymbol{\tau_{12}}$***** about change fractions into decimals or vice versa. Even this group also did not speak about infinity many numbers during the discussion, student J stated on his worksheet that between two numbers there are many numbers, and student K wrote those numbers, fractions and decimals, were just written in different ways.

Analysis for the collective aspects

By this short case study in which only 9 minutes for each group to discuss the HTT about denseness of rational numbers. I realize that it is not so easy to give a deep analysis for the collective aspects emerging during PLSTs' discussion. Based on what Lave & Wenger (2012) introduce the concept of *communities of practice* (CoP) to describe a group of people sharing an interest, a craft or a profession, I interpret this notion as how PLSTs share their mathematical and didactical techniques to their colleagues.

Without any doubt, PLSTs worked together to solve especially the type of task **T*** because I stated clearly on their worksheet to discuss in pairs in 5 minutes. One interesting part is that the way the start the discussion for sharing their thinking. I found that only a group, group 2, started the discussion by asking a question to the other student what he thinks on the case. Other groups directly started by giving his/her mathematical thinking about the task. Both shows the way they build *mutual engagement* for the discussion. Since there were only 2 students for each group except for the group 4, one student shared his/her thinking and the other gave an agreement by saying "yes" or "no" sometimes adding some argumentations or posing a questions. As an example when student F said *" Yes, we could also write 0.75 up here, and 0.75 could be placed between 0.4 and 0.8"*. Then student E said *"Yes, what about the fractions?"*. It is also one of norms that common appears during the discussion and part of *mutual engagement.* Meanwhile, there are various approaches they used to solve the type of task **T*** such as group 5 suggested to relate the problem to the real word mathematics.

During the discussion, PLSTs shared their didactical techniques to solve the tasks. Some of their techniques supported by mathematical explanation, for instance, when student C from group 2 purposed the didactical technique $\boldsymbol{\tau_{11}}$*****, student D gave a mathematical explanation related to the rewrite numbers from decimals into fractions or vice versa. Sometimes, they give justification in the the technology (**θ**) and theory level (**Θ**). It can be seen from the statement of student D that is still related to justify $\boldsymbol{\tau_{11}}$*****. Student D said *"Then they can see that the numbers are the same - they are only written in different notations. This way we see that there are many other numbers in between. There are also decimal numbers, there are infinitely many numbers"*. I categorize this process as positive joint enterprise because they come to the correct mathematical technique with a better mathematical explanation and justification. However, I also found that some students shared a didactical technique without a mathematical explanation. Group 5, for instance, gave a lot of didactical techniques by using various contextual problems such as student J said *"We could also use a plate of chocolate".* She did not give any argumentations how to use it in teaching pupils about density of rational numbers, and also her colleague did not ask for clarification and justification about it.

At the part of a-posteriori analysis, I described some mathematical and didactical techniques to solve the tasks. Those are results from the *shared repertoire* of PLSTs to show their mathematical and didactical knowledge on density of rational numbers. Actually, some mathematical techniques (table 1) were not exactly stated by PLSTs on their worksheet or during the discussion, but I draw based on the idea that when the group purposed the didactical techniques. Then, I found two common mathematical and didactical techniques that are $\boldsymbol{\tau_{11}}$ related to $\boldsymbol{\tau_{11}}$***** (change fractions into decimals or vice versa) and $\boldsymbol{\tau_{12}}$ related to $\boldsymbol{\tau_{12}}$***** (showing for two rational numbers, it can be showed at least a number in between).

Table 1. Mathematical and didactical techniques

Group	Mathematical techniques	Didactical techniques
1	τ_{11}, τ_{16}^{-}	$\tau_{11}^*, \tau_{19}^*, \tau_{20}^*, \tau_{21}^*$
2	$\tau_{11}, \tau_{12}, \tau_{15}$	$\tau_{11}^*, \tau_{12}^*(\tau_{12c}^*), \tau_{15}^*, \tau_{21}^*$
3	$\tau_{11}, \tau_{12}, \tau_{15}$	$\tau_{11}^*, \tau_{12}^*, \tau_{15}^*, \tau_{18}^*, \tau_{22}^*,$
4	$\tau_{12}, \tau_{16}^{-}, \tau_{17}^{-}$	τ_{12}^*, τ_{24}^*
5	τ_{12}	$\tau_{12}^*, \tau_{24}^*, \tau_{25}^*$

Concluding remarks

A-priori and a-posteriori analysis for the of PLSTs' knowledge on rational numbers have not been finished yet. I still realize that mathematical and didactical techniques described based on the praxeological reference models on the a-priori analysis still need to be developed and well organized. Some didactical techniques are probably similar, so it makes quite difficult to distinguee among them especially the didactical techniques from $\boldsymbol{\tau_{18}}$***** to $\boldsymbol{\tau_{25}}$*****. Far from this, I can see that PLSTs showed some mathematical and didactical techniques during the discussion even a-posteriori analysis I did were not really valid and reliable yet. I leave this condition as a challenge for developing a better praxeological reference models to analyse the result.

Meanwhile, the process PLSTs sharing their knowledge about density of rational numbers appears in the sense of collective aspects. I can not make a general conclusion for the process of *mutual engagement, joint enterprise*, and *shared repertoire* because one group has different ways to share their ideas, and of course this happens in a setting of research. Further remarks for this research is that there is a challenge to look at in deep for the collective aspects of PLSTs sharing their knowledge by design a better framework and look at the challenge from the perspective of study and research path (SRP) (Barbequero et.al, 2015)

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Transposing health in a museum exhibition

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The analysis in this paper will show how knowledge on the subject health was translated through a development process into an exhibition milieu. It investigates how the concept of anthropological didactical transposition and, hereunder, co-determination levels can be used to analyse the didactical aspects in a design process of an exhibition.

Introduction

One of the big challenges in the western world today is health, with non-communicable diseases on the rise. Sedentary behaviour, unhealthy food and addictive substances such as alcohol and cigarettes are creating populations with bad health, chronic diseases and short life expectancy (National Institute of Public Health, 2009). Numerous studies in medicine and health on how to battle these diseases have come up with a range of ‘good-practise’ behaviours e.g. the recommendation that every adult should be active min. 30 min per day (Pedersen & Andersen, 2011). However, social science studies point out that the practices of everyday life – e.g. food practices – are more tightly bound to culture and values than to rational arguments from the natural sciences (Halkier & Jensen, 2011; B. B. Jensen, 2004). It is thus possible to identify a gap between health advice based on scholarly knowledge and people’s health behaviour in everyday life.

This gap opens up possibilities for exploring ways of communicating health in alternative settings. Museums are increasingly considered a relevant setting for health interventions, given their position as informal learning environments. Little research has been done on the way health is communicated here and how visitors respond to health messages when they are on a recreational museum visit (Christensen, Bønnelycke, Mygind, & Bentsen, 2015). In this study I take health into a museum context to investigate how the scholarly knowledge is translated through the development process of an exhibition on health and movement. The aim is to uncover aspects determining didactical decisions.

Why health at the museum?

Focus is here on science centres – museums communicating hands-on science and technology experiences (Oppenheimer, 1968). Within science communication there is an increasing interest in health as an important challenge of the 21st century (Zeyer & Dillon, 2014). Science centres are thus relevant settings to investigate how knowledge on health is ‘translated’ in development processes, resulting in exhibitions.

Museums may be considered as:

> [o]bservatories on history and culture, providing a lens on the relationship between health and society over time. (…) They allow us as individuals and communities to connect with our cultures and it is these very cultures, past and present, that set the context for health in the future (Chatterjee & Noble, 2013, p. x).

The focus of Chatterjee and Nobles study is cultural and historical museums, but due to their focus on positive museum experiences with a high degree of interactions, their arguments are also valid for science centres. The hands-on nature of science centres and their natural connections to health (biology, anatomy and physiology) as subject of dissemination, makes it relevant to explore how knowledge on health is translated into an exhibition milieu.

Chatterjee and Noble’s argument that “museums are powerful agents of social enrichment” underlines their view of health as something that is more than just physiological, but also encompasses the concept of well-being (Chatterjee & Noble, 2013, p. 51). Museums are thus seen as settings that can broaden existing knowledge on health and reach new audiences through programmes that are designed in cooperation with healthcare professionals or researchers (Camic & Chatterjee, 2013). In a similar way, Green and Raeburns point out that the “responsibility for health is shared between individuals and systems” (Green & Raeburn, 1988, p. 153) and that health research today should be very much aware of how to enable people to act healthy (Green & Raeburn, 1988). Analysing this from a didactical perspective will add a new dimension to the relatively new field of museums in health.

Theory

In this paper I will use the framework of the Anthropological Theory of the Didactical (ATD), notably the notions of *didactic transposition* (DT) (Chevallard, 1992), and the *levels of didactic co-determination* (Artigue & Winsløw, 2010) to analyse how health is transposed through an exhibition development process in a science centre. The main point of DT (Figure 1) is to frame and describe the deconstruction and reconstruction of the knowledge that is transposed in the journey from (1) research findings and arguments through (2) a policy level deciding what parts of the scholarly knowledge that is important to disseminate further through the school system, (3) how this knowledge is then translated into a teaching context and finally (4) how it is then received in a learning context (Bosch & Gascón, 2006).

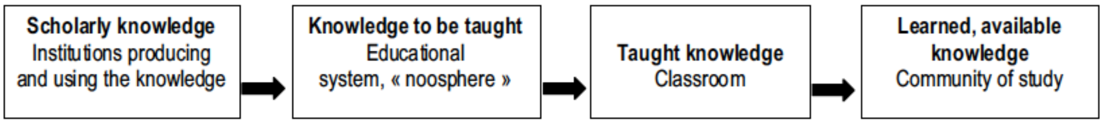

Figure 1. The didactic transposition process, step 1-4.

Working with DT thus becomes a lens for analysing what happens when knowledge is translated from one sphere to another. It emphasises the disciplinary content rather than, for example, pedagogical approaches.

Today DT is also used outside the world of mathematics education, e.g. in the field of museum research. Mortensen’s work shows how DT is a relevant tool for investigating how knowledge is transposed in museum exhibitions (Mortensen, 2010). She takes the original model of DT and adapts it to fit an exhibition development process; here, the scholarly knowledge is transposed through what she calls the curatorial brief (document(s) from the development process documenting the process and choices made) and further into the exhibition milieu (Mortensen, 2010). This approach makes it possible to deconstruct the choices that were made in the development process and reconstruct them; illustrating what scholarly knowledge is communicated in the final exhibition milieu and how.

The focus in this paper is the exhibition development process of an exhibition on health and what kind of knowledge on health is communicated in the final exhibition milieu. Using DT is therefore highly relevant because it offers didactical findings stressing the learning perspectives of the exhibition.

Levels of Co-determination

The levels of didactic co-determination is a framework that can be used to analyse the didactic transposition process with respect to the conditions and constraints that influence the curatorial process, and, notably, at what hierarchical levels these conditions and constraints originate and manifest themselves. It has been adapted to museum contexts where it serves as a way to organise and understand the various constraints and conditions that influence the exhibition design process (Figure 2) (Achiam & Marandino, 2014). It runs from a birds perspective of level 9: Civilisation and goes down to the specific level 1: Task (embodied in an exhibit). Lastly it includes level 0: Visitor knowledge; this level reflects the realisation that all communication in a museum speaks into the everyday context of the visitor and his/her previous knowledge and skills. The rationales and choices made in the transposition process can have different origins, which can be identified through the hierarchy of co-determination levels and it is in this sense an add-on to Figure 1.

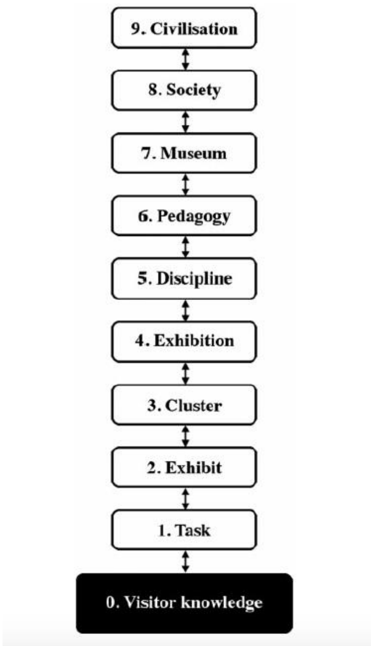

Figure 2. The levels of didactic co-determination as they apply to a museum-based science education context. From Achiam and Marandino (2014).

As part of the DT approach, the process of making the exhibition is an important object of study:

> [T]he point of departure for understanding the way science is represented in exhibitions is the realisation that the *production* and the *product* are disjunctive. That is, the science that goes into an exhibition undergoes a transformation process as it is appropriated from its origin (usually an academic science discipline), adapted to a museum context, and embodied in an exhibition (Achiam & Marandino, 2014, p. 67. Original italics).

In this paper I will use the levels of didactic co-determination to show the hierarchy position of argumentation in the development of the health exhibition and how this influenced the didactical approach chosen in the final exhibition. Following Winsløw and Artigue the levels 1-5 are direct didactical and is thus of special interest (Artigue & Winsløw, 2010). The analysis will focus on one specific exhibit (The Balance Kitchen) with respect to the levels of Exhibit (2) and Task (1).

Methodology

The transposition of knowledge is analysed through documents that have been influential in the development process. The documents used are (1) the original funding application for the exhibition project (Experimentarium & Steno Diabetes Centre, 2012), (2) an internal exhibition design report (Experimentarium, 2014) and (3) an evaluation report made on the final exhibition (Zachariassen & Magnussen, 2016).

The funding application will be used to show how the subject health was conceived of initially in the idea phase of the exhibition. I will look for theoretical approaches to the subject of health to identify the scholarly knowledge that led to the exhibition project.

The development group formulated an exhibition design report after researching the subject, going on field trips to relevant exhibitions at other science centres, having dialogues and collaborations with researchers connected to the project, and carrying out a co-design process with a selection of families (the target demographic for the exhibition). The exhibition design report will therefore be used as a document showing how the scholarly knowledge was transposed into a curatorial brief for the exhibition. Lastly I will use an internal evaluation report on the visitor's use of and experience with the exhibition to give insights into the final stage of the transposition process. I will apply the levels of didactic co-determination to explain at what levels the didactical argumentations in the development process were made.

Results

In order for the analysis of the process to make sense to the reader, it is useful to get an insight into the final product. Even though process and product are disjunctive, the development still leads to a final design, which is ever present in the development process. I will therefore start the analysis by describing the final exhibition and using it as an orientation point for how the transposition ended up.

The Final Exhibition: Taught Knowledge

The exhibition is designed with a 'daily-life narrative', meaning that each exhibit resembles something familiar from everyday life (e.g. *the balance kitchen* and *the obstacle hallway)*. The exhibition has a circular scenography with eight primary exhibits positioned around a midpoint area with screens and stools to gather around the screens: *The Midpoint*. Each of the eight exhibits is painted in a bright colour. The colour from the exhibit extends across the floor towards The Midpoint where all the colours meet to invite the visitor to engage with screens in-between their interactions with the exhibits.

To engage in the exhibition the visitors form teams of 2-5 participants and register with RFID-technology[4] bracelets at The Midpoint. After registering they can engage in the exhibits by checking in together using their bracelets. All exhibits are multi-user; engaging the whole team physically in the exhibits. Using the screens in the midpoint area the teams can take fun-fact quizzes on health with multiple-choice answers. They can see photos of themselves interacting in the exhibits, automatically taken while they where active. And they can nominate the different exhibits for being *most fun, having the greatest learning potential* and *most adaptable to my daily life*. All the information the visitors enter at the screens in The Midpoint is sent to them afterwards by e-mail.

Photo 1+2: The Obstacle Hallway and The Midpoint

The Funding Application: Scholarly Knowledge

The application[5] introduces four educational principles, which are described as central to the development process: 1. Participation and action competences, 2. A broad and positive health concept, 3. Multiple approaches for multiple settings and 4. Equity in health – reaching new target groups (Experimentarium & Steno Diabetes Centre, 2012 p.6). The application focuses on promoting a broad and positive health concept following the WHO definition of health as being "a state of complete physical, mental and social well-being

[4] Technology using radio waves to identify humans and objects. For more see for example http://electronics.howstuffworks.com/gadgets/high-tech-gadgets/rfid.htm

[5] Sent to and granted by the private Novo Nordisk Foundation, Denmark

and not merely the absence of disease or infirmity" (Organization, 2014). Much of the rationale for the project is connected with exploring new settings and approaches in health communication, and the application promotes a theoretical standpoint on health promotion that is rooted in action-based research (Kamper-Jørgensen, Almind, & Jensen, 2009). The references are mainly governmental documents and international organisations (i.e. OECD and WHO).

After introducing the main health problems facing our society, the application cites a report from the Danish national board of health (T. M. Jensen, Andersen, & Olesen, 2004) to problematize the often used moralizing language used when communicating health: "dietary habits, nutrition and exercise can have a distancing effect that fails to entice members of the target group" (Experimentarium & Steno Diabetes Centre, 2012, p. 24). To distance itself from this mode of argumentation, the application argues for communicating health in a way that is closer to daily life contexts, and avoiding a moralising and de-contexualized approach. The participation of the target group, and a cross-disciplinary collaborative approach are emphasised as important – referring to the guiding educational principles. The gap between scholarly knowledge of health and a population that does not respond to the recommendations through behavioural changes can be seen as a challenge of transposition and is something the exhibition curation aims to solve.

The specific content of the exhibition does not play a big part in the application. The only place where specific content is described is when the exhibition concept is formulated as being based on the body's engine, namely the beating heart:

> *Making visitors aware of their own pulse is a pivotal aspect of the exhibition and encourages them to make the connection between their own heartbeat and physical activities that can be easily integrated into their everyday lives* (Experimentarium & Steno Diabetes Centre, 2012, p. 15).

The application is different from many preliminary descriptions of exhibition projects in science centres, since the content is described more in terms of principles than natural phenomena. The active involvement of the target group in the development process and collaboration between research and practice are highlighted as central. The assumption being that this proximity of daily life will lead to a product that will be perceived as relevant and recognizable by the future visitors.

Thus the first step of the transposition is influenced mainly at the top three levels of co-determination and deals with conditions about the exhibition: 9. Civilization, 8. Society and 7. Museum. Going from the challenges of the 21st century, citing WHO, ministerial documents and OECD and over to why the museum is a relevant setting for working with these challenges. It is as such suitable to have these levels in the funding application since it deals with overall argumentations for the exhibition.

The question of how the project should be carried out is influenced mostly by conditions originating at the next three levels and takes up less space in the application. Level 6. Pedagogy is present in the four educational principles of participation and cooperation. Level 5. Discipline is health and I will argue that the level 4. Exhibition in this case is health promotion as exhibition subject, since this is what is most important in the funding application: to communicate health in this specific manner. The lower levels are not present in the application. The didactical aspect is thus not very present in the application.

The Exhibition Design Report: Curatorial Brief

The exhibition design report is a thorough description of the final concept for the exhibition with floor plan, graphical elements, the final exhibits and formalities.

The aim of an ideal user-experience in a visit is described happening:

> *With a whimsical twist, the activities in the exhibition provides the family with a relevant, entertaining challenge and experience of moving, and the physical activity level is high*

(Experimentarium, 2014, s. 4).

It ends the description by inviting the family to go to The Midpoint and work with their own ideas:

> *In The Midpoint there is a lower pace and activity level, the family must in different ways digest the experiences from the activities (Experimentarium, 2014, s. 5).*

Several times in the report, the theme of shared family experiences and identifiable elements are highlighted. The whimsical universe is to be achieved by using shifted size ratio, twisted walls and bright colours.

In regards to the co-determination levels, all exhibits (level 2) are described and the specific tasks listed (level 1). The eight exhibits together with The Midpoint area form the cluster (level 3), given this is how the content of the exhibition is broken down into manageable chunks (Achiam & Marandino, 2014). Especially The Midpoint area is of interest since it should serve as a space where reflections on health can take place (level 5).

The text on the specific exhibit *The Balance Kitchen* has a description of a prospective task for families: a kitchen where you have to turn out as many buttons with light on a batak wall[6] in two minutes without touching the floor. The exhibit is a joint competition on balance and coordination skills. The design of the exhibit (level 2) is the result of a manifestation of the discipline health (level 5) and pedagogy: participation, hands-on activities (level 6). The main health-related objective with the exhibit is described as 1. *"Giving the family an entertaining shared experience that puts focus on the skill of balance (and secondly muscle strength and swiftness)"* and 2. *"Provide inspiration for and dialogue about the possibilities for moving at home/indoor in everyday life."* (Experimentarium, 2014, appendix 3, p. 4). Knowledge from discipline (level 5) and pedagogy (level 6) are thus essential for the final dissemination of the subject health at the exhibit (level 2) and task (level 1). In line with the educational principle of participation, the activity accommodates wishes from the families who had been involved in the development process, such as the children's idea of challenging the arm muscles by climbing in the curtains. It also integrates the mothers' favourite discipline of tightropes, using one's balance. The exhibition design report emphasises didactical aspects of the exhibition, by focusing on the lower 5 levels of co-determination. It is here described how the science centre chose to communicate health.

[6] Concept designed to improve reaction, hand eye coordination and stamina.

Photo 3: The Balance Kitchen

The Evaluation Report: Learned Knowledge

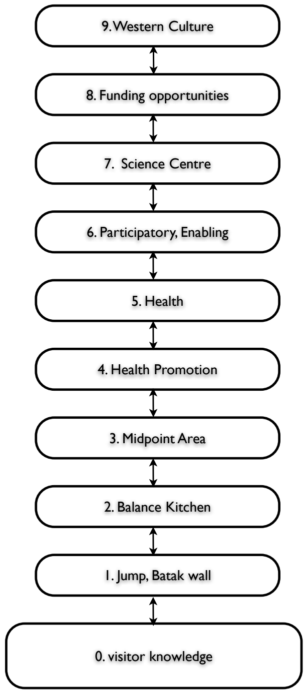

Figure 3. Levels of Co-determination in the health exhibition

An evaluation report on the exhibition was carried out by research staff at the science centre based on a questionnaire survey on visitors at the science centre (Zachariassen & Magnussen, 2016). It was made six months after the exhibition opened and based on answers from 81 families. The questionnaire was made on ipads in connection with a visit at the science centre. The report concludes that the health exhibition is the most popular exhibition with The Balance Kitchen being the most popular exhibit among the visitors (Zachariassen & Magnussen, 2016, p. 30). 73,2% of the family respondents in the questionnaire say the exhibition is relevant for their everyday life; 52,9% state they have gained new knowledge on movement, and 44,1% state they have been inspired about how to move more in everyday life (Zachariassen & Magnussen, 2016, p. 8). When asked if the exhibition is entertaining 95,1% of the visitors respond that it is above average in entertainment. Being together as a family and be active is mentioned as the top reasons for why it is entertaining (Zachariassen & Magnussen, 2016, p. 27).

The exhibition thus seems to be successful in living up to the four educational principles initially formulated in the funding application. The families are having a fun and active shared experience. However, on the question of whether the family has had a discussion of health during their visit only 29,8% respond positively (Zachariassen & Magnussen, 2016, p. 37). The report concludes that one of the reasons for this is that the family dialogue going on during the visit in the exhibition is very much on the practical tasks related to the interaction such as ‘touch here’ (level 1). Also, the results from the report show that the fun-fact quizzes in the Midpoint are not being used as much as intended. This is problematized in the report since the fun-fact quizzes where designed to be a room for a more information and discipline-based communication (level 5). Whether the families talk about health after their visit is not possible to say, but it could be assumed to be a possible outcome of the science centre visit, given that the evaluation report concludes that the families understand that the exhibition is about health, and the report shows that the families spend a third of their total visit time in the health exhibition (Zachariassen & Magnussen, 2016).

The evaluation report concludes that the exhibition is perceived as being relevant to the visitors’ everyday life and that they participate actively. This corresponds with the arguments from pedagogy of participation and hands-on activities (level 6). *The balance kitchen* is reported as being the most popular and it can therefore be assumed that the exhibit (level 2) and task (level 1) are designed in a way where the hands-on principle (level 6) and the approach to the discipline (level 5) through a health promotional directive (level 4) are transposed in a manor attractive and motivational to the visitors.

In figure 3 the co-determination levels specific to the health exhibition is listed true to the original categories as illustrated in figure 2.

The evaluation report mainly responds to the exhibition design report and keeps its scope on the specific exhibits and the visitors personal experiences with them.

Discussion

It seems that a health promotional approach can be tricky when developing a science centre exhibition. In a setting where there often is right or wrong answers it can be difficult to disseminate this more social and mental perspective of health relations and feeling healthy. The way it was done in the present exhibition, where the visitors are forced to experience it together and the physical embodiment is used as a prime facilitator, seems to be successful in disseminating how one can feel healthy. The exhibition thus position itself as being very successful at the level of pedagogy (level 6) through creating shared and enabling experiences and at level of exhibition (level 4) since the health promotional approach is evident in the exhibition. A challenge remains however in the more classic didactical level of cluster (level 3) where the evaluation report shows that visitors do not make very much use of The Midpoint and the connected fun-fact quizzes. The aim of the quizzes was to get families to reflect on their health behaviour and link their own behaviour at home with knowledge on how to gain a healthy body. From a more traditional position within science centre exhibition development it could therefore be argued that the visitors do not learn facts in the health exhibition, but simply have fun together. From a health promotional position however, the positive, shared experience of being active is a first and important step in becoming able in one's own health situation. The positive and shared experience of ones body was also highlighted by for instance a mother who rated the experience 10 out of 10 in relation to being together and she told us she was surprised by her own accomplishments: *"I could jump! I didn't know I could do that."*

Conclusion

The notion of didactic transposition is a valuable lens for analysing how a specific approach to health – health promotion – was ultimately disseminated in a science centre exhibition and how the journey to the final product took place as seen from a didactical perspective. The DT model (figure 1) made it possible to divide the development process into manageable categories for analysis in relation to where in a didactical model they were placed. By adding the levels of co-determination the categories could be placed hierarchic to shed light on what didactic argumentations were present. The subject health (promotion) was ultimately taken from initially focusing on addressing societal health challenges into more tangible and teachable exhibits (level 2) and tasks (level 1) by transposing it into a framework to creating a shared family experience based on using one's body.

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A collective aspect of mathematics textbook production based on the anthropological theory of the didactic

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Mathematics textbook is often seen as a resource in teaching and learning mathematics. However, how mathematics textbook is produced has not been studied intensively. The aim of this paper is to see the development of mathematics textbook production in Japan and in Indonesia. We propose the anthropological theory of the didactic as a theoretical framework, especially using a didactic level of co-determination. Additionally, we used both literature review and interview as data collection. The results show that even though textbooks in both countries are authorized by the government, the application of textbook production are different.

Introduction

Example and exercise is an important part of textbooks because tasks are the bedrock of classroom life Watson and Ohtani (2015). It is also commonly accepted that by focusing on the practice part, students can prepare theirself to face the national examination. Therefore, teachers can consider choosing a textbook based on the quality of tasks. However, beside of practice part, the link between theory and practice in mathematics textbooks is also important. Additionally, the link between a topic and others topic is also important part that should be analysed in choosing textbooks. Otherwise students will only visit on what the framework of the anthropological theory of the didactic is called 'visiting monument' for each of those topics (Chevallard, 2012).

Sure, the aforementioned paragraph is the ideal way to choose a textbook. In reality, sometime teachers focus more on teaching. Thus, it is not commonly secret that the reason for choosing textbooks is more regulation reason rather than professional reason. For example, teacher prefers to use authorized textbooks from government. Does this authorized textbook assure teachers to have a prominent companion? Teachers (and researchers) can do a content analysis of textbooks. However, each textbook's author has different approach to introduce a mathematical content. What does influence the author in writing a textbook? What kind of collective design that is happening in textbook production?

This paper is based on the anthropological theory of the didactic (ATD) as a theoretical framework. The aim of this paper is to develop some related question for analysing collective work of textbooks production. Concretely, we will present some "dimensions" that influence textbook production.

Collective design using ATD

In more broader point of view, Winsløw (2012) investigated two different didactic systems between Japan and Denmark. Lesson study is a well known collaborative study from Japan where a group a teacher can plan, observe, and evaluate the lesson. On the other hand, the noticeable context that can be seen in Denmark is a multidisciplinary teacher team. The major reform in 2005 reform required students to build a combination of at least two major disciplines. Considering lesson study and multidisciplinary teacher team as two different didactic systems, he used three components of paradidactic system, for example predidactic system, didactic observation system and post didactic system as a tool to compare these two didactic systems. One of his results shows that the nonexistence of opportunities to have observation of teaching peer in Danish school is the main differences between strong collaboration in the Japanese lesson study. These two different collaboration didactic systems might because different condition and constraint that establish in two different contexts and institution. Additionally, this knowledge about comparative study is pertinent in didactic research. Obviously, this collective aspect was done by observation, using the anthropological theory of the didactic. Then, this study drives me to wonder how to treat different data (script dialogue) of collective aspect, using ATD.

A recent study about script dialogue analysis is done by Berta Barquero (2015). One of their focus is to analyse dialectic and collective dimension of study research path. The teachers are asked how to teach mathematical modelling. They use specific forum that turns into the transcript. As a result, the schema of question and answer, called the mathematical skeleton is raised. This research shows us that we can use ATD to analyse the collective aspect of study research path, especially, using transcript dialogue. Then, can ATD be used to analyse a collective aspect of data interview?

Textbooks author's perspective

The research in mathematics textbooks is still constrained by textbook analysis, textbooks use, textbooks comparison, electronic textbooks and relationship between textbooks and student achievement (e.g Fan, 2013). These research areas show us that textbook is still treated as a product that is needed to be analysed. Oppositely, research area on textbook author's perspective has not been analysed yet. For example, author perspective is an important part of textbooks production and mostly there is more than one author in a production of textbooks. Then, knowing a shared knowledge/perspective between the textbook's authors in a production of textbooks is also an interesting part to analyse.

Research regarding textbook author's perspective on textbooks in mathematics education area is very limited. A related reference is conducted by Gunstone, Mckittrick, and Mulhall (2005) who interviews textbooks author of senior high school physic textbooks regarding their understanding about direct current (DC) electricity. Gunstone et al. (2005) divide interview into two parts with three focus question in each part. The questions in the first part are related to more general physics textbooks. The authors are asked about how they each saw student learning of physic. They are also asked the ease and difficult of writing, Furthermore, they are asked the model and analogies and the meaning of student learn their textbooks. In the second part, the authors are given questions focused on electricity concept, for example ‘current held and voltage’. The result of the first part shows that the three authors do not have any specific view about student learning physics from textbooks. Also, the authors see the different perspective regarding ease and difficulty in writing the textbooks. The authors also do not convince when they are asked by the meaning of model and analogy. The result of part two shows that the textbook authors have different understanding about current held and voltage. The different perspective of the author shows that there is no shared perspective among textbook author. This factor affected by the way they write a textbook and the way student perception of textbooks. Furthermore, the right understanding about knowledge is important for the student. This research result can be used to develop rules and obligation for author writer. More than that, the theoretical framework part is missing in this research. Thus, a precise theoretical framework is important to see the result. Additionally, research in textbooks production is also conducted by Randahl (2012), Lee and Catling (2016) and Lee and Catling (2016).

ATD and context

In ATD (Chevallard, 1999, 2002) perspective, mathematics cannot be seen as one single identity. Mathematics is learnt in didactic scene due to influence of many aspects for example scholarly mathematics. The authority has to adapt high level of mathematics from scholarly level to pupil by adjusting, simplifying and etc. (c.f Chevallard & Sensevy, 2014)). Thus, in ATD, we recognise on what we call a didactic transposition. Here, mathematics as subject experiences an evolution of scholarly mathematics to knowledge to be taught. This research is located between scholarly knowledge and knowledge to be taught (figure 1).

In order to analyse a topic using ATD, a didactic level of co-determination is needed. we analyse regulation of textbook and textbook authors only in civilisation and society level. In this sense, we were looking for a textbook regulation in ministry of education level. Then, we compare how this regulation is applied in the publishing company. We also analyse the perception of textbook author (publishing company). In this case, we used literature in Japanese case and we did interview with mathematics lower secondary textbooks author (and reviewer) in the Indonesian case (Figure 2).

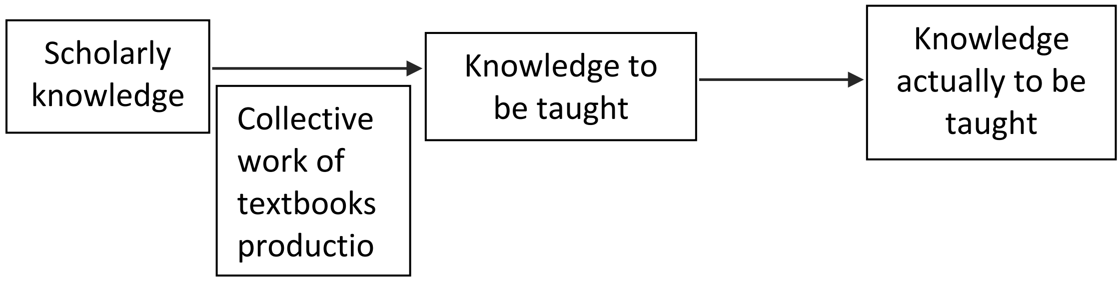


Figure 1. Didactic transposition and area of interest

We mainly asked two question: 1. How do you collaborate with other author (in fact it require more than one author in writing a textbook) 2. What do you think of good textbooks is. We interviewed a textbooks author which is also a lecture in a mathematics education department. Also, we did an interview with a reviewer of a textbook which is also a lecture in a mathematics education department.

Civilisation	Government (ministry of education) Publishing company
Society	
School	How the school choose a textbook
Pedagogy	How textbook is used in classroom
Discipline	Mathematics
Domain	Geometry
Sector	Plane geometry
Theme	Area of polygon
Subject	e.g. given a picture of polygon with the sides, determine the area of triangle.

Figure 2. level of codetermination of textbooks production

Results

We will divide the result in to two categorize. Firstly, a literature review regarding collective design of Japanese textbooks production. Secondly, a literature reviews and interview result pertaining Indonesian textbooks production.

A collective work by Japanese author

Japanese textbooks are authored by government. This can be seen in the regulation that said that only those textbooks that have passed an examination conducted by the Minister of Education, Culture, Sports, Science and Technology may be used in school curriculums (Ministry of Foreign Affairs of Japan, 2016a). Furthermore, the government also require textbook author to make a team. Ministry of Foreign Affairs of Japan (2016b) says that publisher gathers a team of academics and school teachers.

Pertaining to the authors member, Shimizu and Watanabe (2010) stated that each publisher employs a team of authors (university professors, mathematics supervisors for local education agencies or classroom teachers) to create its mathematics textbook series. Shimizu and Watanabe (2010) also added that the fact that textbooks are written by a team that includes a number of experienced classroom teachers ensures that textbooks reflect the reality of classrooms. It also has been said in Lucien Ellington (2003) that teacher collaboration and involvement is a crucial component of publishing company effort throughout the process.

Problem solving has been a trend topic in Japanese textbooks. However, Takahashi (2016) state that a large wave of teacher retirement in recent years has left newly hired teachers without the collegial support they need to develop the expertise to teach through problem solving. Takahashi (2016) added the textbooks developed resources to help teachers teach through problem solving and to help students learn through problem solving. For example, the book provides more alternative approaches for finding the area of the same shape by using diagram and story.

A collective work by Indonesian author

Badan standar nasional pendiikan is a government organisation who authorized textbooks. This organisation also require more than one author in writing a textbook (Badan Standar Nasional Pendidikan, 2014).

However, based on the interview data, we found that there is textbooks author who write individually for whole three series of lower secondary textbooks. Additionally, we asked about their perception on how a good textbook is. He answered that a good textbook focuses on problem solving. He also added a task that also appears in his textbooks:

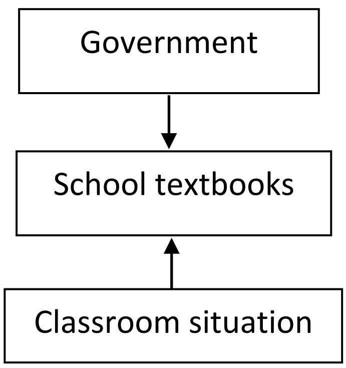
T: given the original area of circle a cm. Determine a new area of circle which has two times area of the original circle.

In other hand, we also did some interview with the reviewer of textbooks and ask for his view about a good textbook. He answered that a good textbook is textbooks that align with curriculum.

Discussion

From the two cases from Japanese textbooks process and Indonesian textbooks process, we can see that, Japanese textbooks consider on what happened in class room situation and adapt in the textbook. Even thought in Indonesia curriculum also focus on problem solving and require more than one authors, the Indonesian author /reviewer more focus on curriculum (especially problem solving task).

It seems that the collective design in japan include both side (government and classroom condition). However, in Indonesia, the textbook production seems focus on government (figure 3 and figure 4).



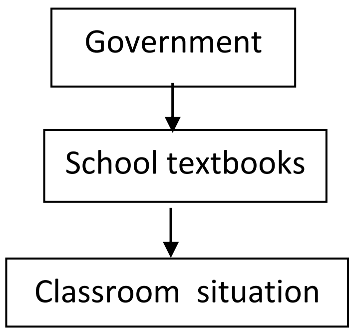


Figure 3. Textbooks production in Japan

Figure 4. Textbooks production in Indonesia

Conlusion

Both Japanese and Indonesian public school textbook are authorized by government. Both of the government also require the authors to do a collaborative work between teacher and university researcher (and professional author). However, both countries have different way to construct their textbook. One Indonesian author state that he prefers to work individually. This condition leads a different priority to define a good textbook. From two interviews, we found that an author more likely to focus on constructing on what he called ‘problem solving’ task. In the other hand, a textbooks reviewer prefers to define a good textbook when it attempt to match the curriculum. Different form Indonesia, teacher collaboration with textbooks author is a crucial part of textbooks production in Japan. As a result, textbooks represent what is needed in the school practice.

This study shows that there are two different ways to construct a textbook in two different countries, even though both country mostly have the same regulation. We presume that collaborative work is easily founded because of lesson study. The collaborative work between university researchers and school teacher is the reason why Japanese textbooks is more adaptive to the school situation. Event thought we can find a collaborative teaching in Indonesia, but it is not largely founded. This two different society could be a reason why the two country has different tradition of collaboration.

Of course, this comparison only seeks the collective process of textbook production. And surely, this task overlooked the comparison on how these textbooks discuses certain topic. Thus, we recommend for further research to consider a comprehensive a didactic level of co-determination to analyse the collective work of textbooks production.

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Appendix A: Schedule of the course

Day 1 **(Monday 16/11)**
9:00-10:00 Welcome and introduction to the course
10:00-11:00 Lecture by Marianne Achiam
11:00-12:00 Group work
13:00-16:30 Participant presentations by David Liam Johnston, Ignasi Florensa Ferrando, and Rune Hansen
16:30-18:00 Seminar led by Marianne Achiam and Carl Winsløw

Day 2 **(Tuesday 17/11)**
9:00-10:00 Lecture by Marianna Bosch
10:00-11:00 Lecture by Dagny Stuedahl
11:00-12:00 Group work
13:00-16:30 Participant presentations by Dyana Wijayanti, Eliza Estrup, and Ingrid Eikeland
16:30-18:00 Seminar led by Marianna Bosch and Dagny Stuedahl

Day 3 **(Wednesday 18/11)**
9:00-10:00 Lecture by Carl Winsløw
10:00-11:00 Group work
11:00-15:00 Participant presentations by Louise Windfeldt, Louise Meier Carlsen, and Yukiko Asami-Johannson
15:30-17:00 Seminar led by Carl Winsløw and Marianne Achiam

Day 4 **(Thursday 19/11)**
9:00-10:00 Lecture by Dagny Stuedahl
10:00-11:00 Group work
11:00-15:00 Participant presentations by Zetra Hainul Putra, Jacob Bahn, and Catharina Thiel Sandholt
15:30-17:00 Seminar led by Dagny Stuedahl

Day 5 **(Friday 20/11)**
9:00-10:00 Lecture by Marianna Bosch
10:00-11:00 Group work
11:00-14:00 Participant presentations by Fernanda Vidal and Britta Jessen
14:00-15:00 Seminar led by Carl Winsløw and Marianne Achiam
15:00-15:30 Closing and assignments

Appendix B: Course readings

Barquero, B., Bosch, M., & Romo, A. (2015). *A study and research path on mathematical modelling for teacher education*. Paper presented at the 9th Congress of European Research in Mathematics Education, Prague, Czech Republic. 10 p.

Calabrese Barton, A., & Tan, E. (2010). We be burnin'! Agency, identity, and science learning. *Journal of the Learning Sciences, 19*(2), 187-229.

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Gueudet, G., & Trouche, L. (2012). Communities, documents and professional geneses: Interrelated stories. In G. Gueudet, B. Pepin & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development* (pp. 305-322). Dordrecht: Springer.

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Appendix C: Course participants

Yukiko Asami-Johansson	Högskolan i Gävle; Sweden
Rune Hansen	University College Syddanmark, Denmark
Ignasi Florensa Ferrando	La Escola Universitària Salesiana de Sarrià, Spain
Ingrid Eikeland	University of Life Sciences, Norway
Fernanda Vidal	University of São Paulo, Brazil
Dave Liam Johnston	Montclair State University, USA
Louise Windfeldt	University of Copenhagen, Denmark
Louise Meier Carlsen	University of Copenhagen, Denmark
Jacob Bahn	University of Copenhagen, Denmark
Zetra Hainul Putra	University of Copenhagen, Denmark
Dyana Wijayanti	University of Copenhagen, Denmark
Eliza Estrup	University of Copenhagen, Denmark
Britta Eyrich Jessen	University of Copenhagen, Denmark
Catharina Thiel Sandholdt	University of Copenhagen, Denmark