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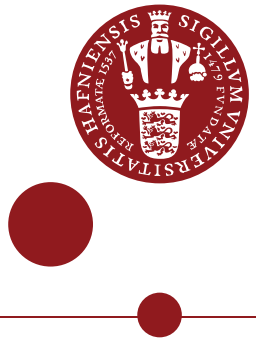

# Master Thesis

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# How many people have ever lived?

A study and research path

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# Abstract

This thesis describes the testing and analysis of a Study and Research Path (SRP) on a classical question from mathematical demography. The hypothesis of the study has been that an SRP on demography can give high school students an insight into how mathematical methods can be used to answer questions in the social sciences. Specifically the SRP concerned the generating question: "*How many people have ever lived?*".

The Anthropological Theory of Didactics (ATD) has been used as a theoretical framework for the study and an epistemological reference model (ERM) has been developed. We have first performed our own study and research of the question and found that it can be answered in multiple ways. The most prominent methods focused either on modelling yearly births as a function of time in order to calculate a definite integral or on modelling the population size as a function of time in order to calculate the integral and divide this with an estimate of an average life expectancy. We have then designed a teaching sequence and made an a priori analysis of it. In the a posteriori analysis we have focused on which questions the students developed, how students used media and students' praxeologies regarding integration and interpolation.

We have found that students developed questions similar to those in the a priori analysis which could be divided into three branches: counting deaths, counting births and counting "person-years". The students mainly studied media to gain information on the origins of man and to find demographic data. Only little media on the methods for calculating an answer was found and it was not studied in depth. A better study of such media might have given a greater variety in successful methods used by students. Regarding both interpolation and integration students almost exclusively used techniques involving CAS. We have found that students know the praxis parts well but that their knowledge of the theory on both regression and integration is lacking. Other studies have suggested that more focus on giving students an interpretation of the definite integral as a Riemann sum may alleviate the problem.

# Resumé

Dette speciale beskriver en afprøvning og analyse af et studie og forskningsforløb (SFF) om et klassisk spørgsmål fra matematisk demografi. Studiets hypotese har været, at et SFF om demografi kan give gymnasieelever et indblik i, hvordan matematiske metoder kan anvendes til at besvare spørgsmål indenfor samfundsvidenskaben. Specifikt har SFF'et omhandlet det genererende spørgsmål: "*Hvor mange mennesker har levet på jorden?*".

Den Antropologiske Teori om det Didaktiske (ATD) har udgjort studiets teoretiske ramme, og en epistemologisk referencemodel (ERM) er blevet udviklet. Vi har først udført vores egen undersøgelse af spørgsmålet og fandt frem til, at det kan besvares på flere måder. De mest fremtrædende måder fokuserede enten på at modellere det årlige fødselstal som funktion af tiden for derefter at beregne et bestemt integral eller på at modellere populationsstørrelsen som funktion af tiden for derefter at beregne integralet og dividere dette med en estimeret gennemsnitlig levelalder. Vi har designet et undervisningsforløb og foretaget en a priori analyse af det. I vores a posteriori analyse har vi fokuseret på, hvilke spørgsmål eleverne udviklede, hvordan eleverne brugte medier og elevernes prakseologier angående integration og interpolation.

Vi fandt ud af, at eleverne udviklede spørgsmål, som svarede til dem fra a priori analysen. Disse kunne opdeles i tre grene: tælle døde, tælle fødsler og tælle "menneskeår". Eleverne studerede hovedsagligt medier for at opnå viden omkring menneskets oprindelse og for at finde demografisk data. De fandt kun få medier omhandlende metoder til beregning af et svar, og disse blev ikke studeret grundigt. Et bedre studie af sådanne medier kunne måske have givet en større varietet i de succesfulde metoder, som eleverne brugte. Angående både interpolation og integration har eleverne næsten udelukkende brugt teknikker, som involverer CAS. Vi har fundet ud af, at eleverne har et godt kendskab til praksisdelene, men at deres teortiske viden omkring både regression og integration er mangelfuld. Andre studier har foreslået, at mere fokus på at give eleverne en fortolkning at det bestemte integrale som en Riemannsum muligvis kan afhjælpe problemet.

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# Chapter 1

# Introduction

Mathematical demography and its mathematical and statistical modelling of human population is an interesting field of research. It's a field that lets us calculate answers to various more or less relevant or interesting questions related to the social sciences. Because of this we thought this field would create a good frame for testing a Study and Research Path (SRP). Our interest was in seeing whether an SRP on a classical question in mathematical demography could help the students understand how mathematical methods could be used to answer questions that are relevant in other subjects than mathematics and as such give the students a reason to study the necessary mathematical methods.

Our own experiment with an SRP was inspired by Kuzuoka and Miyakawa who have tested an SRP in Japanese lower secondary school with the generating question: *When is the number of all people in the world who have lived until the year 1900 equal to the number of people after the year 1900?* [Kuzuoka and Miyakawa, 2018, p. 4]. This question was a modified version of a question that Yves Chevallard and Marianna Bosch proposed at a workshop on the Anthropological Theory of Didactics (ATD): *They say there are more people now alive that all that have lived before 1900. Is that possible? On which year could this possibly be true?* [Kuzuoka and Miyakawa, 2018, p. 5]. We wanted to experiment with an SRP based on our own simple modified version of the question: *How many people have ever lived?*. Our experiment was to take place in a Danish high school setting where the attitude towards the working format may be different than that experienced in the Japanese secondary school and where the older students may apply different techniques in their mathematical treatment of the question.

An SRP based on this question should allow the high school students to work with modelling the population and also integration will be a useful mathematical tool and hopefully the reason to study and learn these tools and methods will be clarified for students as they work with such an SRP.

The thesis contains an a priori analysis of the SRP that is based on our own study and research of the question and an a posteriori analysis and a discussion of the result of testing the teaching sequenc.

# Chapter 2

# Theory

The anthropological theory of didactics (ATD) will constitute the theoretical framework for this thesis. ATD as a program of research was initiated by Yves Chevallard in the 1980s and fits in the Epistemological Program in didactics of mathematics [Bosch and Gascón, 2014]. It provides an epistemological model where mathematical knowledge "*... is seen as human activity of study of types of problems*" [Barbé et al., 2005, p. 236]. It carries the notion of didactic process which models how mathematical activity and knowledge is created [Barbé et al., 2005] and a key characteristic of ATD is that it emphasizes that the didactic transposition of knowledge between institutions such as universities and high schools influences what is taught and learned [Bosch and Gascón, 2014].

## 2.1 Mathematical organizations

According to ATD mathematical activity contains two aspects, praxis and logos, which combine to what is called a praxeology. Praxis refers to the practical block (know how) and logos refers to the knowledge block. The practical block is made of types of tasks, $T$, like solving an equation or making a proof and a set of techniques, $\tau$, which can be used to solve the tasks. The practical block of a praxeology can be noted $[T/\tau]$. According to ATD such mathematical activity will usually be accompanied by an environment that describes, explains and justifies the practical block [Barbé et al., 2005]. This is the logos or knowledge block and it contains two levels. The first level is the technology, $\theta$, which refers directly to the technique describing and explaining it, while the theory, $\Theta$, describes and justifies the practical block on a higher level [Barbé et al., 2005]. So a praxeology also called mathematical organization, MO, is made of the four T's: task $T$, technique $\tau$, technology $\theta$ and theory $\Theta$.

If an MO refers to a unique type of task it is called punctual. Some punctual MO's may combine to a local MO if the same technology can be used to describe and justify the techniques used and at a higher level we have a regional MO where the same theory justifies the technology. One punctual MO may belong to different local MO's depending on which punctual MO's are integrated and what technology is then used to justify the techniques. In the same way local MO's can belong to different regional MO's [Barbé et al., 2005].

As an example we will use this terminology to describe an $MO$ concerning integration as it could appear in HTX (Højere teknisk eksamen). According to the

syllabus as presented in the teaching plan students need to learn about antiderivatives, definite integrals and area- and volume calculations. It is noted that applied aspects of mathematics are important [Undervisningsministeriet, 2017].

The $MO_i$ that we will set up therefore deals with Riemann integrable functions and poses the problem of how to calculate areas and volumes that are defined by them but since the applied aspect is important we will also consider problems of determining the total change in a quantity whose rate of change is known. An example of this is the total number of births when births per year, $B(t)$, is known. The two main types of tasks defining an $MO_i$ are then:

$T_{i1}$: Determine an area defined by the graphs of two given continuous functions.

$T_{i2}$: Determine the total change in a quantity whose rate of change is known.

The techniques for both types of tasks are then similar. A technique, $\tau_{ia}$, for solving the problems could be to determine the definite integral and then use CAS to calculate it. Another technique $\tau_{ib}$ could be to perform a manual calculation of the determined definite integral. Under $\tau_{ib}$ belongs various special techniques like knowledge of antiderivatives of different functions and substitution whose use depends on the functions in question.

The corresponding technology, $\theta_{ia}$ then concerns instructions on how to use CAS for the calculations and for $\tau_{ib}$ it contains a discourse on the use of the rules of antiderivatives of different functions and how to use substitution.

The theory, $\Theta_i$ that justifies it then concerns the Fundamental Theorem of Calculus, definitions of definite integrals and how to interpret areas of $\mathbb{R}^2$. FTC together with the definition of definite integrals justifies that area can be calculated using definite integrals. For $T_{i1}$ it suffices to interpret the definite integral as an area but for $T_{i2}$ it can be advantageous to interpret it as a sum of "very small pieces" (Riemann sum interpretation) which can be motivated by defining the integral as the limit of a Riemann sum. A Riemann sum interpretation is probably more useful for students in most applied contexts [Wagner, 2017].

In relation to the sum interpretation another task within the same regional $MO_i$ that could be interesting to consider is:

$T_{i3}$: Given some points of a monotone function determine a lower and an upper sum approximation.

And since some students may interpret the definite integral as the area bounded by the average value of the integrand a task of type:

$T_{i4}$: Calculate the average value of a function.

With technique

$\tau_{ic}$: use the formula $\frac{1}{b-a}\int_a^b f(x)dx$

is also an interesting task in $MO_i$

We will also set up an epistemological reference model, $MO_r$ relating to interpolation of a set of data. We will here consider the task $T_r$: *Perform interpolation on a set of data.* We will consider two different techniques:

$\tau_{r1}$: Use CAS to perform a regression analysis on the data.

$\tau_{r2}$: Manually determine either the linear or exponential function between each pair of neighboring data points.

The technologies are then respectively about how to perform the regression analysis and about using formulas for determining linear/exponential functions between two points.

The theory is then either to consider nature of the data to determine the kind of regression analysis and then considering the R-value to evaluate the fit or it is to consider nature of the data to determine the kind of function and then use knowledge on the deduction of the used formulas to justify them. There is also an interpretation of what the function is in relation to the data and what it can be used to say something about.

## 2.2 The Didactic Transposition

An important part of ATD is the institutional relativity of knowledge [Bosch and Gascón, 2014]. In order to interpret school mathematics one needs to consider the reconstruction of the mathematics that comes from institutions that produce mathematical knowledge [Bosch and Gascón, 2006]. So one has to consider that knowledge transforms several times as it moves from production by for example researchers at a university to learned knowledge by a student in a classroom. This transformation is referred to as the didactic transposition. The steps between institutions can be seen in 2.1 below.

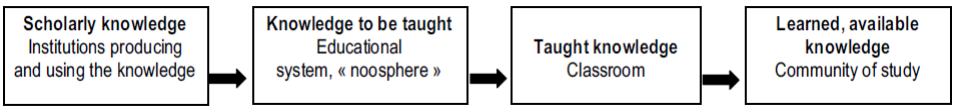


Figure 2.1: Didactic transposition [Bosch and Gascón, 2006, p.56]

The first step concerns the formation and choices of the MO's to be taught. The result of this step can be seen in syllabuses, textbooks, teaching plans etc.

The idea is then that a researcher in didactics of mathematics should not restrict her epistemological model of the relevant body of knowledge to a model seen in one institution. The scholarly knowledge for example should not be the only reference for school mathematics. Instead the researcher needs to create her own epistemological reference model, ERM of the knowledge so that she is freed from epistemological models of the institutions. From this reference model she can then interpret empirical data she might collect from the different institutions [Bosch and Gascón, 2014, Bosch and Gascón, 2006].

As we will see in the next chapter praxeologies relating to both integration and interpolation are very important in the tested teaching sequence. These praxeologies will have to be reconstructed by the students participating. In order to see what the praxeologies within these two areas specifically look like in the noosphere, that is, to see the praxelogies that are usually expected for students to work with we will analyze four randomly chosen exam sets from the last three years. The relevant exercises from the exam sets can be found in appendix D.

The example above about $MO_i$ and $MO_r$ will be our reference models for the analysis.

## 2.3 Integrals in HTX exam sets

In three of the four analyzed exam sets there was one exercise relating to integrals and in one of the sets there were two exercises relating to integrals. The mentioned exercises from the exam sets can be seen in appendix D

Exercise 5, c) from the exam May 2017 and exercise 6, c) from the exam August 2016 were tasks of the type:

> $T_1$:Determine an area defined by the graphs of two given continuous functions.

Exercise 6, c) from the exam December 2016 and exercise 4 a) from the exam May 2015 were tasks of the type:

> $T_2$: Determine the volume of a solid of revolution defined by a given curve in the plane.

Exercise 2, c) from the exam May 15 had a task of the type:

> $T_3$: Given a function determining a plane surface and given functions determining an area of integration determine the plane integral that describes the volume.

So $T_1$ is the same as $T_{i1}$ and typical techniques for solving it are then the same as $\tau_{ia}$ and $\tau_{ib}$. $T_2$ on the other hand is not solved by exactly the same techniques. One technique could be $\tau_2$: *Determine the relevant domain of integration and use the formula* $V = \pi \int_a^b f(x)^2 dx$ *if rotation is around the x-axis and if rotation is around the y-axis write x as a function of y,* $x = g(y)$*, and use the formula* $V = \pi \int_a^b g(y)^2 dy$.

But except for a few extra geometric concerns the technique is still described by the same technology, $\theta_i$ and it belongs to the same local $MO_i$.

From this we see that students must encounter praxis parts concerning calculating integrals while the necessary theory is limited to the claim that the value of a definite integral is the size of an area. These exercises all treat integration as something that allows for calculation of areas and volumes defined by certain functions. They do not require that students are able to interpret the integral or the calculated area as something else when other units are given. So encountering the Riemann sum interpretation as part of the students theory is not necessary.

When working with demographic questions to be posed in the next chapter we will see that students must use the same praxis in order to integrate e.g. a population function. But students must also be able to interpret what this area represents when there are units that are not length. So there must lie some logos that does not seem to be usually required by the students behind the argument to use integration on the function.

## 2.4 Regression in HTX exam sets

Two of the analyzed exam sets contained exercises relating to interpolation. In these exercises from December 2016 and May 2015 students were given data sets with six and eight data points respectively, each point containing two corresponding values. They then contained a task of the type $T_{re}$:*Determine a formula for a function that describes the data.*

In one exercise students were asked to determine both a linear and an exponential function an then determine which function best described data. In the other exercise students were only asked to determine a function that described the data meaning that the evaluation of which functions to choose was already a part of the task.

This task $T_{re}$ is a subtype of the task $T_r$ from our reference model and with several data points and CAS available it encourages the use of technique $\tau_{r1}$ for solving it.

Since students have to evaluate which type of regression to use some of the theory on R-values and the nature and origin of the data seems to be necessary for students to encounter.

So from this we see that students do meet a praxeology concerning interpolation but from the four sets analyzed, regression seems to be the only relevant technique.

In chapter 3 we will see how techniques of regression using CAS instruments are relevant in answering demographic questions but much more popular are the techniques of determining different functions for pairs of chosen data points to interpolate. The latter does not seem to be as relevant to students based on the brief analyzis of the four mentioned exam sets from the last three years.

The evaluation of which functions best describe data may be very relevant praxis in answering questions in chapter 3.

## 2.5 Study and Research Paths

The notion of a "Study and Research Path" also sometimes "Study and Research Course" (from here on SRP) was introduced by Chevallard around 2004 [Chevallard, 2005]. It was meant as a tool for didactic design in "the paradigm of questioning the world" where students over time would both do autonomous study of material and do their own research to answer big questions. SRP can together with a certain representational tool also be used for analysis of didactical processes [Winslow et al., 2013].

### 2.5.1 Motivation for SRP

Chevallard introduced the tool SRP as a way of avoiding some problems he saw in mathematics teaching. In "Teaching Mathematics in Tomorrow's Society: A Case for an Oncoming Counter Paradigm" he discusses a transition from one paradigm in mathematics education to another. He says we have been in a paradigm that in the framework of ATD is called "visiting monuments". In this paradigm students learn pieces of knowledge as very isolated works and are supposed to admire them without knowing the reason for their existence. Knowledge here comes in chunks that are determined by tradition and it therefore tends to be unknown for students why the work is there and what it is useful for [Chevallard, 2015]. As a consequence students tend to develop a relation to knowledge as something institutionally driven that can be forgotten after exams and not as something functional based on actual utility. Another problem is that the monuments visited are not chosen on an experimental basis of what is useful [Chevallard, 2015].

The new paradigm he calls "Questioning the World". In this paradigm education is considered a lifelong process and importantly students have a receptive attitude towards questions that arise which have no obvious answer. When students face

such a question they will consider it and study it in order to arrive at an answer instead of just discarding it like they would in the old paradigm. This attitude he calls Herbartian and he compares it to the scientists' attitude in her field of research [Chevallard, 2015].

Chevallard's introduction of SRP was motivated by a work format where students should work autonomously on transdisciplinary problems where "*... the aim was to put important and meaningful questions at the forefront of teaching*" [Winslow et al., 2013, p. 269] . Answering meaningful questions in an autonomous study can be a way of avoiding monumentalism. Today educational researchers seem to agree that for students to learn well the teaching should promote autonomous student work and not just transmit knowledge [Winslow et al., 2013]. But Winslow et al. also explain that besides acquiring certain modes of work students must also learn specific pieces of knowledge and in particular they claim it is important for learning that students see how these pieces are structurally related i.e. that it is important to avoid monumentalism. But also they emphasize that students must learn to perform study of existing knowledge in various media and must be able to use this knowledge [Winslow et al., 2013] .

### 2.5.2 The didactic design

So an SRP as a didactical design departs in one or more meaningful or important questions posed by the teacher and then revolve around the students' study and research of these questions. The main questions are sometimes called generating questions since they under the paradigm of "questioning the world" should enable students to pose related questions whose answers can help them in their inquiry and Winsløw et al. explain that the students' work with the generating questions involves three different moves [Winslow et al., 2013]. As the name is "Study and Research Path" study and research are two of the moves. During "study" students must consult existing knowledge through different media such as the internet and books and they must evaluate the use of it towards an answer to questions. The research part includes inquiry and problem solving. Here students use reasoning to construct and justify answers to questions. The third move is to elaborate new questions. This can be both subquestions whose answers give partial answers to the generating question and it can be derived questions that are motivated by the work done but which do not help answering the generating question. So the idea is that students construct knowledge as answers to explicit questions through their work towards an answer to the initial generating question by using these three moves. The dynamic between questions and answers is thus essential to the SRP.

This can be connected to the ATD terminology since the questions posed gives rise to an identification with certain types of tasks for which a technique may be applied. The generating questions can be chosen strategically such that students will have to attain certain pieces of knowledge, or praxeologies, while they still have the chance to find their own paths and media in the search for answers [Winslow et al., 2013]. The learned knowledge from the works studied should then be functionally coherent because it is related and relevant by the inquiry into the generating question. So the reason for and use of the studied works is clear [Chevallard, 2015]. So by using an SRP the students should learn to work autonomously and acquire pieces of knowledge. Furthermore because these pieces of knowledge stem from the students'

own study and research of a meaningful question the relation between them should be clearer and by strategically choosing the meaningful question students can be forced to consult existing knowledge and learn to use it.

An SRP can range from being very open to being very targeted. The initial question for an open SRP could be "What is a tsunami". Such a question would allow students to work with a large variety of questions and answers across disciplines. But smaller questions which only give space for a smaller variety of derived questions to appear can be used and can then allow the teacher to direct the work towards specific targets [Winslow et al., 2013].

In relation to this Kuzuoka & Miyakawa claim that much inquiry based learning resorts to "fake inquiring" because the teacher already knows what works the students must encounter and the generating question is then just a trick to get the students to find it [Kuzuoka and Miyakawa, 2018]. This problem situation arises because of the structure of the curriculum as monuments to be visited in the old paradigm.

### 2.5.3 Representational tool

So SRP's depart from questions and students construct knowledge as they work on them. The work with a question can often produce several answers and also questions which can then again produce new answers and questions. This dynamic of questions and answers and construction of knowledge can be visually represented by a tree-diagram as shown in figure 2.2. The figures can be made with different variations in order to emphasize different things. Here the boxes are for example of different darkness to illustrate whether questions are supposed to be posed by teachers, students or in collaboration and one could also change the shape of the arrows to indicate whether one will mainly use study or research to follow the arrow [Winslow et al., 2013].

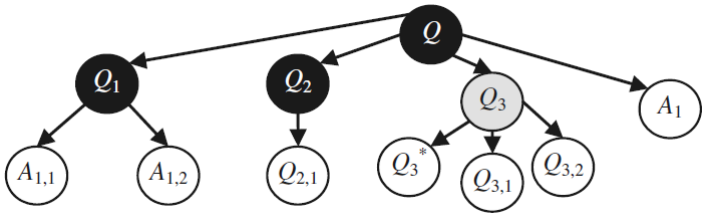


Figure 2.2: Example of a Q&A tree-diagram [Winslow et al., 2013, p.271].

This representational tool can be useful in the a priori analysis of the didactic design but it is also a useful tool for an a posteriori analysis of the didactic process. In fact Winsløw et al. suggest that it can be used to analyse any didactic process [Winslow et al., 2013].

# Chapter 3

# Mathematical demography as the topic for a study and research path

In this section the topic of mathematical demography will be explored. We will pose a specific question related to the topic and then try to answer this question as thoroughly as possible. This answer will mainly be based on an exploration of the literature on the topic and the methods that are used will be discussed.

## 3.1 The history of mathematical demography

Demography is the scientific study of human populations [IUSSP, 2017] and mathematical demography can be said to be a specialization that deals with "*... the articulation, analysis, and empirical application of theoretical models or representations of populations and demographic processes via the use of mathematics, including mathematical statistics.*" [Land et al., 2005, p. 1]. It draws from the kind of numerical analysis used in e.g. engineering and it uses interpolation and curve fitting. Also in India and other places the data is obtained by probability samples so that the theory of estimation from samples is applied [Keyfitz, 1976]. The core consists of two basic classical population models; the life table/stationary population model and the stable population model [Smith et al., 2013].

It is difficult to outline this specialization precisely from the overall subject of demography and the origins of both things are close since mathematical questions are implicit in the early demographic works. Mathematical demography can be divided into three periods with their own questions and attempted answers [Keyfitz, 1976].

John Graunt is generally acknowledged to have made the first publication in the field of demography [IUSSP, 2017]. In 1662 he presented numbers of christenings and burials, presented life tables and made calculations of survivorship but when Leonard Euler introduced stable age distributions in a paper in 1760 mathematical demography had its true beginning [Keyfitz, 1976, Smith et al., 2013].

In the middle period in the late 1800's Alfred J. Lotka among others did great work. He spent most of his time devising and solving a renewal equation where the number of births in a generation was related to the number in the previous generation. He also "*...raised many of the questions that make up the core of our subject and provided the tools for answering them*" [Keyfitz, 1976, p. 2].

In the contemporary period mathematical demography has spread in many directions. Nathan Keyfitz is said to have been the leading figure in the second half of the twentieth century and his books have defined the subject for many students and scientists [Smith et al., 2013].

## 3.2 How many people have ever lived

An interesting question to explore using mathematical demography is "*How many people have ever lived?*". The last century has seen a magnificent growth in the total human population and a rumor has spread that there are more people alive today than there are people who have ever died and this has spurred some demographers to tackle the question [Haub, 2011]. The question is also of some relevance in the study of mutational variation and assessment of the evolution of human racial variation [Weiss, 1984]. A few different approaches to answer the question have appeared but common for them all is that they need some estimates and assumptions:

- Estimates of the total human population at different times.
- Estimates of and a definition for how long humans have existed for.
- Estimates of either the birthrates or the life expectancy at different times.
- Assumptions on the growth of the population that allows interpolation of either the population estimates or the estimates of the number of births.

Estimates on population sizes have been carried out by various historians and demographers based on different things. First of all available census data has been considered. But also archaeological findings such as the number of rooms in buried houses can be used to determine the density of a population in some era. For very old populations fossil records are used and comparisons are made to modern hunting tribes to estimate the population density for inhabited areas [Deevey, 1960]. Estimates by researchers vary greatly. For the period around 1 AD estimates range from 170 millions to 300 millions [Manning, 2008].

To determine where to count from is also a matter of definition and of genetics. Some count from 4.5 million BC to account for all members of the genus Homo [Weiss, 1984]. Others use for example a start of two people at 50.000 BC to count all Homo sapiens although it may now seem that Homo sapiens appeared already around 200.000 BC [Smithsonian, 2017].

The growth rate of the population has not been constant over human history but many have argued and utilized that this number may have been approximately constant over shorter intervals [Keyfitz and Caswell, 2005, Weiss, 1984, Vaughan, 2011]. This is due to assumptions about the constancy of the elements of growth e.g. successive groups of women having children at the same age and death occurring at the same time, so that the absolute growth of the population is proportional to the population's current size. If the birthrate, which is the number of births per person per time unit, is the constant $b$ and the deathrate is constant $d$ then the rate of change $N'(t)$ of the population can be expressed by the differential equation $N'(t) = kN(t)$, where $k = b - d$ and $N(t)$ represents the population as a function of time [von Foerster et al., 1960]. So if for example the growth in population per

year is the percentage $k$, then the population size $N_t$ after $t$ years is $N_t = N_0(1+k)^t$ where $N_0$ is the initial value. Thus under such assumptions on $b$ and $d$ the total population would develop exponentially [Keyfitz and Caswell, 2005]. Since we will deal with very large numbers we will, to simplify calculations, assume that compounding happens continuously instead of in some discrete time interval . So we set $(1 + \frac{r}{j})^j = (1 + k)$ and since $\lim_{j\to\infty}(1 + \frac{r}{j})^j = e^r$ a starting point for a calculation based on population sizes is to assume that the population, $N$, at time, $t$, is described by $N(t) = N_0e^{rt}$ which is the solution to the mentioned differential equation. Thomas Ramsay argues that this is especially accurate when an organism is in an environment that has plenty of capacity e.g. humans at the discovery of agriculture. He claims that it works well with most human populations since they "*... rapidly shift from one environmental equilibrium to another*" [Ramsey, 1999]

### 3.2.1 Calculating "person-years"

In the article *How many people have lived on the earth?* Nathan Keyfitz demonstrated how one could find a simple formula for the calculation using estimates of the total human population at different times, an assumption on the average lifespan of a human being and assuming uniform exponential growth between population estimates [Keyfitz, 1966].

CALCULATION OF PERSON-YEARS LIVED, WHEN PERIOD IS IN FOUR INTERVALS, AND IN ONE INTERVAL

| $t$ by Calendar years of the Christian Era | $n$ | ln $n$ | $\frac{(n_2-n_1)(t_2-t_1)}{\ln n_2-\ln n_1}$ |
|---|---|---|---|
| −1,000,000 | 2 | 0.698 | |
| | | | $0.34\times10^{12}$ |
| −5,000 | 5,000,000 | 15.425 | $.31\times10^{12}$ |
| 0 | 250,000,000 | 19.337 | |
| | | | $.63\times10^{12}$ |
| 1650 | 545,000,000 | 20.116 | |
| | | | $0.45\times10^{12}$ |
| 1960 | 3,000,000,000 | 21.822 | |
| | | | $1.72\times10^{12}$ |

Table 3.1: Table with population estimates [Keyfitz, 1966, p.581]

The two left columns of the table above shows the five estimates of the human population that Keyfitz used for his calculation. Because of the arguments above he assumed that there was a uniform exponential growth between any two consecutive estimates which therefore determines an exponential function. So the function describing the population over time is a piecewise function comprised of one exponential function for each of the time intervals between estimates:

$$N(t) = \begin{cases} N_1e^{r_a(t-t_1)} & t_1 \le t \le t_2 \\ N_2e^{r_b(t-t_2)} & t_2 \le t \le t_3 \\ N_3e^{r_c(t-t_3)} & t_3 \le t \le t_4 \\ N_4e^{r_d(t-t_4)} & t_4 \le t \le t_5 \end{cases}$$

For each of these functions the growth rate must be determined and if the human population is $N_1$ at time $t_1$ and $N_2$ at the next estimate at time $t_2$ then we have $N_1 = Ce^{rt_1}$ and $N_2 = Ce^{rt_2}$ so

$$\frac{N_2}{N_1} = \frac{Ce^{rt_2}}{Ce^{rt_1}} = \frac{e^{rt_2}}{e^{rt_1}} = e^{r(t_2-t_1)} \tag{3.1}$$

From this we take the natural logarithm on both sides and isolate $r$ to find the continuously compounded growth rate:

$$r = \frac{\ln N_2 - \ln N_1}{t_2 - t_1} \tag{3.2}$$

So at any given time $t$ between $t_1$ and $t_2$ the total population is calculated as $N(t) = N_1 e^{r(t-t_1)}$, with $r$ as given above and $N_1$ as the population at time $t_1$.

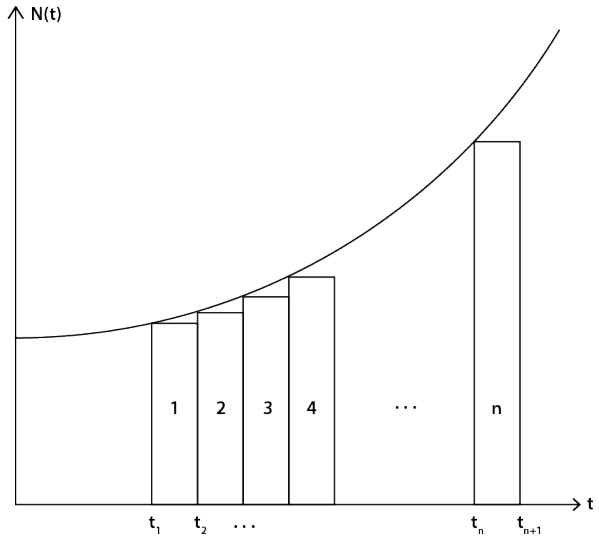


Figure 3.1: Riemann sums

The graph in figure 3.1 represents such an exponential function and some rectangles of equal width are drawn under the graph. The area of each rectangle is then calculated by $N(t_i)\Delta t$, where $\Delta t = \frac{t_{n+1}-t_1}{n}$. These areas represent the number of "person-years" lived. If for example $N(t_1) = 10,000$ and $\Delta t = 5$, that is 10,000 people live for 5 years, then the area of the first rectangle represents 50,000 "person-years" lived. So the geometrical interpretation of the area under the graph is "person-years". We can calculate the area of all $n$ rectangles by $\sum_{i=1}^{n} N(t_i)\Delta t$. Since we have already chosen to assume that $N(t)$ is a continuous exponential function of t we can keep decreasing the width of the rectangles to get better approximations of the area under the curve. When we now take the limit of this sum as $n$ approaches infinity we will per definition get the Riemann integral whose geometrical interpretation is exactly the signed area under the graph which as mentioned represents "person-years": $\lim_{n\to\infty} \sum_{i=1}^{n} N(t_i)\Delta t = \int_{t_1}^{t_n+1} N(t)dt$. This demonstrates how a Riemann sum interpretation of the definite integral can be helpful in determining the unit of the calculated number.

So the total number of "person-years" lived in the first piece of the piecewise function is then found by integrating the exponential function over the interval:

$$\int_{t_1}^{t_2} N_1 e^{r(t-t_1)} dt = \frac{N_1}{r}(e^{r(t_2-t_1)} - 1)$$

From (3.1) we can see that this is the same as $\frac{N_1}{r}(\frac{N_2}{N_1} - 1) = \frac{N_2 - N_1}{r}$. $r$ is now substituted with the right side of equation (3.2). Keyfitz proposed to use an average life expectancy for human history of 25 years. So by dividing by 25 we get the final formula for the number of people who lived in the interval:

$$N_{t_1}^{t_2} = \frac{(N_2 - N_1)(t_2 - t_1)}{\ln N_2 - \ln N_1} \frac{1}{25} \tag{3.3}$$

His result with the last population estimate in 1960 was 69 billion people. He argues that this number is not very sensitive to the chosen average life expectancy which he believes to be between 20 and 35. That is a decrease of $1 - \frac{25}{35} = 29\%$ or an increase of $1 - \frac{25}{20} = 25\%$. This gives a range of 49 billion to 86 billion. However the number of population estimates may have significant influence. Using just a start of two people and the 3 billion in 1960 one would reach a result of staggering 5690 billion. Keyfitz assumed two people started 1,000,000 years BC. Had he only used 500,000 years BC the result would have been 62 billion and calculating from just 50,000 BC would give 56 billion.

In 1999 Thomas Ramsey used this method to update the number. Also calculating from 1,000,000 BC and using a life expectancy of 25 but with more and different benchmarks and including the time from 1966 to 1999 his result was that 96.1 billion people had ever lived [Ramsey, 1999].

If we use numbers from the UN to add an interval from 1999 to 2016 we arrive at 100.4 billion [UN, 2017a]. The life expectancy of the world in those last periods was probably higher though making the actual number smaller. A life expectancy of 75 years for the last period would make the last contribution of around 4.3 billion into around 1.4 billion.

This method of integrating a piecewise function is illustrated in the graph below:

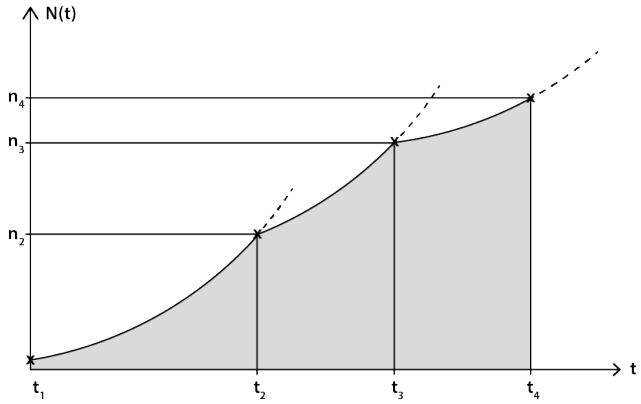


Figure 3.2: Representation of a definite integral of a piecewise defined function

Instead of assuming exponential growth between estimates could we have used a simpler model? A linear function between estimates would still be monotonic increasing and it would be even simpler to use in calculations but it would give a bigger result. But how much bigger? Using the data in table 3.1 Keyfitz' exponential method gave around 69 billion whereas the linear method gives around 173 billion. A comparison of the numbers for each interval can be seen in table 3.2.

| Year | Linear | Exponential | Difference |
|---|---|---|---|
| 1,000,000 BC to 5,000 BC | 99.5 | 13.6 | 85.9 |
| 5,000 BC to 0 | 25.5 | 12.4 | 13.1 |
| 0 to 1650 | 26.2 | 25.2 | 1.04 |
| 1650 to 1960 | 22.0 | 18.0 | 3.98 |
| All | 173.2 | 68.8 | 104.4 |

Table 3.2: Comparison table. Numbers of people are in billions

This is quite a big difference but 82 percent of the difference comes from the first interval where the linear method gives a number more than 7 times that from the exponential method. Had the interval been from 500,000 BC the numbers would have been 49.5 and 13.5 for the linear and exponential method respectively, that is just little over 3.5 times as much. The absolute value of the difference depends on the size of the interval and the relative difference also depends on the values of the population estimates. So for the later smaller intervals the differences between the methods are smaller.

Kenneth M. Weiss has also used this method to answer the question, although his perspective was to calculate how many times the initial population is replaced. He used estimates of population sizes to calculate person-years using the same method as Keyfitz of assuming exponential growth between estimates. But Weiss divides

with different estimates of life expectancy for the different epochs starting with 20 year generations and then slowly rising [Weiss, 1984]. His result was that 150.9 billion individuals of the Homo genus had been born in the last 4.5 million years but if we count from the latest assumed race split at 50.000 BC it is 57.2 billion. Using UN population estimates and a life expectancy of 75 as Weiss suggests for a contemporary industrial nation we can add a period from 1983 to 2016 and then the final result is 59.8 billion.

### 3.2.2 Calculating births

In the book *Applied mathematical demography* Keyfitz used another method to answer the question [Keyfitz and Caswell, 2005]. Knowing how many people have ever lived is the same as knowing how many people have ever been born so this method used estimates of the number of births in different years to calculate the total number of births throughout human history. Estimates on number of births per year can be based on estimates of population size and of birthrates whose product is births per year [Weiss, 1984]. Keyfitz used estimates by the Population Reference Bureau of births in different years throughout human history and assumed a uniform exponential increase in population size between estimate years. This is again due to assumptions on the constancy of elements of growth such that death- and birthrates are assumed constant between estimate years so that the size of the population is described by $N(t) = N_0 e^{rt}$. Since the number of people born per time unit is calculated as the birthrate $b$ times the population size $N(t)$ we see that births per time is:

$$B(t) = bN(t) = (bN_0)e^{rt}$$

So the number of births per time as a function of time, $B(t)$, is also an exponential function between estimate years. So if the number of births in year $t_1$ is estimated as $B_1$ and the number of births in year $t_2$ is estimated as $B_2$ and we assume that the total number of births per year is exponentially increasing over the considered time interval we can calculate as before: For each two consecutive points we determine the exponential function through those points and integrate it between them. This time though the Riemann sums represent the number of births. A rectangle could have height "5 births per year" and width "10 years" so that the area represents 50 births. So the resulting formula is similiar to the one before. This time we have births per year $B_1$ and $B_2$ instead of population size and we omit dividing by an average life expectancy:

$$B_{t_1}^{t_2} = \frac{(B_2 - B_1)(t_2 - t_1)}{\ln B_2 - \ln B_1} \tag{3.4}$$

Using numbers from the Population Reference Bureau as shown in the table below, Keyfitz calculated that the total number of people ever born in 1962 was 70.9 billion.

| $t$ | $n$ |
|---|---|
| 600,000 B.C. | 1 |
| 6000 B.C. | 250,000 |
| A.D. 1650 | 25,000,000 |
| A.D. 1962 | 110,000,000 |

Table 3.3: Estimated number of births in four different years [Keyfitz and Caswell, 2005, p. 12].

This time he set the starting point as 1 birth in the year 600,000 BC. The United Nations has published an estimate of the number of births from July 2010 to June 2015 [UN, 2017b] and so if we use a fifth of this as the number of births in 2015 we can add a period from 1962 and will then get 77.4 billion. Had he calculated with one birth at 1,000,000 BC the number would have been 8 billion higher and from 50,000 BC it would have been 11 billion lower.

A much more recent and very popular article on the topic using the same method of calculating has been published by Carl Haub [Haub, 2002]. He has written for the Population Reference Bureau a popular article on the topic giving his estimated answer and he has updated the number as late as 2011 [Haub, 2011].

His method of calculating is the same meaning that he considers the number of people born in different years, assumes exponential growth between estimate years and integrates to get the number of births. His starting point of 50,000 BC was based on UN estimates of when the first Homo sapiens appeared and for population sizes he considered averages of estimates by the UN and other sources [Haub, 2011]. Arguments about different life expectancies in different periods lets him make estimates of birthrates at the time.

From the the birthrates and the population numbers the number of people born in benchmark years can be calculated and Keyfitz' method can then be applied. He estimated that as of mid-2011 108 billion people had ever been born.

The difference in the two methods, calculating births and calculating person-years, that have now been discussed is not big. The basis for both methods is assumptions of constancy of elements of growth that allows us to assume exponential growth for both $B(t)$ and $N(t)$ between estimate years. It is of no relevance for the result if one multiplies by a constant before or after the integration so in one method one could say we multiplied the integrated result with the birthrate of the period whereas in the other method we divided by the life expectancy (either with different expectancys for each period or with one for all of human history). So the difference is whether estimates of birthrates or of life expectancys are used.

### 3.2.3 Discrete calculation

The number of people who have ever lived is a discrete number developing discretely over time and the calculations of the number must be based on discrete phenomenons such as birth and death. Therefore one can wonder why the methods above base calculations on continuous functions and not discrete functions. But any attempt to make a precise (as far as the precision of the data allows) calculation is obscured

by the fact that we do not know how long time there is between one death or birth and the next.

As mentioned earlier the calculations were based on the assumption that the birthrate and deathrate were constant on the considered intervals so that the population size could be modelled.

In the discrete reality they cannot be constant not even in discrete time intervals (except when they are equal to each other) but on average they might have been. But if they were so on average then an immediate guess on what the population size is at any specific moment could be calculated from the continuous function that represents this assumed average. In the next section, 3.2.4, we will show why this is a good guess.

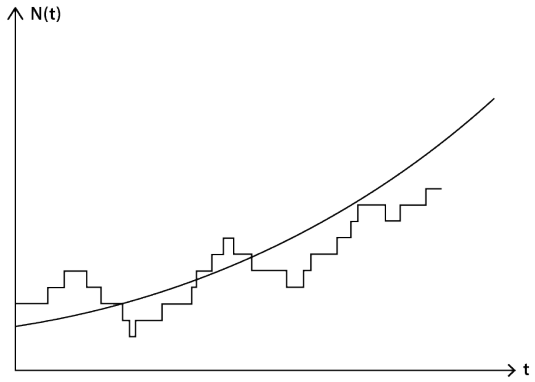


Figure 3.3: Graphs of a discrete function and an exponential function

But Keyfitz and others must have assumed such a constant average of $b$ and $d$ which brought us to a continuous model of the population size. We deduced that $N = N_0(1 + k)^t$ which when $t$ is treated continuously can be rewritten as $N = N_0e^{rt}$. Because of the way the numbers are given such as birthrates being births per thousand people per year one might want to treat time discretely in steps of one year. So for the purpose of demonstration we will show an example of a calculation where compounding happens yearly.

In the calculation above we used the number of births per whole year. The estimate said that 250,000 people were born in the year 6000 BC but in the calculation we let time increase continually and said that 250,000 people per year was the rate with which people were born at the very start of the year 6000 BC

Alternatively we could have treated time discretely so that $t$ would always increase with whole years. In that case we would consider a discrete piece-wise function of the form:

$$B(t) = \begin{cases} b_1 N_1 (1+r)^{(t-t_1)} & t_1 \leq t \leq t_2 \\ b_2 N_2 (1+r)^{(t-t_2)} & t_2 \leq t \leq t_3 \\ ... \end{cases}$$

For each interval between benchmark years we would then calculate $r$ as $r = \sqrt[t_{n+1}-t_n]{\frac{B_{n+1}}{B_n}} - 1, 0 \leq n$, and to sum all the births in an interval we calculate:

$$\sum_{t=t_n}^{t=t_{n+1}-1} b_n N_n (1+r)^{(t-t_n)}$$

Notice that to get the sums in the time interval $t_n$ to $t_{n+1}$ we summed from $t = t_n$ to $t = t_{n+1} - 1$. The sum can be rewrtten as:

$$\sum_{t=0}^{t_{n+1}-1-t_n} b_n N_n (1+r)^t = \frac{b_n N_n (1+r)^{(t_{n+1}-t_n)}}{r} - \frac{b_n N_n}{r}$$

Here we used that the fact that $\sum_{t=0}^{M} (1+r)^t = (1+r)^M + \frac{(1+r)^M}{r} - \frac{1}{r}$ and thus $\sum_{t=0}^{M-1} (1+r)^t = \frac{(1+r)^M}{r} - \frac{1}{r}$.

In the calculation above we simply find a lower sum of the function with a fineness of 1 year in $t$. The result is therefore expected to be slightly lower than what Keyfitz found. With Keyfitz' method calculating from 600.000 BC to 1962 the result is that 70.961 billion people lived and with this discrete method the result is 70.906 billion people. So as expected there's a relatively small difference of around 55 million.

### 3.2.4 Exponential average growth from a stochastic process

Samuel Goldberg has described in the book *Probability in the Social Sciences* why we can interpret exponential growth as the average behaviour of a population size that varies stochastically [Goldberg, 1983]. He begins with two assumptions:

> 1: In a small time interval $\Delta$t each person has probability $\lambda$t to give birth to one person and probability $\mu$t to die.
>
> 2: Occurrences of births and deaths are independent of each other.
>
> [Goldberg, 1983, p. 20]

We will use the notation $P_k(t) = P(N(t) = k)$ for the probability of $k$ persons being alive at time t.

Goldberg (p. 20) poses a lemma along the lines of:

*Let the size of the population be $k$ at time $t$. In a small interval from $t$ to $t + \Delta t$, the population can undergo transitions from $k$ persons, to the following number of persons with the following probabilities*

$k+1$: $k\lambda\Delta t + o(\Delta t)$

$k-1$: $k\mu\Delta t + o(\Delta t)$

$k$: $1 - k(\lambda + \mu)\Delta t + o(\Delta t)$

$k \pm n, (n \geq 2\text{: } o(\Delta t)$

To see where this comes from we show a proof of the first part: With $k$ persons alive at time $t$ we need to consider all possible cases where there are $k+1$ persons alive at time $t + \Delta t$. Either exactly one person gives birth and no one dies in $\Delta t$. The probability for this must be:

$$k(\lambda\Delta t)(1-\lambda\Delta t)^{k-1}(1-\mu\Delta t)^k$$

Expanding this gives $k\lambda\Delta t$ plus some terms with a factor of $\Delta t$ to at least a power of 2. Thus the extra terms are each $o(\Delta t)$ and so is their sum.

Otherwise $n$ persons give birth and $n-1$ persons die, where $2 \leq n \leq k$. Considering we now have $n$ choices from $k$ people to give birth we get:

$$\binom{k}{n}(\lambda\Delta t)^n(1-\lambda\Delta t)^{k-n}\binom{k}{n-1}(\mu\Delta t)^{n-1}(1-\mu\Delta t)^{k-n+1}$$

But for $n \geq 2$ each term in the expansion will be $o(\Delta t)$. So the probability of going from $k$ persons to $k+1$ persons in $\Delta t$ is $k\lambda\Delta t + o(\Delta t)$.

We will now use the lemma to express $P_k(t+\Delta t)$, $k \geq 1$, in another way since we can break $P_k(t+\Delta t)$ into sub-events. At time $t$ there must with probability $P_{k+n}$, $n \in \mathbb{Z}$, have been $k+n$ people alive and we can then use the lemma to find $P_k(t+\Delta t)$. So we get:

$$\begin{aligned} P_k(t+\Delta t) = P_k(t)\big(1-k(\lambda+\mu)\Delta t + o(\Delta t)\big) + P_{k-1}(t)\big((k-1)\lambda\Delta t + o(\Delta t)\big) \\ + P_{k+1}(t)\big((k+1(\mu\Delta t + o(\Delta t)\big) + o(\Delta t) \end{aligned}$$

This is equal to

$$P_k(t) - k(\lambda+\mu)P_k(t)\Delta t + (k-1)\lambda P_{k-1}(t)\Delta t + (k+1)\mu P_{k+1}(t)\Delta t) + o(\Delta t)$$

From which we can get:

$$\frac{P_k(t+\Delta t) - P_k(t)}{\Delta t} = -k(\lambda+\mu)P_k(t) + (k-1)\lambda P_{k-1}(t) + (k+1)\mu P_{k+1}(t) + \frac{o(\Delta t)}{\Delta t}$$

This is recognized as a difference quotient and since $\frac{o(\Delta t)}{\Delta t} \to 0$ as $\Delta t \to 0$ per definition, we see that:

$$\frac{dP_k(t)}{dt} = -k(\lambda+\mu)P_k(t) + (k-1)\lambda P_{k-1}(t) + (k+1)\mu P_{k+1}(t)$$

For $k = 0$ the case where $-1$ person is alive at time $t$ must be given probability 0 and the result is then the same.

This gives a system of infinitely many differential equations which can be used to identify the mean population size as something that grows exponentially.

We get the expectation, $m_1(t) = E[N(t)] = \sum_{k=0}^{\infty} kP_k(t)$ and we will then consider:

$$\frac{d}{dt}m_1(t) = \sum_{k=1}^{\infty} k\frac{dP_k(t)}{dt}$$

This can be expanded with the formula we found above and then reduced to give

$$\frac{d}{dt}m_1(t) = (\lambda - \mu)m_1(t)$$

which is the differential equation we have considered earlier which when $\lambda - \mu$ is set equal to $r$ will have the solution $m_1(t) = ce^{rt}$ [Goldberg, 1983, p. 23]. Therefore under the assumptions 1 and 2, we can interpret the population as growing stochastically such that it's size at a time $t$ is a random variable, but whose average size at time t grows exponentially [Goldberg, 1983, p. 23].

### 3.2.5 Logarithmic growth, hyperbolic growth and regression

As argued for it is reasonable to assume that the rate of change of the population is, on average, proportional to the current number of people:

$$\frac{dN}{dt} = aN$$

But we might also consider other situations since the environment may have an influence on birth- and deathrates. With an increasing population and density of people one could suspect $a$ to be decreasing when population grows because of increased interspecific competition for resources such as food. An example of $a$ being a monotonic decreasing function of N is seen when we let $a$ be a linearly decresing function of N: $a(N) = a_0 - a_1 N$. This can be rewritten by introducing some constants such that $a_0 = r$ and $a_1 = \frac{r}{K}$. We get $a(N) = r - \frac{r}{K}N = r\frac{K-N}{K}$ and recognize that $K$ represents a carrying capacity for the population in the environment. If the differential equation above is solved with such an $a$ one gets a classic S-shaped logarithmic growth curve. Such a curve effectively describe fruitflies in a milk bottle or bacteria on a petrisdish [von Foerster et al., 1960].

However there may be other factors for humans which could make this model a bad fit for the development of the human population. Von Foerster et al. argue that other things like more readily available mates may counter the negative effects from the environment as density is increased. He argues that effective communication will allow a species to form a coalition such that instead of competing with each other in a zero-sum game members can collaborate in ways that make environmental hazards have less effect on the development in population size. This is seen in humans with the strong social build-up, urbanization and mass-communication and Von Foerster et al. calls it coalition growth. They claim that $a$ in a sense represents living standard and so the human population may have had a monotonic increasing $a(N)$ [von Foerster et al., 1960].

In a book about examples of applied mathematics Robert B. Banks considered the rapid growth in population from 1650 to 1950 and like Von Foerster he experimented with an increasing $a$ [Banks and Banks, 1999]. Banks lets $a$ be proportional to $N$ such that $a(N) = \frac{\alpha}{N_0}N$. Thus $\frac{dN}{dt} = \frac{\alpha}{N_0}N^2$. So the rate of change is here proportional to the current value squared. This is a separable differential equation which can be written as $\frac{N'}{N^2} = \frac{\alpha}{N_0}$ and when we solve it for general solutions we get

$$N(t) = \frac{N_0}{CN_0 - \alpha t}$$

for some constant C, and we let $C = \frac{1}{N_0}$ such that $N = \frac{N_0}{1-\alpha t}$ and so $N_0$ denotes the population at time $t = 0$.

This is hyperbolic growth and it diverges to $\pm\infty$ as $t \to \frac{1}{\alpha} =$ so it definitely has limits in its forecasting abilities. By rewriting the equation to $\frac{1}{N} = \frac{1}{N_0} - \frac{\alpha}{N_0}t$ Banks can plot 11 data points on populations in billions from 1650 to 1990 so $t = 0$ in year 1650 and he makes a linear regression. The correlation coefficient is close to 1 and he gets the values $\alpha = 0.002675$ and $N_0 = 0.525$. Then $\frac{1}{\alpha} = 374$ meaning that the world population would approach infinity as we approach the year 2024. As a way of evaluating the function he uses this formula to make calculations of the population at different times from 1,000,000 BC to 0 AD and compare them with historians' estimates for those periods. His results fit fairly well with the historians' estimates. To answer the initial question he integrates the function from almost 1,000,000 BC to 1990 AD. That is from $t_0 = 1650 - 1,000,000$ to $t_1 = 1990 - 1650$ and he divides with an average lifespan of 25 years as Keyfitz did:

$$\int_{t_0}^{t_1} \frac{N_0}{1-\alpha t} dt = N_0 \int_{t_0}^{t_1} \frac{1}{1-\alpha t} dt = \frac{N_0}{\alpha} \log \frac{1-\alpha t_0}{1-\alpha t_1}$$

The result of this calculation was 80.8 billion [Banks and Banks, 1999].

Von Foerster et al. considered the situation when $a = a_0 N^{1/k}$ for constants $a_0$ and $k \approx 1$. The general solution for this differential equation is

$$N(t) = \frac{k}{C - a_0 t}^{k}$$

They were interested in finding "doomsday", $t_0$, that is the time at which the function approaches $\pm\infty$. We see from the denominator that doomsday occurs when $a_0 t = C$ so $t_0 = \frac{C}{a_0}$. By letting $N_1$ denote the population at the time $t_1$ we find that $C = N_1^{\frac{-1}{k}} k + a_0 t_1$ and then $t_0 = \frac{C}{a_0} = t_1 + \frac{k}{a_0} N_1^{\frac{-1}{k}}$ and so the whole thing can be written as $N(t) = N_1 \frac{t_0 - t_1}{t_0 - t}^{k}$ which clarifies the time until doomsday.

By letting $\tau = t_0 - t$ and $K = \frac{k}{a_0}^{k}$ we can write $N(t) = \frac{K}{\tau^k}$. Von Foerster et al. proceeded to carefully choose estimates of the world population from the year 0 AD to 1958 and by the method of least squares they achieved the values: $t_0 = 2026.87 \pm 5.50$ years, $K = (1.79 \pm 0.14)10^{11}$ and $k = 0.990 \pm 0.009$ So the resulting function is:

$$N(t) = \frac{1.79x10^{11}}{(2026.87 - t)^{0.990}}$$

where time $t = 0$ is year 0 AD. As mentioned this function diverges when $t = 2026.87$ and this time is aptly named "doomsday".

It is now possible to try to use this function to calculate the number of people who have ever been born. We do this as earlier by integrating the function and dividing by the life expectancy. If we calculate from 1 million BC to 1960 (when Von Foersters article was published) and use a life expectancy of 25 years for the entire period like Keyfitz the result is that 75 billion people (65-88) have ever lived.

This extrapolation of the function so far into the past is a stretch and according to the function around 200.000 people lived in 1 million BC.

Calculating from 50.000 BC would yield a result of 51 billion people (45-59).

If we forecast the function to 2017 we get close to "doomsday" but do not get ridiculous numbers yet. According to the function today's population would be 18.5

billion and the calculation gives that 90 billion people (82-100) would have lived from 1,000,000 BC to 2017.

So Von Foerster et al. and Banks did not make piecewise functions of the population over time. Instead they argued for what kind of function would suit the development of the population and then they used regression to get the proper values for the constants in those functions.

### 3.2.6 The fraction of people alive

The question of what percentage of all the people who had ever been born were alive at the time of asking can be answered from the results above and Cohen estimates that it as of 2013 was at most 9.3% [Cohen, 2014].

Cohen et al. has considered the theoretical question; under which conditions does this fraction rise? This question can be considered where the time $t$ changes continuously. Let the following functions of $t$ be

$F(t)$: The fraction

$A(t)$: People alive

$B(t)$: The number of people ever born

$b(t)$: Births per time

$d(t)$: deaths per time

It is reasonable to assume that all numbers are strictly positive which will allow for rearrangements of formulas. The fraction $F(t) = \frac{A(t)}{B(t)}$ and we are then interested in seeing when the derived function $F'(t)$ is positive. Now $B'(t) = b(t)$, and let the cumulative deaths be $\int d(t)dt = D(t)$ so that $D'(t) = d(t)$. We have $A(t) = A(0) + B(t) - D(t)$ so that $A'(t) = b(t) - d(t)$ Now using the quotient rule to differentiate $F(t)$ we get:

$$F'(t) = \frac{B(t)A'(t) - B'(t)A(t)}{B^2(t)} = \frac{b(t)\Big(B(t) - A(t)\Big) - B(t)d(t)}{B^2(t)}$$

This must be positive exactly when the numerator is positive and we see

$$b(t)[B(t) - A(t)] - B(t)d(t) > 0$$

if and only if

$$-\frac{b(t)}{B(t)} > \frac{d(t) - b(t)}{A(t)}$$

which is equivalent to:

$$\frac{b(t)}{B(t)} < \frac{b(t) - d(t)}{A(t)} or \frac{B'(t)}{B(t)} < \frac{A'(t)}{A(t)}$$

This simply and obviously says that the fractional change in the numerator of $F(t)$ must exceed the fractional change in the denominator for $F(t)$ to rise when t rises. We can isolate the number of people ever born and get:

$$\frac{b(t)A(t)}{b(d)-d(t)} < B(t)$$

Today we can get pretty good estimates of births and deaths in a whole year and the number of people alive so if we consider this discretely instead of continuously such that t moves in increments of one year and thus determines whole years after the date the first humans appeared we get that $F(t) = \frac{A(t)}{B(t)}$ rises from $t$ to $t+1$ if and only if:

$$F(t) = \frac{A(t)}{B(t)} < F(t+1) = \frac{A(t+1)}{B(t+1)} = \frac{A(t)+b(t)-d(t)}{B(t)+b(t)}$$

This can be rearranged as $\frac{b(t)}{B(t)} < \frac{b(t)-d(t)}{A(t)}$. which is similar to what we saw in the continuous case, so as before we can isolate $B(t)$ as:

$$\frac{b(t)A(t)}{b(d)-d(t)} < B(t)$$

With 2013 numbers of $b(t)$, $d(t)$ and $A(t)$ Cohen finds that $B(t)$ must be at least 12.1 billion for $F(t)$ to rise which is significantly lower than the estimates given above, so the fraction is likely to be rising at the moment [Cohen, 2014].

### 3.2.7 Methods and Results

So we have now seen different ways to answer the question of how many people have ever lived. According to Cohen a plausible range as of 2013 was that between 50 and 150 billion people had ever lived and Cohen reported that the fraction of people alive would in 2013 be between 0.1% and 9.3% and most likely rising at the moment [Cohen, 2014].

Different results on how many people have been born up until almost present time have been discussed above. They differ because they use different methods for calculating and because they use different estimates for the calculations. The results brought here vary from a low of 59.8 billion using Weiss' method and numbers combined with present UN numbers to a high of 108 billion calculated by Haub in 2011. As mentioned Keyfitz thought the life expectancy, which is closely related to birthrates, could make results almost 30% higher or lower. If one agrees on a definition of counting Homo sapiens then the error one can get from choosing a different starting point seems to be at most a few tens of billions. The population estimates, especially old estimates can vary much and the choice of these estimates can have a significant influence on ones result. The given results are definitely very crude ball-park estimates but they differ from each other by less than a factor 2.

The two most popular methods were both presented by Keyfitz in the 1960's and they revolved around knowing estimates of populations and of either the birthrates or the life expectancy. It was then about finding the exponential function between two points of either a population or a birth function and integrating each segment before adding them all together. The integrated population functions would either have each segment divided by a different life expectancy as Weiss did or they would all be divided by the same life expectancy like Keyfitz did.

Less serious methods for this calculation was the one by Banks and the one based on Von Foerster et al.'s work. These used arguments about the nature of propagation to suggest types of functions to suit available population data followed by a regression to find the population function before integrating and dividing by life expectancy.

All calculations were based on attempts to model some development in world population and generally there are two contrasting approaches one can use for a modelling task. The one is theoretical modelling where the model is based on assumed mathematical relations of the objects of the system that is modelled. The other approach is empirical modelling where the model is based on empirical observations. For example the assumptions of constancy of elements of growth allowed for a deduction of exponential functions which could be integrated. Also the ideas of coalition growth gave an idea of what the population function could look like. But a satisfying function could not have been made by entirely theoretical modelling and so empirical data consisting of population estimates and of birthrates or life expectancy entered the models. For Keyfitz and others these data served to set the starting value and growth rate for each part of the function. Banks and Von Foerster had a more experimental approach where the data was used to perform regression in order to determine the constants and the fit of the proposed function. They could therefore be said to have had a more empirical approach than Keyfitz did although both used a mix of theoretical and empirical modelling.

# Chapter 4

# Design

The analysis in the previous chapter shows that the question *"How many people have lived on earth?"* can be explored in different ways and that it may have potential as a generating question for an SRP. In this Chapter we we will describe the context in which our teaching experiment takes place. We will describe the purpose of the teaching sequence and we will present the lesson plan.

## 4.1 Context of the teaching

The teaching sequence was tested in a third year class at HTX in Denmark. It was a class with A-level mathematics and physics and the mathematics teacher claimed that it was a very strong class socially and academically. It consisted of 24 students, all male. They had already learned about integration and they had learned about the relevant functions including exponential functions and piecewise functions and they had even learned a bit about differential equations. So basically they had encountered all the MO's necessary to reach a meaningful answer to the question.

The teaching sequence spanned five hours over three different days. There was two hours on the first day (Lesson 1 + 2), one hour on the second day of teaching (Lesson 3) and two hours on the last day (Lesson 4+ 5).

The teaching of the sequence was carried out by the usual mathematics teacher. This way the influence of having someone new teaching the class was avoided and it allowed us to observe and take notes during the teaching. However this meant that we had less control over the teachings and the didactic process but to ensure that the teaching plan was followed as closely as possible we discussed it thoroughly with the teacher beforehand and in between lessons.

## 4.2 Purpose of the teaching sequence

The purpose of this teaching sequence was to test whether studying some elements of mathematical demography in high school could be used to give students an insight into how mathematical methods can be used to answer "big questions" in the social sciences.

The specific aim of the Study and Research Path was thus to fulfill parts of the curriculum for HTX A-level mathematics in a way that could give students this insight. The SRP design was used with the purpose of making students acquire an

Herbartian attitude so that they in collaboration could work with the "big question" and so that they could construct new knowledge corresponding to parts of the curriculum as answers to explicit questions posed mainly by themselves. This process should hopefully reveal to them how mathematical methods can be used to answer the question and that should also make some of the reasons for learning these parts of mathematics curriculum clear to the students.

The syllabus (læreplan) for HTX A-level mathematics dictates that "*the application-oriented dimension of the subject has great importance and consists of describing and analyzing problems by use of mathematical theories and models ... and afterwards develop and asses solutions*" [Undervisningsministeriet, 2017, p. 1](our translation). It is an academic objective to be able to analyze and solve a practical problem and to document and interpret the solution and its validity. Students should also be able to formulate themselves and to change between symbolic mathematical language and everyday written or oral language. On didactics the syllabus demands that students encounter projects concerning concrete problems with open exercises [Undervisningsministeriet, 2017].

A Study and Research Path with the generating question *How many people have lived on earth?* could be well suited to fulfill these things. The generating question is a real question about the real world and it has relevance and should intrigue students. It allows for a real world application of math. Students will encounter the project as a big problem to explore and they will have to document what they find and formulate it in a way that lets them share it with each other in order to asses each other's results and solutions. The syllabus contains a list of topics that must be taught and among these are:

- *"Characteristic properties of functions... determination of formulas for functions".* (Red: This includes exponential functions and piecewise defined functions.)
- *"Use of regression to determine formulas for functions that describe a certain set of data."*
- *Integral calculus; antiderivatives, definite and indefinite integrals... calculations of area"*

[Undervisningsministeriet, 2017, p. 2](Our translation) To answer the generating question students will have to model the growth of the world population throughout human history or the growth in births per year. As seen there are more ways to do this. The aim is that they will either argue for a type of function and then determine a specific function's formula by use of regression or that they will argue for the function consisting of piecewise exponential functions. The latter should allow students to use their knowledge on the characteristic properties of exponential functions and piecewise defined functions and let them determine the function's formula. When arguing for the behavior of population development and for the type of function the concept of differential equations is hoped to emerge. In order to calculate how many people ever lived students must integrate the function and should therefore use integral calculus to calculate the area under a curve and interpret what that area represents.

## 4.3 The lesson plan

Our analysis of the question *"How many people have lived on earth?"* showed its potential as a generating question. It encourages asking sub-questions and does not just have one certain answer. Therefore this will be the generating question, $Q_0$ for the teaching sequence. The SRP will be quite open and students should be allowed to use all media they have available. We anticipate that having sessions where students share their findings may help groups that get "stuck" or encourage groups to expand their "tree" of questions.

A table of the lesson plan can be seen here:

| **Time** | **Time accumulated** | **Action** |
|---|---|---|
| **Lesson 1+2** | | |
| 5 min | 5 | Introduction |
| 7 min | 12 | Appetizer to the topic |
| 10 min | 22 | Working format presented |
| 3 min | 25 | Dividing into groups |
| 5 min | 30 | Initial exploration of $Q_0$ without use of media |
| 10 min | 40 | Conference |
| 20 min | 60 | Exploration of $Q_0$ with all media |
| *Break* | | |
| 40 min | 100 | Further exploration |
| 20 min | 120 | Conference with focus on presenting the media they found. |
| **Lesson 3** | | |
| 35 min | 35 min | Continued study of the media they found |
| 25 min | 60 min | Conference: Presenting a short calculation based on data. |
| **Lesson 4+5** | | |
| 75 min | 75 min | Finish work on arguments for a formula and prepare a poster and a presentation. (*There is a short break after 60 minutes*). |
| 45 min | 120 min | Conference: Final presentation as a "poster-session". |

Table 4.1: Lesson plans (For a more detailed lesson plan see appendix E)

According to the plan the course is introduced with a small appetizer for the theme and then the working format and the generating question is introduced. The focus of the first lesson should be to have the students adopt the Herbartian attitude and make them use questions to explore the generating question. Therefore they should first pose questions without consulting any media and then share their questions with each other.

Afterwards they should use this shared library of questions to investigate the problem with the use of whatever media they can find online. Lesson two ends with a conference where they share and describe the relevant media that the groups find.

On the second day the work on the problem should continue. They will now have a shared "*library*" of useful media so focus here should be on studying this and understanding how to use it to answer the generating question. This lesson ends with a conference where the students share the results they have achieved so far. The last day consists of two lessons where the focus should be for students to finish their line of arguments justifying an answer and to prepare a polished presentation of their results. The end of the lesson then consists of these presentations.

# Chapter 5

# A priori analysis

We will here bring an analysis of how we expect the students to work with the generating question "*How many people have ever lived?*". This analysis of the didactical design draws from our own study of the question as presented in chapter 3 and SRP will be used as a representational tool.

Below is a tree-diagram that presents the questions we expect to be explored by the class. A solid line indicates that answers to the question below partially answer the question above and a dotted line indicates that the question is motivated by work on the question above but does not help answering it. The first subquestions create four branches:

$Q_1$ is about the initial conditions

$Q_2$ represents counting deaths but it is not expected to be a fruitful method.

$Q_3$ represents the method of counting human-years.

$Q_4$ represents the method of counting births

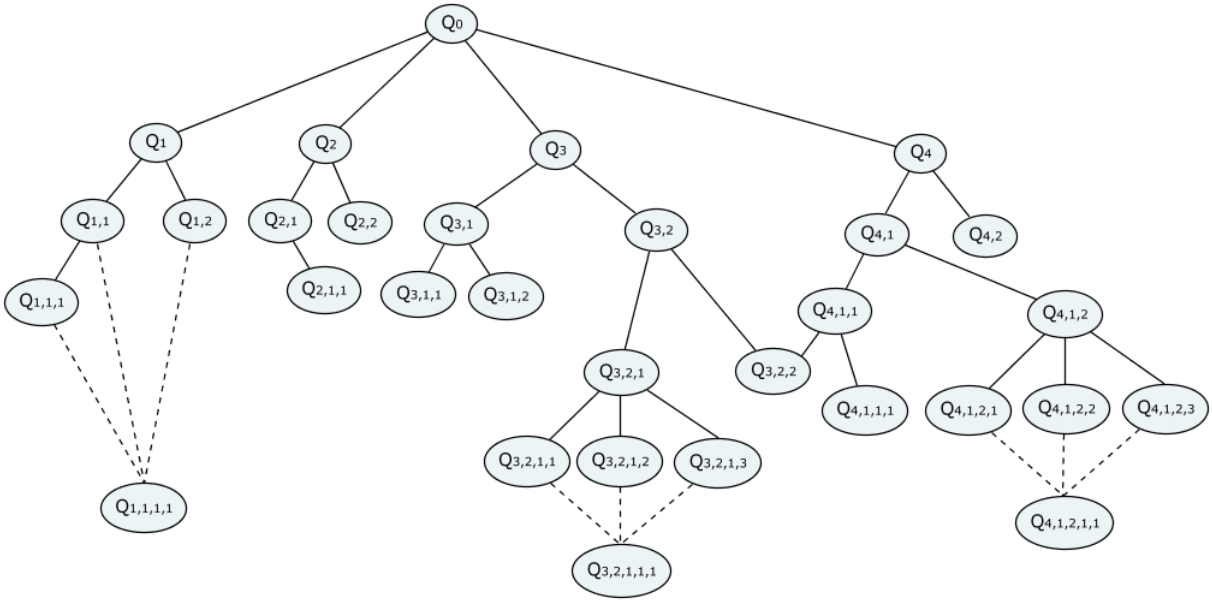


Figure 5.1: Tree diagram of a priori analysis

$Q_0$: How many people have lived on the earth?

$Q_1$: What should the initial conditions for the calculation be?

$Q_{1,1}$ When did the first human live?

$Q_{1,1,1}$: How do we define a human being?

$Q_{1,1,1,1}$: How big of an influence do the choices in $Q_{1,1}$ and $Q_{1,2}$ have on the final result?

$Q_{1,2}$: How big should the initial population be in our calculation.

$Q_2$: Can a calculation be based on a model of the number of deaths per year as a function of time?

$Q_{2,1}$: How can we model this?

$Q_{2,1,1}$: What data can we find on the number of deaths throughout history?

$Q_{2,2}$: How many people are alive today?

$Q_3$: Can a calculation be based on a model of the population size as a function of time, $N(t)$?

$Q_{3,1}$: How can we use such a model to sum all the people without counting the same person more than once?

$Q_{3,1,1}$: What does the area under the graph represent?

$Q_{3,1,2}$: What has the average life expectancy been over human history? What has it been at different times?

$Q_{3,2}$: How can the population as a function of time be modelled?

$Q_{3,2,1}$: How can we interpolate from a set of data for the world population at different times?

$Q_{3,2,1,1}$: Is linear interpolation between years with data the best way?

$Q_{3,2,1,1,1}$: How big of an influence do the choices have on the final result?

$Q_{3,2,1,2}$: Is exponential interpolation between years with data the best way?

$Q_{3,2,1,3}$: Is exponential regression on all years with data the best way?

$Q_{3,2,2}$: What data can we find on world population size throughout history?

$Q_4$: Can a calculation be based on a model of the number of births per year as a function of time?

$Q_{4,1}$: How can the number of births per year as a function of time be modelled?

$Q_{4,1,1}$: What data can we find on the number of births at different times in history?

$Q_{4,1,1,1}$: What data can we find on birthrates at different times in history?

$Q_{4,1,2}$: How can we interpolate from a set of data for the number of births in different years.

$Q_{4,1,2,1}$: Is linear interpolation between years with data the best way?

$Q_{4,1,2,1,1}$: How big of an influence do the choices have on the final result?

$Q_{4,1,2,2}$: Is exponential interpolation between years with data the best way?

$Q_{4,1,2,3}$: Is exponential regression on all years with data the best way?

$Q_{4,2}$: What does the area under the graph represent?

We expect that several groups will early on find the article *"How Many People Have Ever Lived On Earth?"* by Carl Haub [Haub, 2011]. This article suggests answers to many of the questions that the students can pose. It might turn out to be a central media for the process and we therefore present in figure 5.2 a form of flow-chart that presents how we expect the students will progress if they find the article.

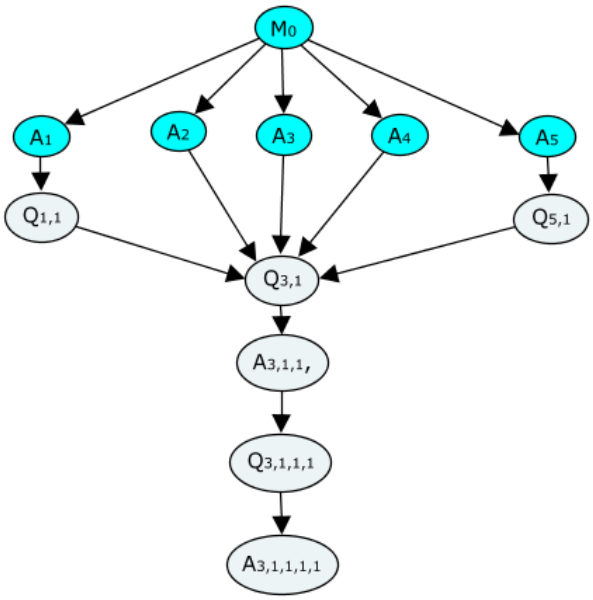


Figure 5.2: Diagram of expected SRP after the article *"How Many People Have Ever Lived On Earth?"* by Carl Haub [Haub, 2011] is found

The blue boxes represents things that are explicit in the article.

$Q_0$: How many people have ever lived on the earth?

$A_1$: 108 billion people have ever lived.

$Q_{1,1}$: How is this number calculated?

$A_2$: Start counting the population with 2 people at 50,000 BC.

$A_3$: Assume that the human population grows exponentially between estimate years.

$Q_{3,1}$: How can we use $A_2$, $A_3$ and $A_4$ to say something about births?

$A_{3,1,1}$: From $A_2$, $A_3$ and $A_4$ we can model the population size as a function of time with a piecewise exponential function, $N(t)$. If we multiply this by the birthrates we get from $A_4$ we get the births per year as a function of time, $B(t)$.

$Q_{3,1,1,1}$: How do we count the accumulated number of births?

$A_3, 1, 1, 1, 1$: We integrate $B(t)$ to get the accumulated number of births.

$A_4$: Estimates of population size and birthrate in different years.

$A_5$: A calculated number of births between estimate years.

$Q_{5,1}$: How is this calculated?

On the diagrams: A question students can consider is *"What should the initial conditions for the calculation be?"*. The generating question suggests we count all time but we know that humans have not been around for all of time so one has to ask:*"When did the first humans live?"* which will maybe lead to *"How do we define humans?"* and also *"How big should the initial population be?"*. Each of these questions have several possible answers and in the end one can ask: *"How big is the influence of these assumptions on the final calculation?"*.

We have outlined three branches starting at $Q_2$, $Q_3$ and $Q_4$ corresponding to three different approaches to solving the problem. Students can focus on modelling deaths, births or population size, but we expect that attempts to model the number of deaths will be an unfruitful endeavor. As we saw in chapter 3 this is also not a method that is typically used to carry out the calculation.

Each branch will instigate questions regarding how to make the models. This could give questions about whether to create a piecewise function that fits data for certain years, or to make some regression over a large dataset. Questions for evaluating the models can then be posed.

Going from these models to an answer to $Q_0$ necessitates that students consider what the areas under the graphs represent. For the people-years method they can ask: *"How do we avoid counting the same person more than once?"*. Here students must realize the idea to consider the life expectancy and find data on it. And most importantly they must realize that the integral of the function represents the accumulated number of lived years (or accumulated number of births for the birth-method). These ideas students could see in e.g. [Ramsey, 1999] or [Vaughan, 2011].

If one searches online for number of births or if one searches directly for an answer to $Q_0$ one will probably find different short articles without much explanation on calculations but which reference the article *"How Many People Have Ever Lived On Earth?"* by Carl Haub [Haub, 2011]. Or one may find this article directly. This article gives many relevant answers:

It suggests to start counting from 50,000 BC.

It suggests to start with a population of 2 people.

It gives estimates of the number of births over different periods.

(Here students should ask how these numbers are found/calculated.)

It justifies interpolating data exponentially between estimate years.

It gives numbers on population size and birthrate for different years.

To the last parts students may ask *"How can the number of births be calculated form this?*

# Chapter 6

# Research Questions

To guide our analysis of the collected data we have set up the research questions below. We are first of all interested in exploring the different paths students take in order to find an answer to the initial question and we will compare these to each other and to our own a priori analysis.

We have also collected data that can give insight into the way students use different media and also group presentation to find answers and to direct their work.

Lastly we found in our a priori analysis that the generating question may lead to working with integration and especially interpreting the meaning of integrals as sums so we want to explore students' math-praxeologies related to this.

The research questions are:

## 6.1 RQ1

What questions do the groups develop from $Q_0$?

What variation if any is there among groups?

How do these differ/agree with our own a priori analysis?

## 6.2 RQ2

How are questions and answers generated by the students? In particular, what media do they find and how do they use them?

How do the groups use presentations of the other groups?

## 6.3 RQ3

What can be said from the observations about the students' math-praxeologies related to:

Integrals (as "sum")?

Interpolation/making functions from data?

# Chapter 7

# Methodology

We will now describe what kinds of data we have collected and why and we will explain the methods we used to analyze the data.

## 7.1 Data collection

As mentioned in 4.1 the teaching sequence spanned five lessons over three days and was executed in a third year HTX class with A-level mathematics. The teacher thought the teaching sequence would fit their teaching plan well because they had already learned about integration and they were soon to write their *Studieretningsprojekt* (a large autonomous interdisciplinary assignment).

We collected sound recordings, logbooks, web histories and a final assignment which included a poster each group made.

- Sound recordings: The 24 students were divided in six groups and we have sound recordings for the 5 lessons from five of the groups. That gave a little more than 21 hours of recordings so it was a large data set.

  This was the primary data in our description and presentation of what the students did during the lessons such as posing questions and searching for relevant media.

- Logbooks: Each group was told to compile the questions they worked with and to describe their work during the lessons in a logbook. The logbook was to be uploaded to Lectio (a web based organizational tool) by the end of each lesson.

  The logbooks would then supplement the sound recordings and make it easier to understand and interpret what is talked about and why.

- Web history: Furthermore students were to find their web history for each lesson, and include it in their logbooks. This way we could see what media students found. This would not be possible from sound recordings and logbooks alone. This was quick to do during lessons and gives a good overview of the media visited. We considered using screen capturing software to get a more detailed insight in the use of media such as how long time they spent on each webpage. We chose not to do this since it would give a large amount of data that would be very time consuming to analyze. It would also take too

much time from the lessons to set up and it might interfere too much with the students' behaviour.

- Assignment: Lastly we asked the students to hand in a final assignment about their project which included images of the poster they made in the lessons. This would have more details than the logbooks about the mathematical praxeologies they have developed.

## 7.2 Referencing

Names of students have been redacted from logbooks and assignments in the appendix so that the participating students remain anonymous.

Transcriptions of selected parts of the audio files can be found in Appendix A. The appendix is organized with a table for each group containing time stamps and for some of the stamps the questions that are posed during that time are marked. the table for a group is followed by the transcriptions from that group.

Assignments can be found in appendix B. The logbooks can be found in Appendix C and the web histories can be found in the logbooks.

Future references to these appendices will use the following format:

L: Lesson

Log: Logbook

Ass.: Assignment

There was audio from five lessons and there were three days where groups could hand in a logbook. So, as an example, a reference to something said by group five in lesson 4 that can be found in appendix A will be noted with (G5, L4, 34:40), where the last part is a time stamp indicating the time in the audio file where the quote begins. A reference to the logbook from group three on day 3 is (G3, Log3) and a reference to the assignment by group two would be (G2, Ass.).

It should be noted that for some groups lesson four and five were recorded on one audiofile. Therefore L will concretely refer to the number of the audiofile so if a reference has a late time stamp in L4 it may have been said in lesson five.

## 7.3 Diagrams

In chapter 8 we present several tree-diagrams that are based on the collected data. These represent the SRP's performed by the students. In these diagrams questions are written in the form $Q_{x,x,x,x}$ where the x's are an index of numbers and letters. A $Q$ with the same index in different diagrams always refers to the same question. If the index consists only of numbers it comes from the a priori analysis. If it also contains letters then it was not present in the a priori analysis. A dotted line between questions means that the question below does not directly answer any questions above but is merely a question that is motivated by the work on questions above.

In the creation of the diagrams we have made different choices of what to include and what not to include. First of all only things relevant to the SRP have been considered so talk about free time and other things is not included. We have also made choices on the amount of details. In some areas students asked many questions like *"How long has it taken since the beginning that it has fallen from more than six children per woman to less than three?"*(G3, L3, 21:35). This amount of detail would make the rest of this thesis unreadable so we have interpreted to what extent the details are necessary and show something relevant about the students' progress in the SRP.

Questions are also not always said explicitly even though students were asked to pose questions. As an example consider the following dialogue:

> *A: we would like to use the one that's called polynomial*
>
> *B: logarithm*
>
> *C: it looks good doesn't it*
>
> *B: well it is not bad but the other one is even more precise, the nine-th degree polynomial*
>
> (G5, L4, 34:40)

This dialogue and it's context hinted that the students worked with the question $Q_{3,2,1}$*: How can we interpolate from a set of data for the world population at different times?* and the subquestion $Q_{3,2,1,d}$ regarding whether polynomial regression is the best way to do it. Clues from logbooks and assignments on what they worked on could also be cause to draw branches on the diagram.

Questions were not always asked the same way either so someone could say: *Yes the first human but the question is when is it a human and when is it a developed monkey so when*(G4, L1, 04:45) and we would classify it under the question $Q_{1,1,1}$*: How do we define a human being?*

A few questions will be marked with a * to signify that we did not see clearly in the data where the question was posed or worked on but that students probably had thought about it at some points based on them working on certain subquestions.

We did not make a tree-diagram for group one (the only group we did not have audio recordings from), because we did not have enough data to make it of the same detail as the diagrams of the other groups.

## 7.4 Transcriptions

The sound recordings were recorded in the period 08.11.2017-14.11.2017 and an oral permission to use them in the thesis was given by all students and the teacher.

The 21 hours of audio were first listened through and notes were taken as to where the groups worked with different questions and where they talked about integration or regression. Afterwards the material that proved relevant was transcribed.

The transcription is made after *Dansk standard for udskrifter og registrering af talesprog, 2. udg. 1992, minimale udskriftskonventioner/DANSK STANDARD 1* [Gregersen, 1992]. The audio was recorded in a room with a lot of background

noise so it was difficult to distinguish group members from each other. For one transcription in the same time stamp the lettering A,B... to distinguish persons may be wrong a few times. For different time stamps one can not expect the same letter to represent the same person. For the purposes for which we have used the data we do not think it is very relevant exactly who said what. The relevant part is what is being discussed in the groups and therefore our use of the data is still valid.

Some dialogue from the audio recordings and quotes from logbooks and assignments are presented in the thesis. These have all been translated from Danish to English by us.

# Chapter 8

# Data and a posteriori analysis

## 8.1 The realization of the teaching sequence

Some parts of the teaching did not follow the lesson plan (fig. 4.1) very closely. Lesson two started with a small conference although it was not initially planned, as students had already had enough time in lesson one to explore the question and various media to be able to present their findings in the form of relevant webpages (shown on projector) and the questions and answers they supply. Afterwards they continued their work and towards the end each group briefly made a purely oral presentation of what they had worked on and what they had found out.

Lesson three began with group two having their logbook shown on the projector as they presented their work for the rest of the class and afterwards group four did the same. It seemed like several groups had spent the entire time studying chromosomes, species, vaccines etc. This is an essential part of the SRP but the teacher discussed with us how it might be necessary to move on from that part so there would be enough time to make calculations. Therefore to encourage all groups to begin calculating, the groups that had done some considerations on calculations were chosen to present and the class was afterwards told that in the next lessons they should be able to present a calculation for an answer to the generating question.

Lesson four began with group one presenting the work they had on the white-board (preserved from the lesson the day before)(G1, Ass.). It showed a piecewise function of the population. Some places they interpolated linearly and others with a logistic function and a discussion about the meaning of the area under the graph ensued. After this the groups worked on the presentations and then they performed the presentations.


Figure 8.1: Visualization of the teaching sequence

## 8.2 Data collected

Not all groups produced/handed in all the material we requested. Out of six groups we made sound recordings of all groups except for group one and the following table shows which groups handed in what:

| | Group 1 | Group 2 | Group 3 | Group 4 | Group 5 | Group 6 |
|---|---|---|---|---|---|---|
| Audio, all lessons | ÷ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Logbook day 1 | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Logbook day 2 | ✓ | ✓ | ÷ | ✓ | ✓ | ✓ |
| Logbook day 3 | ✓ | ✓ | ✓ | ÷ | ÷ | ÷ |
| Assignment | ✓ | ✓ | ÷ | ÷ | ✓ | ÷ |

Table 8.1: Data collected

The amount of details in the logbooks varied greatly. Some groups made quite thorough descriptions of their work while others were very scarce.

## 8.3 A posteriori analysis

With the data collected we were able to perform an a posteriori analysis of the teaching sequence guided by our research questions. We will first focus on the paths that each group took to find an answer to $Q_0$. Afterwards we will investigate the websites visited by the students and we will consider how the conferense let the

groups use each other as media. Lastly we will describe in detail how they used and worked with the concepts of integrals and interpolation.

### 8.3.1 The realized SRP's

We here bring a brief description of each group's work and show a tree-diagram of the realized SRP (except for group one for which we did not have sound recordings and thus did not have enoguh data to construct a diagram properly). A question marked with a * in the diagrams is one that we could not find cearly in the empiri but they still seem to have worked with or worked with sub questions to.

Group one

On day one they worked with the some of the initial conditions for the calculation such as $Q_{1,1}$ where their formulation was: *"For how many years have humans existed?"*(G1, Log1). They considered counting births as they asked about parameters that affects births and said they needed to find the number of births per year (G1, Log1).

In lesson two they found data on the population size at different points and used this data to model population as a function of time by dividing human existence into four periods. They used linear and other functions between data points (G1, log2).

In lesson three they used maple to find the integral of their function and presented this as their final result. During the last conference it was pointed out by other students that they should divide this number with an average life expectancy. Their result was then 137 billion people (G1, Ass., p. 4).

Group two

In the first lesson it only took group two a few minutes before they began to search for a function that could describe the population as a function of time with the intention to integrate this function and divide the result by an average life expectancy (G2, L1, 02:15).

In lesson 3 they began to see if they could find a function that describes births per year as a function of time in order to integrate this and get their final result. They looked for birthrates and found data from year 1960 to 2015 and chose to perform linear regression in order to extrapolate these data back to the year 10.000 BC.

After this they went back to work with their first method again and got the result to be between 80 and 100 billion people (G2, Ass.).

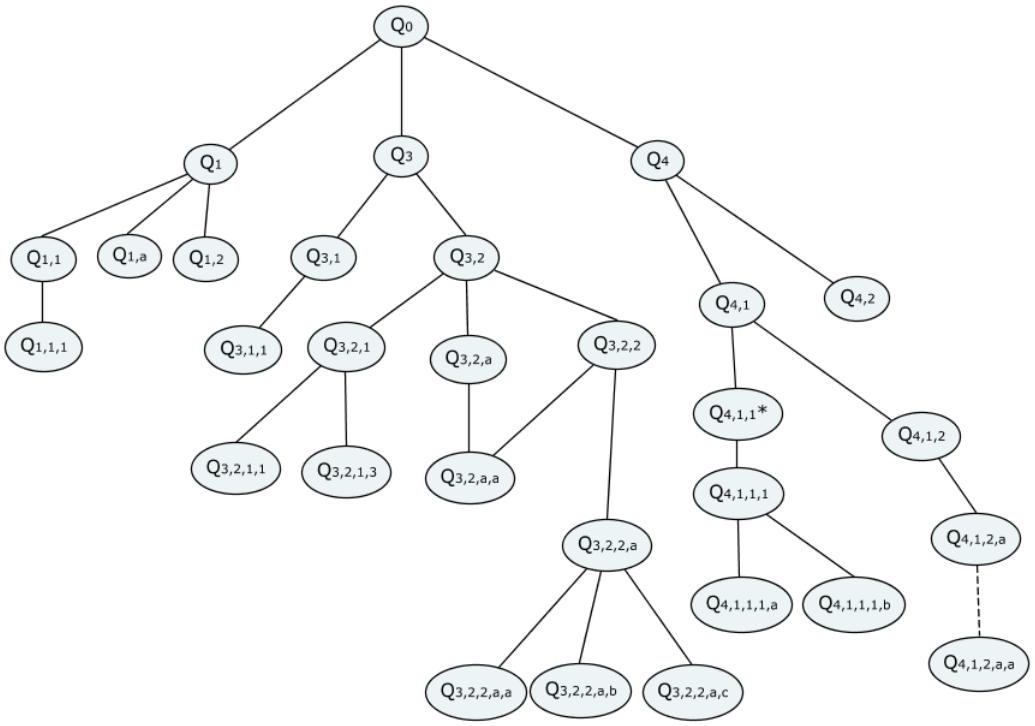


Figure 8.2: Tree diagram group 2

$Q_1$: What should the initial conditions for the calculation be? (G2, L1, 39:50)

$Q_{1,1}$: When did the first humans live? (G2, L1, 39:50)

$Q_{1,1,1}$: How do we define a human being? (G2, L1, 39:50)

$Q_{1,2}$: How big should the initial population be in our calculation? (G2, L3, 00:55)

$Q_{1,a}$: Do we count stillborns? How long should a person live before it counts? (G2, L2, 13:18)

$Q_3$: Can a calculation be based on a model of the population size as a function of time, N(t)? (G2, L1, 02:15)

$Q_{3,1}$: How can we use such a model to sum all the people without counting the same person more than once? (G2, L1, 02:15)

$Q_{3,1,1}$: What does the area under the graph represent? (G2, L1, 2:15)

$Q_{3,2}$: How can the population as a function of time be modelled? (G2, L1, 2:15)

$Q_{3,2,2}$: What data can we find on the world population size throughout history? (G2, L1, 2:15)

$Q_{3,2,a}$ Should the function be piece wise? (G2, L1, 2:15)

$Q_{3,2,a,a}$ What data can we find on the population size before and after 1850? (G2, Log1)

$Q_{3,2,2,a}$ What effect has different factors had on the population size? (G2, Log1)

$Q_{3,2,2,a,a}$ What effect has different diseases had? (G2, Log1)

$Q_{3,2,2,a,b}$ What effect has medicine had? (G2, Log1)

$Q_{3,2,2,a,c}$ What effect has wars had? (G2, Log1)

$Q_4$: Can a calculation be based on a model of the number of births per year as a function of time? (G2, L3, 12:00)

$Q_{4,1}$: How can the number of births per year as a function of time be modelled? (G2, L3, 12:00)

$Q_{4,1,1}*$: What data can we find on the number of births at different times in history?

$Q_{4,1,1,1}$: What data can we find on birthrates at different times in history? (G2, L3, 12:00)

$Q_{4,1,1,1,b}$ How many women between age 25 and age 29 have lived in the last century? (G2, L4, 03:31)

$Q_{4,1,1,1,a}$ What has the fertilityrate been? (G2, L4, 03:31)

$Q_{4,1,2}$: How can we interpolate (or extrapolate) from a set of data for the number of births in different years. (G2, Ass.)

$Q_{4,1,2,a}$: Is linear regression on all years with data the best way? (G2, Ass.)

$Q_{4,1,2,1,a,a}$: Why do we get such a big number? (G2, L4, 06:48)

$Q_{4,2}$: What does the area under the graph represent? (G2, L3, 12:00)

Group three

The group began by considering $Q_1$ and discussing whether to go with $Q_2$ or $Q_3$ (G3, L1, 02:00). In the next lessons they searched for birthrates but later on looked for data on population size which they found and used. They plotted data points in excel and discussed what kind of regression to use (G3, L3, 33:10). On the last day they were inspired by a presentation by group one and turned to make a piecewise linear function in maple. They prepared to integrate it and considered the average life expectancy but did not manage to finish the calculations in time (G3, L4, 01:14:54).

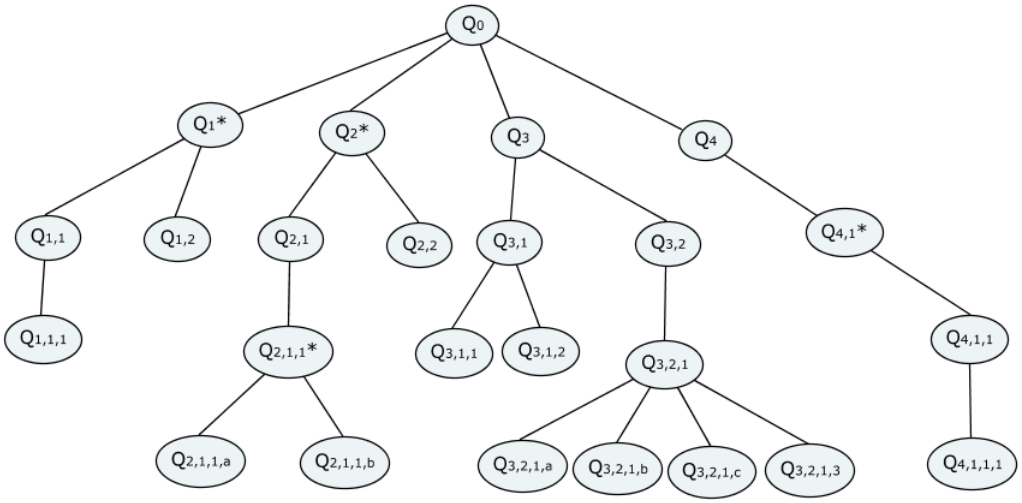


Figure 8.3: Tree diagram group 3

$Q_1*$: What should the initial conditions for the calculation be?

$Q_{1,1}$ When did the first human live?(G3, L1, 02:00)

$Q_{1,1,1}$: How do we define a human being?(G3, L1, 08:10)

$Q_{1,2}$: How big should the initial population be in our calculation.(G3, L3, 23:05)

$Q_2*$: Can a calculation be based on a model of the number of deaths per year as a function of time?

$Q_{2,1}$: How can we model this?(G3, L2, 01:20)

$Q_{2,1,1}*$: What data can we find on the number of deaths throughout history?

$Q_{2,1,1,a}$: Which factors influence the number of people who die each year?(G3, L1, 34:55)

$Q_{2,1,1,b}$: How many people die each year?(G3, L2 01:20)

$Q_{2,2}$: How many people are alive today?(G3, L2, 01:20)

$Q_3$: Can a calculation be based on a model of the population size as a function of time, $N(t)$?(G3, log3)

$Q_{3,1}$: How do can we use such a model to sum all the people without counting the same person more than once?(G3, L4, 00:25)

$Q_{3,1,1}$: What does the area under the graph represent?(G3, L4, 01:43)

$Q_{3,1,2}$: What has the average life expectancy been over human history? What has it been at different times?(G3, L4, 01:14:54)

$Q_{3,2}$: How can the population as a function of time be modelled?(G3, l3, 28:45)

$Q_{3,2,1}$: How can we interpolate from a set of data for the world population at different times?(G3, L3, 33:10)

$Q_{3,2,1,3}$: Is exponential regression on all years with data the best way? (G3, L3, 33:10)

$Q_{3,2,1,a}$: Is logistic regression on all years with data the best way? (G3, L3, 33:10)

$Q_{3,2,1,b}$: Is power regression on all years with data the best way? (G3, L3, 33:10)

$Q_{3,2,1,c}$: Is linear regression on all years with data the best way? (G3, L3, 33:10)

$Q_4*$: Can a calculation be based on a model of the number of births per year as a function of time?

$Q_{4,1}*$:How can the number of births per year as a function of time be modelled?

$Q_{4,1,1}$: What data can we find on the number of births at different times in history? (G3, L2, 21:35)

$Q_{4,1,1,1}$: What data can we find on birthrates at different times in history?(G3, L2, 19:57)

Group four

This group began as the other groups with answering some of the questions under $Q_1$ (G4, log1). They then considered counting the number of deaths but decided on conting births (G4, L1, 19.40). They claimed that they wanted to model the birthrate but with that word they actually meant the number of births per year (G4, L1, 44:50 og 46:55). To do this they found data on the population size at different times and tried to first model the population as a function of time(G4, L2, 8:55 and 12:00). From that model and from an estimate of the average life expectancy they tried to find the number of births per year (G4, L3, 2:25 and 4:00). They are not succesful in carrying out these calculations but did consider that to find an answer to $Q_0$ they should integrate the function for births per year that they sought (G4, L1, 46:55)

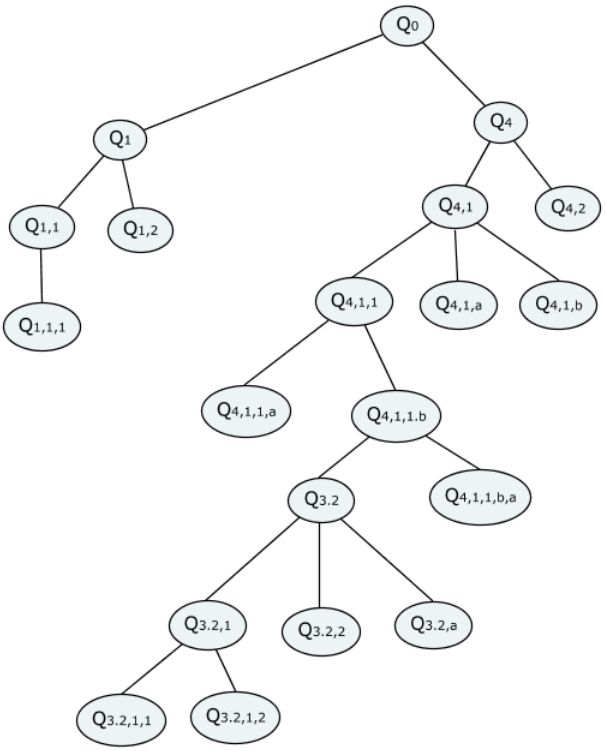


Figure 8.4: Tree diagram group 4

$Q_1*$: What should the initial conditions for the calculation be?

$Q_{1,1}$ When did the first human live?(G4, L1, 04:45)

$Q_{1,1,1}$: How do we define a human being?(G4, L1, 04:45)

$Q_{1,2}$: How big should the initial population be in our calculation.(G4, L2, 13:05)

$Q_4$: Can a calculation be based on a model of the number of births per year as a function of time?(G4, L1, 44:50)

$Q_{4,1}$: How can the number of births per year as a function of time be modelled?(G4, L2, 19:05)

$Q_{4,1,1}$: What data can we find on the number of births at different times in history?(G4, Log1)

$Q_{4,1,1,b}$: Can the number of births be found from data on populations size at different times?(G4, L3, 09:32)

$Q_{4,1,1,b,a}$: What perentage of people die each year?(G4, L3, 10:30)

$Q_{3,2}$: How can the population as a function of time be modelled?(G4, L2, 19:05)

$Q_{3,2,1}$: How can we interpolate from a set of data for the world population at different times?(G4, L2, 25:58)

$Q_{3,2,1,1}$: : Is linear interpolation between years with data the best way?(G4, L2, 12:00)

$Q_{3,2,1,2}$: Is exponential interpolation between years with data the best way?(G4, L2, 25:58)

$Q_{3,2,2}$: What data can we find on world population size throughout history?(G4, L2, 02:30)

$Q_{3,2,a}$ Should the function be piece wise? (G4, L2, 25:58)

$Q_{4,1,1,b,a}$: How many percent of people die per year?(G4, L3, 10:30)

$Q_{4,1,a}$: Should the function be piece wise? (G4, L1, 44:50)

$Q_{4,1,b}$: When does the population begin to grow?(G4, Log1)

$Q_{4,2}$: What does the area under the graph represent?(G4, L1, 44:50)

Group five

Group five first considered basing their modelling on the number of deaths or the population size corresponding to $Q_2$ and $Q_3$ and they chose to go with the latter (G5, L1, 13:30 and 14:55). But then they found data on the number of births in Denmark from 1977 to 2015 and used polynomial regression and integrated the polynomial. This was their calculation of the number of births in Denmark from 1977 to 2015, which made them trust their method (G5, Log1). They then sought after data on births for the whole world. This proved difficult but they did find data on the population size at different times (G5, L3,47:00). In the last lesson they needed more time so they decided to use regression on their data to model the population size as a function of time, integrate the function and divide the result by an average life expectancy (G5, L4, 11:00 and Ass.). They used polynomial regression again but also logistic regression and an average life expectancy of 40 to 50 years. This gave them a result around 28-33 billion people (G5, Ass.).

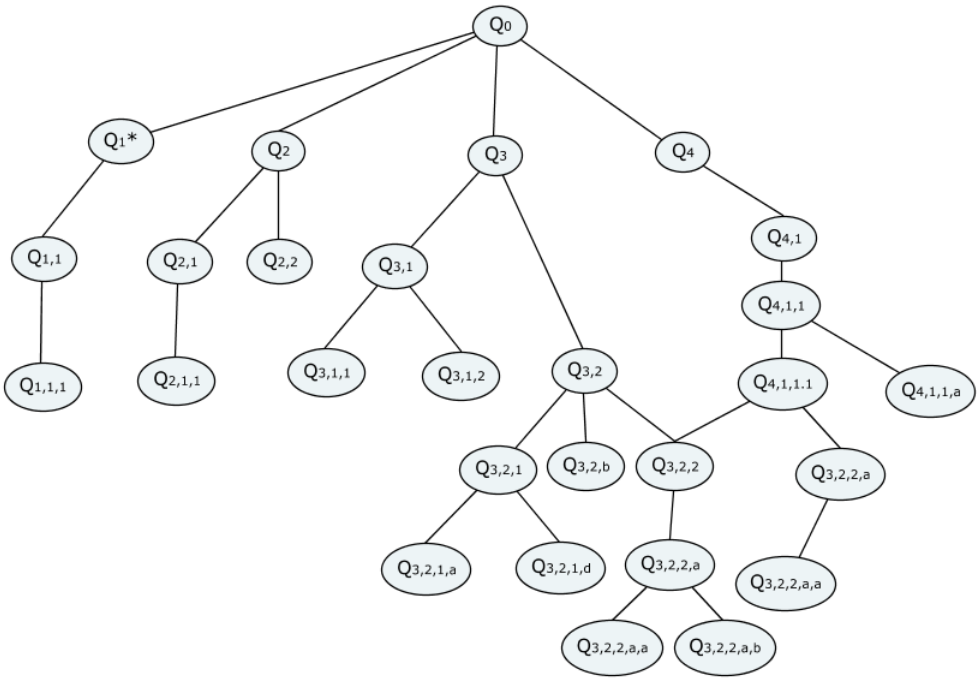


Figure 8.5: Tree diagram group 5

$Q_1*$: What should the initial conditions for the calculation be?

$Q_{1,1}$ When did the first human live?(G5, L1, 34:20)

$Q_{1,1,1}$: How do we define a human being?(G5, L1, 57:20)

$Q_2$: Can a calculation be based on a model of the number of deaths per year as a function of time?(G5, L1, 13:30)

$Q_{2,1}$: How can we model this?(G5, L2, 01:42)

$Q_{2,1,1}$: What data can we find on the number of deaths throughout history?(G5, Log1)

$Q_{2,2}$: How many people are alive today?(G5, Log1)

$Q_3$: Can a calculation be based on a model of the population size as a function of time, $N(t)$?(G5, L3, 45:32)

$Q_{3,1}$: How do can we use such a model to sum all the people without counting the same person more than once?(G5, L3, 51:00)

$Q_{3,1,1}$: What does the area under the graph represent?(G5, L3, 51:00)

$Q_{3,1,2}$: What has the average life expectancy been over human history? What has it been at different times?(G5, Ass.)

$Q_{3,2}$: How can the population as a function of time be modelled?(G5, L3, 45:32)

$Q_{3,2,1}$: How can we interpolate from a set of data for the world population at different times? (G5, L4, 34:40)

$Q_{3,2,1,a}$: Is logistic regression on all years with data the best way?(G5, L4, 34:40)

$Q_{3,2,1,d}$: Is polynomial regression on all years with data the best way? (G5, L4, 34:40)

$Q_{3,2,b}$: How much data should we use? (G5, L4, 16:00)

$Q_{3,2,2}$: What data can we find on world population size throughout history?(G5, L3, 14:59)

$Q_4$: Can a calculation be based on a model of the number of births per year as a function of time?(G5, L1, 13:30)

$Q_{4,1}$: How can the number of births per year as a function of time be modelled?(G5, L1, 13:30)

$Q_{4,1,1}$: What data can we find on the number of births at different times in history?(G5, L1, 13:30)

$Q_{4,1,1,a}$: How many people where born in Denmark from 1977 to 2015?( G5, L1, 13:30)

$Q_{4,1,1,1}$: What data can we find on birthrates at different times in history?(G5, L3, 08:15)

$Q_{3,2,2,a}$: What effect has different factors had on the population size?(G5, L3, 10:50)

$Q_{3,2,2,a,a}$: What effect has different diseases had? (G5, Log1)

Group six

This group also focused on modelling the population size as a function of time (G6, Log2). They considered how they could not just integrate the function but also need an average life expectancy. They found some data but did not finish their modelling for the final presentation. Instead they found data on the number of people alive and the birthrate for each year from 1960 to 2015 (G6, L5, 42:50). They multiplied these numbers for each year and added them all together to calculate the number of births since 1960 (G6, L5, 57:15)

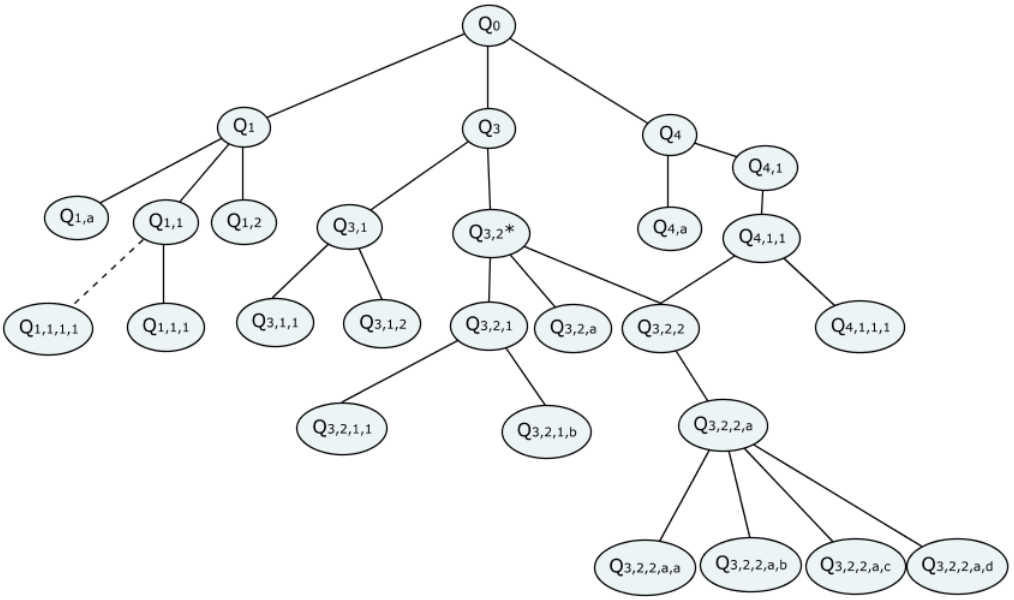


Figure 8.6: Tree diagram group 6

$Q_1*$: What should the initial conditions for the calculation be?

$Q_{1,a}$. Do we count stillborns? How long should a person live before it counts?(G6, Log1)

$Q_{1,1}$: When did the first humans live?(G6, Log1)

$Q_{1,1,1}$: How do we define a human being?(G6, Log1)

$Q_{1,1,1,1}$: How big of an influence do the choices in $Q_{1,1}$ and $Q_{1,2}$ have on the final result?

$Q_{1,2}$: How big should the initial population be in our calculation?(G6, L1, 02:15)

$Q_3$: Can a calculation be based on a model of the population size as a function of time, $N(t)$?(G6, Log1)

$Q_{3,1}$: How do can we use such a model to sum all the people without counting the same person more than once?(G6, L3, 05:28)

$Q_{3,1,1}$: What does the area under the graph represent?(G6, L3, 05:28)

$Q_{3,1,2}$: What has the average life expectancy been over human history? What has it been at different times?(G6, L3, 23:30)

$Q_{3,2}*$: How can the population as a function of time be modelled?

$Q_{3,2,1}$: How can we interpolate from a set of data for the world population at different times?(G6, L5, 08:53)

$Q_{3,2,1,1}$: : Is linear interpolation between years with data the best way?(G6, L5, 08:53)

$Q_{3,2,1,b}$: Is polynomial regression on all years with data the best way? (G6, L5, 20:44)

$Q_{3,2,a}$ Should the function be piece wise?(G6, L5, 26:18)

$Q_{3,2,2}$:What data can we find on world population size throughout history?(G6, Log1)

$Q_{3,2,2,a}$ What effect has different factors had on the population size? (G6, Log1)

$Q_{3,2,2,a,a}$ What effect has different diseases had? (G6, Log1)

$Q_{3,2,2,a,b}$ What effect has medicine had? (G6, Log1)

$Q_{3,2,2,a,c}$ What effect has different wars had? (G6, Log1)

$Q_4$: Can a calculation be based on a model of the number of births per year as a function of time?(G6, L5, 50:50)

$Q_4, a$: How do we sum the number of births from each year?(G6, L5, 54:20)

$Q_{4,1}$: How can the number of births per year as a function of time be modelled?(G6, L5, 52:30)

$Q_{4,1,1}$: What data can we find on the number of births at different times in history?(G6, L5, 50:50)

$Q_{4,1,1,1}$: What data can we find on birthrates at different times in history?(G6, L5, 50:50)

### 8.3.2 Comparing the SRP's

The following diagram shows all questions that were worked on in the teaching sequence. The coloring indicates how many of the groups 2-5 that worked on each question.

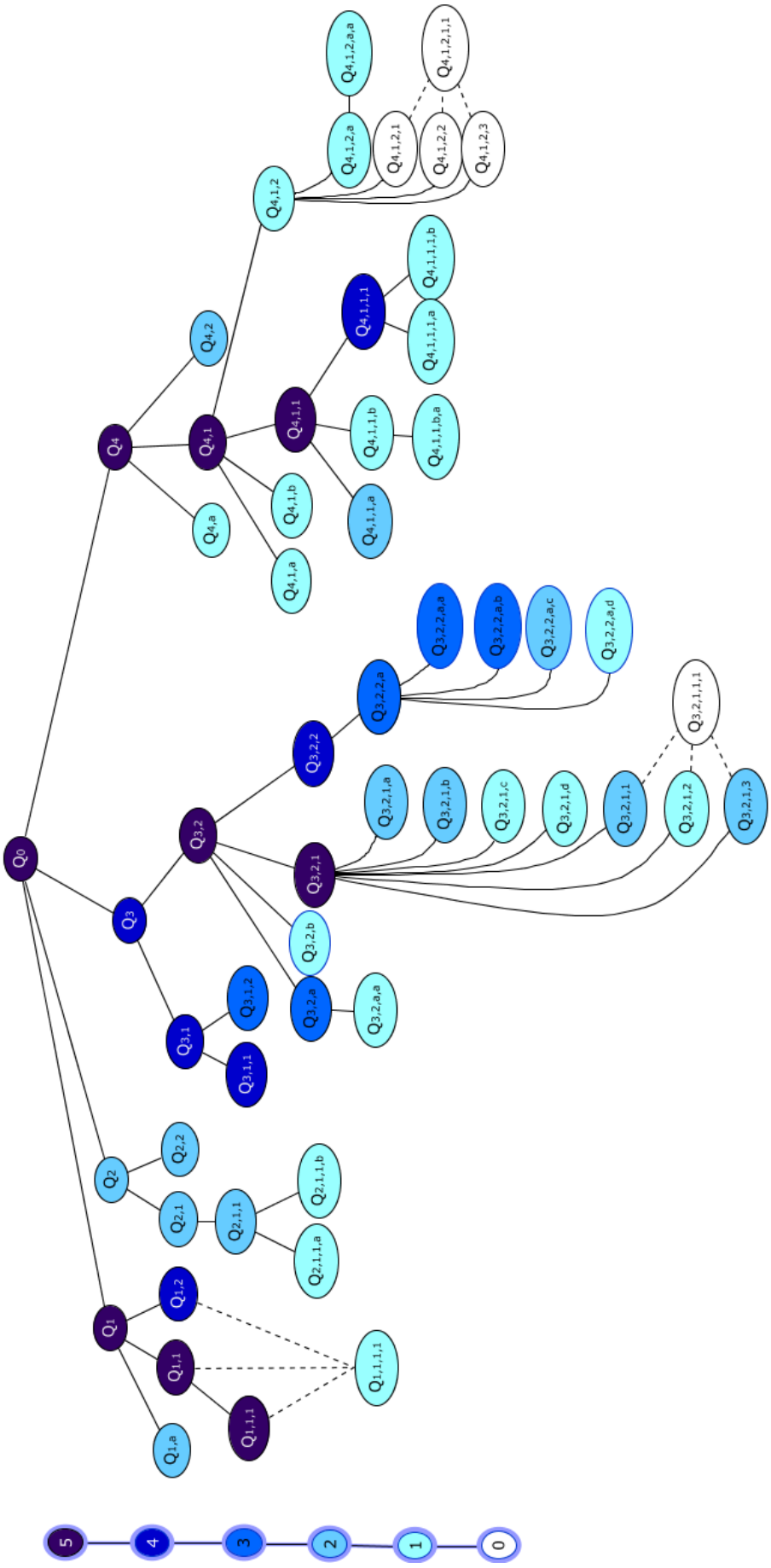


Figure 8.7: Tree diagram of groups combined

First of all we see that almost all questions from the a priori analysis were worked on by some students. $Q_{3,2,1,1,1}$ about comparing different interpolating methods was not posed and the different methods for interpolating with the birthmethod, $Q_{4,1,2,1}$, $Q_{4,1,2,2}$ and $Q_{4,1,2,3}$ were not worked on.

Compared to the a priori map the students went into more detail in some areas. For example when estimating the effects of factors such as war, disease and medicine on the population size and number of births as seen in $Q_{3,2,2,a}$ and subquestions. They also used different ways of interpolation namely different regression methods mentioned in $Q_{3,2,1,a-d}$.

In general all groups investigated the initial conditions thoroughly and so the branch from $Q_1$ is dark. Only two groups seemed to really consider counting deaths and in the end they couldn't follow through with that method.

All groups did at some point consider the idea of counting births and how to model births per year as a function of time. This is seen in the dark colors on branch four. Only group four kept to this method though.

The dark colors throughout branch three shows that most groups worked on modelling the population as a function of time in order to find an answer. Although group four used the method of counting births they also modelled the population first to calculate births from that model.

In the end of the sequence only three groups, one, two, and five, actually showed a calculation that gave an answer. All three groups used the person-year method, as described in chapter three, to calculate an answer and group three was also very close to finish calculations of an answer as they switched to the person-year method in the last lesson. The students' answers were 28-33 billion, 87-100 billion and 137 billion which seems to be fairly realistic and reasonable answers given the uncertainty of such a calculation. The answers we found in chapter 3 varied between 60 and 108 billion.

### 8.3.3 The students' use of media

All groups used the internet to search for media that could help them answer some of their questions. The groups included screenshots of their web history in the logbooks. The following analysis of this material shows what kind of media the groups searched for and found, and how they used it.

In general what the students searched for and found can be divided into at least three categories:

1. Searches for and use of media related to the initial conditions. In the first lessons the groups made a very extensive investigation of the definition of man, and when the first humans lived.

2. Searches for and use of media related to data on especially population size but also on births, deaths and fertility. The media they found here can be divided into two categories:

    - Media that gives a large amount of data from around 1950 until present time.
    - Media that gives data mainly on population size and that also contains data from periods long before 1950.

3. Searches for and use of media related directly to $Q_0$ or to the methods used to calculate it. It was easy for groups to find articles giving an answer to $Q_0$. Students rarely searched for the methods used for the calculation.

Examples of the use of media in relation to category 1:
Group two used searches such as *"The origin of man"*(our translation), *"differences between homo sapiens and homo sapiens sapiens"* and *"homo sapiens sapiens timeperiod"*. They found results with titles such as *"Homo sapiens – The Smithsonian Institution's Human Origins Program"* and *"the human evolution"* (our translation) (G2, Log1). This media enables students to answer some subquestions related to $Q_1$ as for example The Smithsonians Insitute claims homo Sapiens evolved 200,000 years ago [Smithsonian, 2017]. Other groups had very similar searches and results.

Examples of the use of media in relation to category 2:
Group four searched for *"World population graph"* and *"world population 10000BC"* and found the Wikipedia article *"World population estimates"* which gives 33 estimates of the world population from 70,000 BC. to 2015(G4, Log1) [Wikipedia-contributors, ]. Group five found the page worldpopulationhistory.org which provides an estimate for each year from year 0 (G5, Ass.) [Wasserman and Bliese, 2016]. Group three made searches as *"yearly births"*(our translation) and found information on fertility rate on the page data.worldbank.org (G3, Log1), a page on which one can find a plethora of population data from around 1950 and onwards [World Bank Group, ]. Also group two explored the data from this page (G2, Log1). Related to the search for data for the modelling some groups also researched things like diseases, medicine and war which might have affected the population numbers but no groups seemed to be able to incorporate this in their models (G2,Log2, G6,Log1, G4,Log1).

Important pages that the groups used for data were:

Data.worldbank.org: Data on many things related to demography from circa 1950.

Ourworldindata.org [Roser and Ortiz-Ospina, 2017]: Estimates of population size for each year form 10,000 BC.

Wikipedia.com – World population estimates: 33 estimates of population size from 70,000 BC.

Worldpopulationhistory.org: Estimates for population size for each year from year 0.

Examples of the use of media related to category 3:
From the data it appears that only group three made a direct search query for $Q_0$ (G2,Log1). Yet several groups stumbled upon media that gave answer to $Q_0$. Specifically groups one, two, three and six saw pages from either illvid.dk or historienet.dk that contained very short articles which gave a number around 107 billion people as an answer to $Q_0$ [Videnskab, 2009, Historienet, 2012]. The articles gave few or no answers on how to calculate the number but they stated explicitly that the number comes from the Population Reference Bureau in Washington D.C., which contains

the article *"How many people have ever lived on earth?"* mentioned in the a priori analysis [Haub, 2011].

Group two shortly after visited prb.org but did not use it further (G2, Log2). Group three made a search for the Population Reference Bureau and later found the article discussed in the a priori analysis (G3, Log1). They were skeptical about the article and its results because of its use of the words *"guesstimates"* and *"semi-scientific"* and they did not consider how the PRB had found their result (G3, L2, 13:18). Group six also searched for and found the article by PRB and considered how it was calculated: *"where are your sources mate "*(G6, L1, 16:37). But they did not figure it out and instead discussed further how they should do.

So it seems that the methods for calculating where mostly students' own ideas. Four groups found the article from PRB which gives some hints to how to calculate it but students do not seem to pick up on these hints and they do not seem to use the data it provides.

Besides using the internet to search for media the groups also held presentations/conferences where they shared their findings with each other. It has been difficult to determine to what extent the groups have used each other as media. The groups did all work on similar ideas and do not seem to get stuck which may have been because they had listened to the ideas of other groups. Despite there being multiple presentations only one person explicitly mentions the work of another group: *"I think their idea is very good (Red: about the presentation by group one)"*(G3, L4, 00:25). Following this the group actually changed their method to one similar to what they heard in the presentation.

During some presentations the methods for the calculation were discussed. An example is seen in the dialogue below from the first presentation in the first lesson. A member of group two had just presented an idea of using integration and an average life expectancy and another student asked:

> *A: I just have a question to that part how long do humans live uhm in relation to what relevance it has here*
>
> *B: you have to. You can't just when we take the area below the curve you can't just take it all over the place because it's. Humans live like longer time so if you take it from one day to the other then it's not like there suddenly is seven billion the one day and then seven billion the other because it is the same seven billion*
>
> *A: but is it not easier to describe with the birthrate then that is with how many there are born. Then you don't need to account for uhm*
>
> (G2, L1, 16:05)

This shows that some students did engage in the methods presented by other groups.

### 8.3.4 The students' praxeologies of integration

In order to answer $Q_0$ students were faced with tasks of the types described in the ERM in chapter two. A task like $T_{i2}$: *Determine the total change in a quantity whose*

*rate of change is known* was met when they wanted to estimate the number of people who had ever lived from a function of births per year as a function of time. A task of the type *Calculate and interpret the definite integral of function that describes a quantity as a function of time* was met when the students calculated from a model of the population size. This is a slightly more general task than $T_{i2}$ but the theoretical discourse belonging to it is the same. So students had to apply either instrumented techniques like $\tau_{ia}$ or manual techniques like $\tau_{ib}$ but in order to realize that this was necessary they probably had some idea of the theory, $\Theta_i$ justifying the technology and techniques. The FTC and the knowledge that the definite integral gives the size of an area doesn't explain in itself that integration must be applied to solve this task. But for example the definition of the integral as the limit of a Riemann sum may encourage a Riemann interpretation of the integral, that is, understanding the integral as a sum of "very small pieces" which could justify its application in this context.

Group one did well on modelling the world population as a function of time and they realized that they had to do something with integration in order to find an answer but it seems that they did not understand what this integral represented.

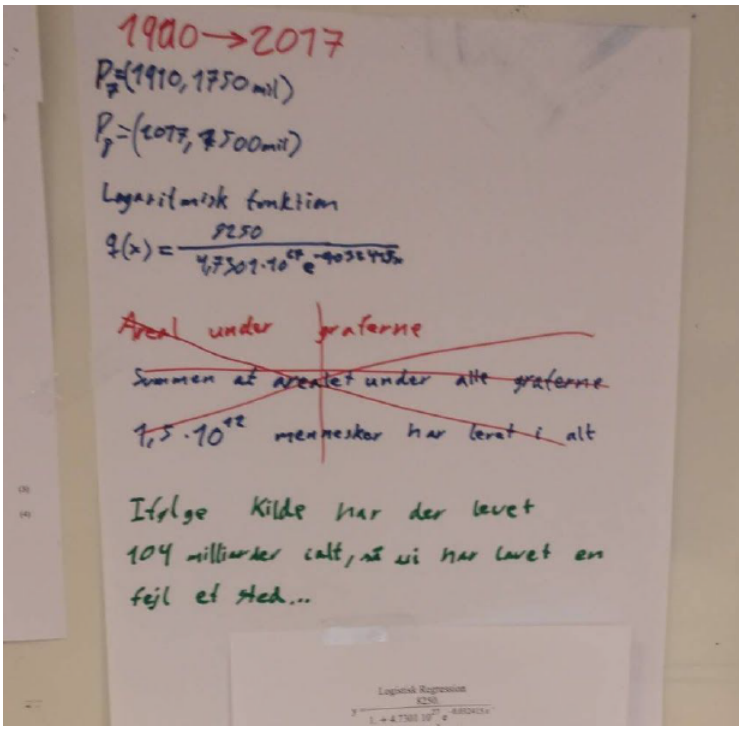


Figure 8.8: Picture from the assignment from group one (appendix B.1).

As is seen in figure 8.8 of a picture from their assignment they first wrote: *"Area under the graphs – The sum of the areas under all the graphs –* $1.5 * 10^{12}$ *people have ever lived "* (G1, Ass., p.4) . So they thought that the area under the graph was an answer to how many people had ever lived. They were critical about their own result though as they write below: *According to source there have lived 104 billion in all so we have made a mistake somewhere.* The incorrect conclusion was crossed out and corrected during presentation as other students pointed out that they needed to divide this number by an average life expectancy. So we see that group one was able to perform the praxis part of the encountered praxeology. They used $\tau_{ia}$ when

they used CAS to calculate the definite integral. But they did not interpret the result correctly which shows that they may have lacked some understanding of the theory. They did understand to interpret it as the size of the area below the graph but they did not understand what the area represented in the units given. This rises the questions of how they got the idea to integrate in the first place and the answer could be because they saw in presentations that other groups did that.

The first discussion on how to answer $Q_0$ happened in the groups in the first lesson without the use of any media. After a few minutes of discussing how to model the population a member from group two mentioned integration and the group realized that a function of population per time needed to be integrated and that an average life expectancy should be accounted for. The discussion about integration of the population function went:

> *C: uhm so if we can find that one about it then if it is piecewise or not we can integrate under it*
>
> *B: yes yes so*
>
> *A: all right yeah cool*
>
> *B: so we'll get a huge area*
>
> *A: because that's a way to find the number of people*
>
> *B: you know it's how you would do it mathematically*
>
> *A: no no because then you have to realize that the number of people below the graph then you must realize that one person on average live a generation yes but now we live about three generations right*
>
> *B: yes but you can't say that then every day right then there will there lives this many people and then they die that day and then there's born this many the next day. You have to have a sort of average age right. We have to like put some sort of average age in as a parameter so it is very different*
>
> (G2, L1, 02:15)

It is hard to say exactly how C came up with the idea of integrating the function. B mentions that they get an area and A mentions "*people below the graph*" which indicates that their interpretation of the integral as an area is strong. Yet they realize that they are not done. The fact that B says that you can't say that all people die one day and are born the next indicates that he also has some idea of interpreting the integral as a sum. Group two presented these ideas of integration and average life expectancy to the class in the first lesson. From their assignment it is evident that group two also understood what the integral represented when integrating births per year as a function of time as they write " *We could then calculate the integral of the function and that would be the number of people who had lived*"(G2, Ass.).

One other group, group five, mentioned integration in a group discussion before the presentation of group two. In the dialogue below student A opened the group's first discussion on how to solve the problem by proposing integration of a birth function:

*A: what we do right we find the birth number for as many years back as we can right and then we make curves over it more or less and then we integrate that fucking curve*

*B: so we have linear over year and then how many people who lived at that time*

*A: no how many were born at that time.*

*C: but you don't consider that some of them are the same people that lived the same year*

*A: sure sure it is just the birth number*

*C: yes okay*

(G5, L1, 13:30)

This followed a discussion on how to find data on births. They do not give clues to an area interpretation in the short dialogue. Getting the idea to use integration in this task of type $T_{i2}$ might indicate a reasonable understanding of the theory $\Theta_i$ and student C, probably thinking about integrating a population function, mentions the problem of it being the same people living over time. This idea could come from seeing the definite integral as sums. In their logbook from the first lesson they show a calculation of the number of people born in Denmark in the period 1977-2015 by taking the integral of a function of births per year as a function of time (G5, Log1). In the final assignment they had changed method: *"By integration we found the number of people-years lived since year zero. By dividing this number with the average life expectancy we found the number of people that have lived."*(G5, Ass.).

Group three also got the idea that the integral of the population gave them the number of years ever lived. In the last logbook they wrote: *"The plan was to take the integral for the combined function to find the number of years that had been lived to thereafter divide this with an estimated average life expectancy which will give a good guess on an answer to how many there have lived on earth."* (G3, Log3). But it seems this idea was inspired by group one at a very late stage as one student a presentation in lesson four said *"I think their idea is very good (Red: about the presentation by group one)"*(G3, L4, 00:25). They then considered what units they will get when integrating:

*A: yes then we have some data points here right. Then we say in year one hundred there was zero point three billion right. So we just take that because then we get the unit when we take the area under the graph*

*B: because then we get which unit*

*A: then we get person-years and then we take the average life expectancy for a person*

(G3, L4, 01:43)

This group also considered using the instrumented technique $\tau_{ia}$ to solve the task but the theory that justifies why they should calculate the definite integral and what that integral represents may not have been clear to them from the beginning since they did not consider using that technique until they heard it from others late in the process.

Group four didn't manage to find an answer but it seems that some group members did have an idea about how to interpret the integral of the functions. They said: *"A: but I think that if you have the number of people in that year then if you integrate you will find how many have been born."*(G4, L3, 04:30). So A introduces the idea of integration to the group, perhaps because he heard from the presentations that other groups used integration, but his incorrect conclusion indicates that he does not fully grasp the theory behind and therefore interprets it wrongly. But a discussion later continues:

> *A: but if you have how many there are in that specific year then you can integrate it and then you can get how many there have ever lived.*
>
> *B: Yes but the problem is that the problem is that what you do, right. You can actually not do that. And the reason why you can't do that is because that uhm you have to have the birthrate because otherwise they go into the statistic multiple times so it's like uhm. If you live a hundred years then you are born a hundred times do you see. So it will be way off.*
>
> (G4, L3, 06:20)

B mentions that *"they go into the statistics multiple times"* which is similar to the way group two argued and indicates some idea of interpreting the integral as a sum. The student does not necessarily have an interpretation of the integral as a sum product since he does not mention the unit it will give but the quote indicates an idea of summing the function values.

The same kind of arguments were also seen in group six. They first discussed integration after hearing other groups present in the beginning of lesson three and one member questioned the use of integration:

> *A: I don't understand it yet*
>
> *B: why they look at the area under the graph*
>
> *A: yes*
>
> *B: you make a graph over how fast it grows and then everything below is all those who have also lived*
>
> (G6, L3, 05:28)

This is again a clear indication of an area interpretation. Later in the same lesson they bring arguments like group two and four did on why an average life expectancy is necessary:

*A: couldn't you say that if you look at the rise in population between two data points and divide it with the average life expectancy the we will know how many have lived –ish*

*B: yes roughly speaking*

*A: but why do you have to have the part with average life expectancy. Because for example here there's fifty thousand two hundred years. There's no one that lives that they live forty years and then another person lives forty years perhaps*

*B: we are looking at how many there are each year*

*A: that's of course right*

*B: it's just about how many are there now how many are there now how many are there now. But is that not it. That if you just put all these together and make it into a graph and then calculate how many there are below the graph then you will get all those who have lived.*

*A: because there are also a whole lot that dies*

*B: yes okay*

(G6, L3, 39:26)

This idea of "*...how many are there now how many are there now...*" could come from the idea of seeing the integral as a sum but he continues with *...calculate how many there are below the graph...* and makes a wrong conclusion which his partner corrects. There seems to be again an understanding of the definite integral as a sum of function values but not an understanding of the integral as sum products. As discussed a Riemann-sum interpretation, that is, interpreting the definite integral as a sum of very small pieces would probably make it more clear to students what unit they get when they integrate and this would make the use of an average life expectancy easier to understand.

So all groups where met with a task of type $T_{i2}$ or of its more generalized version and all groups approached it using the instrumented technique $\tau_{ia}$. No group did at any point consider to carry out the calculation manually as in $\tau_{ib}$. Furthermore many students showed signs of interpreting the integral as an area and many had trouble with the interpretation of the integral in the context. This could indicate that the only part of the theory $\Theta_i$ they know well is about the definite integral of a function being the signed area below the graph of the function.

Yet a few students did manage to use arguments about the use of an average life expectancy which could indicate that they interpreted the integral as a sum which could have helped them realize that integration was a viable technique to use in the context.

The idea of using integration appeared first in groups two and five, most likely independently of each other. Later use of integration by the other groups may have come from them hearing group two present the idea. In some groups the idea may have appeared from an understanding of integration as a sum of function values which some students showed signs of.

### 8.3.5 The students' praxeologies of regression

All groups used some sort of interpolation or extrapolation for their modelling that is they encountered the task of the type $T_r$.

An example was seen when group two extrapolated very far back from a small data set. Group two found data on birthrates going back to year 1960 and then tried to use excel to apply linear regression, $\tau_{r1}$ on the data in order to extrapolate to 10,000 BC (G2, Ass.). This is illustrated on the picture from their assignment in figure 8.9.

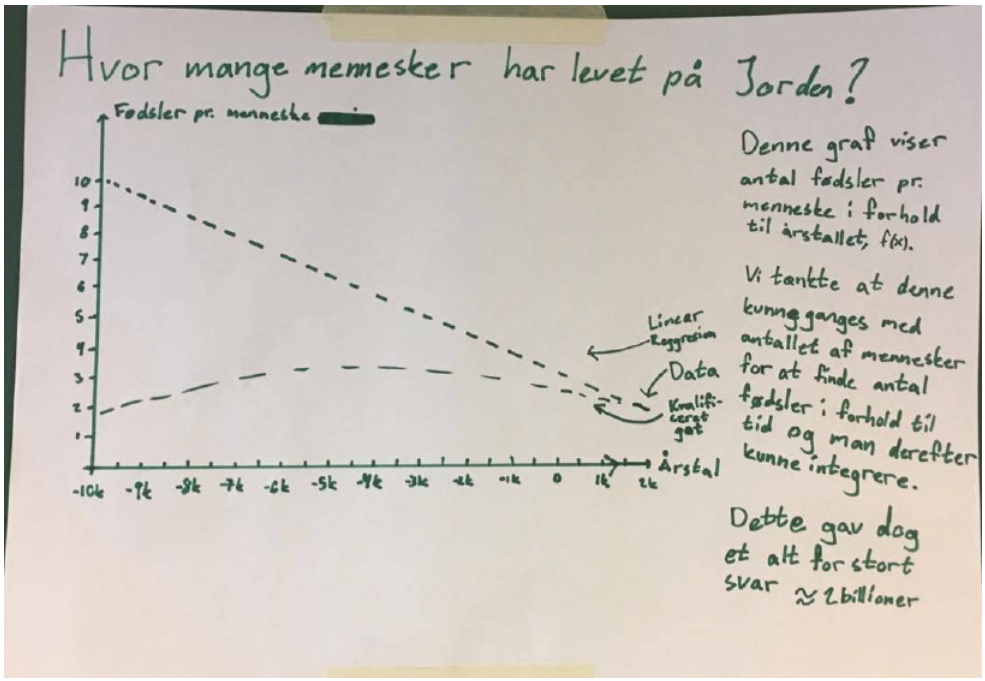


Figure 8.9: Picture from the assignment from group two (appendix B.2).

Afterwards they realized though that this gave an unrealistic result:

> *A: do you know what the problem with this is guys*
>
> *B: what is the problem*
>
> *A: that our data lies like here*
>
> *C: oh yes you draw that one. I don't know why you assume you have made that one. oh yes it will be hard to extend it*
>
> *B: births per human per year. Holy shit yes we have reproduced*
>
> (G2, L4, 31:15)

This was the only case of extrapolation in the teaching sequence. It was more common that groups used interpolation on their data points. Group two also did this. On data for the population size in different years they used exponential regression as they stated in their assignment: *"We found that the function was an exponential function but as you can see on figure 1 below it doesn't fit perfectly."*(G2,

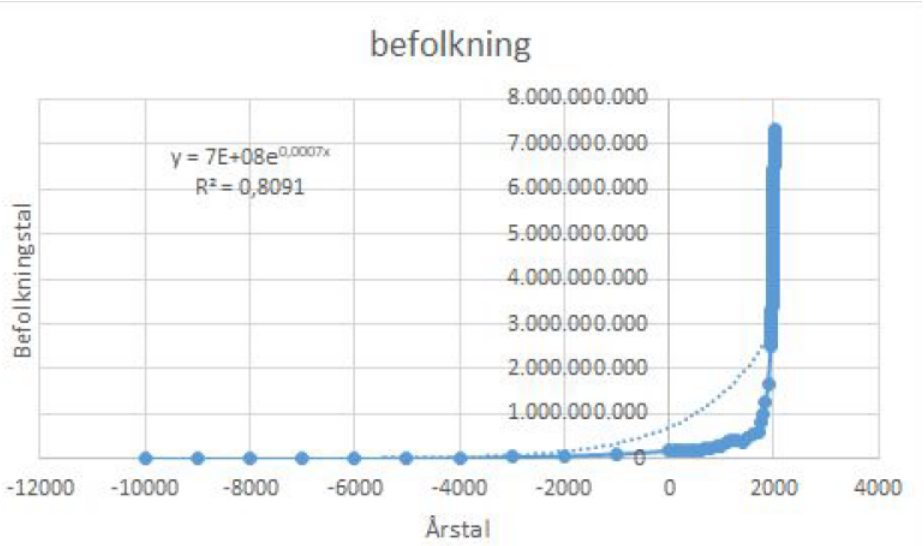


Figure 8.10: Picture from the assignment from group two (appendix B.2).

Ass.). Their figure can be seen in figure 8.10. Their reasoning for this choice does unfortunately not appear in the data.

In general the students tended to use technique $\tau_{r1}$. They used CAS to perform a regression analysis but they did not give many arguments on why they chose to interpolate the way they did. When group three modelled population as a function of time they also initially attempted to use regression: *"no no open the calculation window and then we make a regression analysis on it."*(G3, L3, 30:35). So they plotted their data and discussed which regression analysis was better:

> *B: yes wow that is exciting then you just need to find something that fits it*
>
> *C: linear no*
>
> *B: think*
>
> *C: okay logarithm*
>
> *A: I think it is power*
>
> *B: uhm that is not so good power is better uhm okay exponential is better uhm okay*
>
> *A: it doesn't fit that well*
>
> *B: doesn't' fit well actually*
>
> (G3, L3, 33:10)

So it seems that they used the technique $\tau_{r1}$ properly but they tried to evaluate which function fits data better by merely looking at the graph of the function and the plotted data points. They do not say anything about where the data comes from in order to make this evaluation. Another example is seen when group four wanted

to use CAS to make regression analysis on a larger set of data. As the following dialogue shows it appears that they tried different types of regression not knowing how they work and then evaluated by seeing how well the graph for the function fits the data points and their expectation for the development of data:

*A: but you can choose another kind of*

*B: then try*

*A: what should we try*

*B: uhm not exponential. Try polynomium*

*A: polymonial*

*B: well the population is not going to drop again I can tell you that*

(G5, L4, 35:55)

They tried polynomial regression and did probably not realize that a high enough degree polynomial can fit a finite set of points arbitrarily well. Their regression analysis gave a polynomial that dropped after the last data point and this was the group's reason to discard this regression. Other groups also used polynomial regression. From the first logbook of group five we see that they initially used polynomial regression on a dataset so they could afterwards integrate it (G5, Log1) and in the end of the lessons they used both polynomial and logistic regression (G5, Ass.). Their discussion went:

*A: we would like to use the one that's called polynomial*

*B: logarithm*

*C: it looks good doesn't it*

*B: well it is not bad but the other one is even more precise, the nine-th degree polynomial*

(G5, L4, 34:40)

Also group six tried polynomial regression but one member understood it better as the following dialogue shows:

*A: what is a first degree polynomial. Then it is just a line right*

*B: yes*

*A: can it not go higher than sixth degree*

*C: apparently not*

*B: just give me something*

*A: then take that one. That's where there is least well it has the best uhm fit on it well you can see there is more where it just follows.*

*B: what does the power look like*

*A: it looks fucked up because it's a sixth degree polynomium*

*C: guys you should take a polynomium and then go really many degrees up*

*B: yes but it won't give us more degrees than six*

*A: what does a sixth degree polynomium look like*

*B: there*

*C: it is the one that fits best of all of them*

*D: it always does. If you have a polynomium with enough degrees you can fit any data set*

(G6, L4, 25:08)

So group six also tried whatever regression that would give a function that was close to the data but one group member knew that with a high enough degree polynomial they could fit a data set arbitrarily well.

In general it appears that students very uncritically perform $\tau_{r1}$ by applying the regression analysis provided by CAS and their arguments for which type of analysis is better seem to be mainly based on the visuals. Almost no consideration is done about how a population might develop. This theory, $\Theta_r$, about considering R-values and the nature/context of the data is lacking. Yet by applying a high degree polynomial regression they will get a function which they can then integrate using CAS but they do not have proper arguments for their choice of type of regression.

Besides using CAS for regression on large data sets, some groups did at some point also use linear interpolation between each two neighboring points of smaller data sets. Group one did this on the early intervals in their calculation and performed a manual calculation, corresponding to $\tau_{r2}$, as seen in figure 8.11 (G1, Ass.).

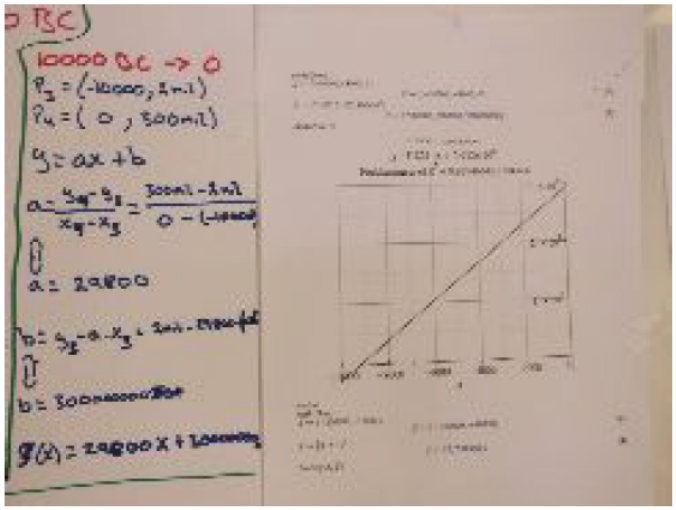


Figure 8.11: Picture from the assignment from group one (appendix B.1).

As mentioned earlier group three changed method in the last lessons as they were inspired by a presentation by group one. They then chose some data points to define intervals for which they used CAS to make linear functions based on the endpoints of the intervals (G3, Log3). This is seen in figure 8.12.

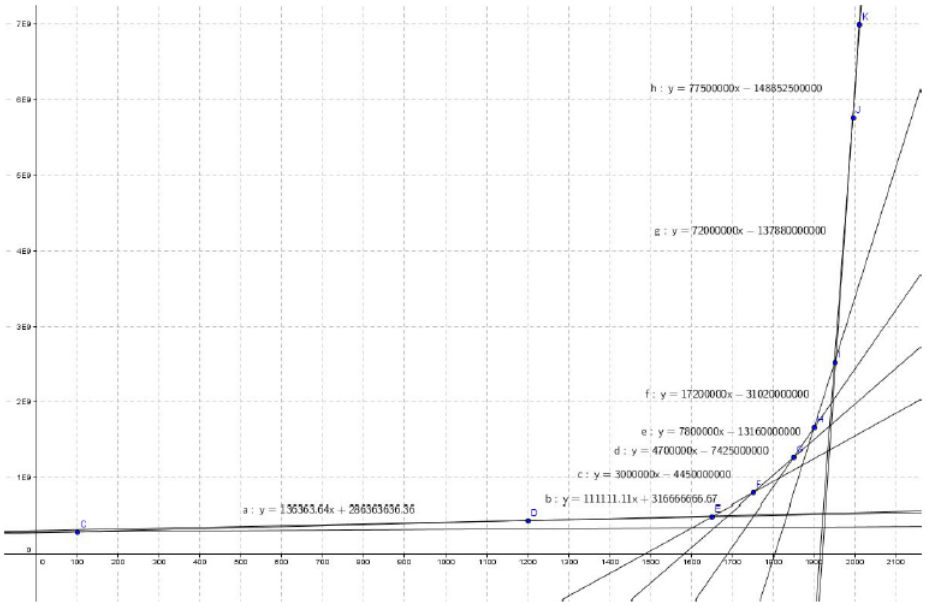


Figure 8.12: Picture from the logbook from group three (appendix C.3).

Their choice of using a linear function was not explicitly stated but one could think that it was chosen for simplicity and for being monotonically increasing (when using their data) which may fit their idea of population growth.

Also group four did this in lesson two and they actually discussed and gave arguments for why they chose a linear function and not an exponential function for the interpolation:

> *A: can't you just say that it is linearly increasing there*
>
> *B: I can say that it is linearly increasing and then from zero and then just over to that*
>
> *A: yes precisely what I thought*
>
> *B: and to two million. Then there comes more and more people. Because they have hard living conditions so it doesn't go so fast it is not exponential as it is done with the rest otherwise it is a little exponential in the end right*
>
> *A: and they have not invented agriculture yet so that is why*
>
> (G4, L2, 12:00)

So they actually used arguments on how populations develop to evaluate which monotonic function, linear or exponential, to make it. But again the their theoretic knowledge on the behaviour of functions is weak as he says it's not exponential

since the growth should not be too fast except for in the end. But when making the functions from the same two points the linear will grow faster in the beginning and the exponential will grow faster later on which sounds like his idea of population growth.

So most groups used CAS to make regression analysis of their data and some groups determined linear functions between neighboring data points in order to interpolate. The students had a good grasp of the instrumented technique $\tau_{r1}$ and the manual tehcnique $\tau_{r2}$ but in general their understanding of the theory $\Theta_r$ was bad and thus their choice of functions were bad.

# Chapter 9

# Discussion and conclusions

All students participated actively in the SRP on answering the question "*how many people have ever lived?*" and they were engaged in a real Study and Research Path. There was no single predetermined answer to be found and there was no unique way of finding an answer. So it was necessary for the students to consult different media which they found online and study it and the students performed research as they adapted their praxeologies on for example interpolation and integration to solve the problem.

Guided by RQ1 our a posteriori analysis showed that the different groups went on several different paths in their pursuit of an answer yet all of them remained close to the three main branches laid out in the a priori analysis: counting deaths, counting births and counting person-years. As such the a priori analysis could be used as a tool to predict which praxeologies the SRP would allow the students to encounter. Jessen also reports that an a priori analysis is an important tool for the design of the SRP and for the choice of the generating question [Jessen, 2017]. This predictability makes it easier to reach some intended parts of the curriculum such as integrals and application of mathematics while still working on an SRP with an Herbatian attitude in a manner much like researchers working on big questions.

Regarding RQ2 we found that in order to determine different parameters for their calculation and get useful data students had to study media regarding things like history and demography. The reason for and meaning of studying these parts came from the quest to answer the big relevant question $Q_0$. Likewise the reason to actually apply mathematical techniques on a real world problem came from this quest. As such the sequence avoided *visiting monuments* like definite integrals and regression since the reason for using these techniques was obvious from the context and brought up by the students themselves. So the sequence could give students an insight into how mathematics could be used in combination with other subjects to answer big questions in the social sciences. Kuzuoka and Miyakawa have tested a similar SRP in Japanese lower secondary school and they also explained that such a multidisciplinary inquiry is a useful way to provide the rationale for the mathematical knowledge as well as the knowledge from other disciplines [Kuzuoka and Miyakawa, 2018].

In the first lessons students spent a lot of time on finding information on the initial conditions for their calculation. They spent so much time that the teacher encouraged the students to move on from that part of the quest. Studying these things are an important part of the SRP. Students must have an Herbartian attitude

and as mentioned the study of these things in part gives reason to the use of mathematics. But time spent on this also means that less time is left to spend on the mathematical parts and therefore it can be necessary to limit the time spent here. Kuzuoka and Miyakawa similarly reported *"However, as a consequence of spending a lot of time for finding and understanding data, there was a small amount of time left for developing their own answer, which was not fully elaborated and verified at the end of inquiry."* [Kuzuoka and Miyakawa, 2018, p. 11]. So it seems that in this SRP it may be necessary to evaluate how much time should be spent inquiring into certain things. Questions on our origin were some of the first that were posed by students so one could early on in the sequence devote a certain amount of time for students to inquire on just this branch. This could be followed up by a conference in which the class has to agree on exactly where to count from and how many people to start with.

The students were able to keep working on the problem without coming to a stop and the conferences may have been a good didactical tool in order to achieve that. We saw that the conferences were used to discuss methods for calculating and we saw that at least one group used what they heard in a presentation to change tactics completely. This sharing of ideas at conferences has also been experimented by others. Thrane has for example experimented with SRP's in 2009 and also included presentations. In these presentations misconceptions could be corrected and the relevant praxeologies were discussed and validated [Thrane, 2009]. This sharing of ideas and discussion of praxeologies may help students realize the use of and limitation of the discussed praxeologies.

But in the end only three groups actually presented a number as an answer although all groups were able to present reasonable methods for calculating. Furthermore the only method students successfully applied was the person-year method. Part of an explanation to this might come from our analysis of the students' use of media. We found that students used media for three things namely facts on the origin of man, collecting demographic data and lastly methods for calculating. The analysis showed that the last category was not studied in depth. Our own study and research of $Q_0$ showed that much media detailing how to calculate could be found both on the method of counting births and counting person-years. Yet only four out of six groups found media explaining how to calculate. These groups found the article by Carl Haub which in part explained the method of counting births. Yet none of the groups were able to use this media to carry out that method or search for related media. This may indicate that the students simply did not study that media thoroughly enough perhaps because they are not used to perform in depth studies on their own.

The students were also bad at posing the questions they worked with explicitly even though they were asked to do so and in general the change in the division of responsibilities between students and teacher that the SRP brings may sometimes be difficult for students to adapt to. Britta Jessen has also tested SRP's and similarly reports that students might be used to normally being able to simply apply answers in given media and not having to study it further [Jessen, 2017]. It is possible that the students could have come through with Haub's method had they studied his article in depth and searched for related media. Therefore it would be interesting to see if the teaching sequence when tested on students who are more used to the working format would result in a more thorough study of media regarding methods

for calculating and in the end a use of other methods than the one we saw.

The results of our teaching sequence can be compared with the SRP tested by Kuzuoka and Miyakawa with 13-14 year olds in Japan. They asked the question *When is the number of all people in the world who have lived until the year 1900 equal to the number of people after the year 1900?* [Kuzuoka and Miyakawa, 2018, p. 4]. So this question also calls for some calculation of how many humans have lived until a certain point in time. Kuzuoka and Miyakawa give a description of two groups' work towards an answer. One group used a method very similar to the one used by the Danish high school students. It was to model the population as a function of time and then compare the area below the function until year 1900 with the area below the function after year 1900. They used only three data points where the Danish students used many more and they interpolated by manually finding a linear function between neighboring points where the Danish students typically used CAS for calculating and often used it to make a regression analysis. Also the Japanese students found the starting point for their calculation by extrapolating their first linear function where the Danish students relied on studied media to determine a starting point. The Japanese students used geometrical arguments to manually calculate the area below the graph of the function where the Danish students used CAS to perform integration. So the Danish students used more data and more advanced instrumented techniques which may have let them model the population in greater detail.

The use of different techniques may be contributed to the different level of the students but it is interesting that the method used by this group of Japanese students was in general similar to the one used by many Danish students. The interpretation of the area below the graph was an essential part of this method and the Danish students had to divide this by an average life expectancy. When group one for example did not do this it showed that they were not able to interpret the area correctly. The Japanese students did not necessarily have to divide by an average life expectancy since if they assumed it was the same for all time their comparison of areas surmount to a comparison of person-years which would then result in the same as a comparison of people. Therefore it is unfortunately not possible to determine, from the information in the article, whether the Japanese students were able to interpret the units of the calculated integral correctly.

Another group from the Japanese study got a result that was extremely unrealistic when they set up an equation using births per year and *"... in a strange way used the data..."* [Kuzuoka and Miyakawa, 2018, p. 11]. Kuzuoka and Miyakawa report that in many groups students were not critical towards the data they found. This contrasts the Danish students who remained fairly critical towards both data and towards their own results.

In our search into RQ3 we found that all students without trouble mobilized the praxis parts of the praxeology $MO_i$ from the ERM. The theoretical discourse was more doubtful though. Several students showed clear signs of interpreting the definite integral as an area. Some students did also indicate an understanding of the integral as a sum of function values but not necessarily as a sum product of "very small pieces". The students did not say anything that indicated an understanding of the integral as a sum of products but they did several times indicate an understanding as a sum of function values that are "very close" to each other. This seems to have been enough for some students to realize that their result should be divided

by an average life expectancy while other students had difficulties understanding this and had difficulties with determining the units of the definite integral.

Wagner has published a study on beginning and upper-level undergraduate physics students' conceptual knowledge of integration and he similarly finds that "*What typical students do know about integration is often confined to the procedural knowledge necessary for evaluating certain integrals symbolically, while their conceptual understanding is limited to interpreting the definite integral as an "area under a curve".*" [Wagner, 2017, p. 3] So students typically know how to apply the necessary techniques (manually) but their theoretical knowledge is limited to an area interpretation. He also explains that research suggests this interpretation is not the most useful in applied contexts whereas a Riemann sum interpretation of the definite integral as a sum of "very small pieces" is better for making sense in applied contexts. Wagner used students' abilities to interpret the units as evidence for whether a student used Riemann sum interpretation. So in order for students to have a more useful interpretation of definite integrals in applied contexts such as our tested SRP one could have more focus on Riemann sums and on the definition of the integral as a limit of Riemann sums. Wagner explains though that there are more factors that prevents students from adopting a Riemann sum interpretation.

Much like with integration the students did well on mobilizing the praxis parts of the praxeology $MO_r$ on interpolation whereas the knowledge parts were not always properly mobilized. Some students used manual techniques when finding linear functions between two points and some students used CAS for the same task. Mostly though students used CAS to perform a regression analysis and our analysis of their arguments, or lack thereof, when choosing which regression analysis to use revealed that they do not have a proper understanding of the theory behind. The students did not consider the R-values and they rarely considered the context of the data but merely looked at how close the graphs were at the points. As a result some groups ended up presenting a function found by high degree polynomial regression.

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# Appendix A

# Selected transcriptions

| Group 2 – Transcribed parts and the questions they identify | | |
|---|---|---|
| Lesson | Time stamps | Questions |
| 1 | 02:15- 05:50 | Q3.2.2, Q3.2., Q3.2.a, Q3.1, Q3.1.1, Q3.2.2 |
| 1 | 16:05-16:42 | |
| 1 | 39:50-40:13 | Q1, Q1,1 |
| 2 | 13:18-13:47 | Q1.a |
| 3 | 00:55-01:36 | Q1,2 |
| 3 | 12:00-16:06 | Q4.1.1.1., Q1.a, Q4 |
| 4 | 03:31-04:23 | Q4.1.1.1.a, Q4.1.1.1.b |
| 4 | 06:48-07:04 | Q4.1.2.1.a.a |
| 4 | 31:15-31:38 | |

**G2, L1, 02:15-05:50**

A: Jeg tænker et åbenlyst spørgsmål det ligesom sådan med hvor mange i de sidste jeg ved ikke 100 eller sådan frem til øh den industrielle revolution ikke derfra og til nu øh det er sådan rimeligt hvor mange der har? (boet)? Der fordi det er der den ligesom har vokset mest helt klart
B: Ja, jeg tror stadig det udgør en væsentlig del altså alt det andet
A: Ja ja
C: Altså man kan kigge på populationen gennem tiden
A: ja ja men altså ligesom at dele det op
C: men der kan man jo også se der burde man kunne? (tænke)? Det i hvert fald hvis man laver det regionalt der burde man kunne finde ud af om der har været et outbreak af en sygdom der har dræbt folk
A: ja, ja
C: altså hvis man gør det regionalt. Det er ret jeg tror det ret svært at det er ret svært at se det
A: altså hvis man havde en masse tid og åndssvagt mange ressourcer så kunne man jo gå ind i folkeregistre i hele verden og øh fra men øh
B: ah det tror jeg ikke
C: nej det kan man ikke
A: Altså jeg tænker mere hvis man kunne beskrive hastigheden altså
B: jamen vi kan jo bruge
C: jamen vi kan jo lave en matematisk modellering for at lave en funktion af, hvordan den
A: ja og det er derfor jeg er lidt jeg tænker at dele det op i nogle tidsperioder så du har stykvis funktioner fordi at hvis du kigger på det ikke så er det sådan så er det nærmest sådan at i starten da har du sådan en meget meget svag lige ret linje ikke og så på et tidspunkt så
B: et af de store punkter hvor den begyndte at stige rigtig hurtigt det var da vi begyndte at få penicillin
A: ja
B: ikke. Da vi begyndte at få antibiotika da skød det bare opad.
A: jamen specielt det der med at lave sådan stykvis
C: øh altså også øh måske
A: vi laver ligesom fire stykker papir her
C: fair nok
B: øh hvad er det
A: det er hvad vi skal lave præcis
B: nå nice
C: øh altså hvis vi kan finde den der om det så er stykvis eller ej så kan vi jo integrere under den
B: ja ja så
A: sådan mand year fedt
B: så får vi et kæmpestort areal
A: for det er nemlig en måde at finde antallet af mennesker
B: du ved det er sådan du ville gøre det matematisk
A: nej nej fordi så skal du jo tænke på at antallet af mennesker under grafen så skal du jo tænke på at et menneske i gennemsnit lever altså en generation ja men nu lever vi sådan cirka tre generationer ikke

B: ja men du kan ikke sige at så til hver dag ikke så vil der så lever der så mange mennesker og så dør de den dag og så bliver der født så mange til næste dag. Du skal j have en eller anden gennemsnitsalder ikke. Vi bliver nødt til ligesom at putte en eller anden gennemsnitsalder ind som en parameter altså det er jo meget forskelligt
C: jamen det er jo det der med stykvis funktioner

**G2, L1, 16:05- 16:45**
A: Jeg har lige et spørgsmål til den der hvor lang tid lever mennesker øhh i forhold til hvad det har af relevans til det her
B: Du skal jo have. Altså du kan jo ikke bare når vi tager arealet under kurven du kan jo ikke bare tage det ud over det hele fordi der er jo. Mennesket lever jo sådan der i længere tid så hvis vi tager det fra den ene dag til den anden så er det jo ikke fordi at der pludselig er syv milliarder den ene og så syv milliarder den anden for det er jo stadig de samme syv milliarder
A: Men er det ikke lettere at beskrive så med fødselsraten altså med hvor mange der bliver født. Så behøver man ikke tage højde for øh.

**G2, L1, 39:50- 40:13**
A: hvornår definerer vi ligesom at vi har mennesker. Er det fra homo sapiens eller
C: det er det jeg kigger på her. Hmm jamen det er det jeg sidder og kigger efter et eller andet årstal. Jeg tror vi snakker fem seks millioner år men jeg er ikke sikker.

**G2, L2, 13:18 – 13:47**
A: fordi hvor lang et liv skal du have før du har levet
C: det er også det der med om dødsfødsler tæller med som fødsler ikke
A: abort øh nej
C: nej mon øh altså så skal man til at snakke om hvor mange undfangelser der er sket ikke

**G2, L3, 00:55-01:36**
B: øh jeg ved ikke. Siger vi befolkningen starter med to fordi at øh
A: nej
C: nej
B: nej ok
C: nej den starter ved nul ikke og så er der en eller anden der har fået et muteret barn
B: ja det er der vi er
C: fair nok
A: og så har det muterede barn fået børn med en anden ikke muteret og så har de fået muterede børn
B: så det er det ene muterede barn vi går efter

**G2, L3, 12:00-16:06**
C: hvis vi laver et estimat over hvad hedder det hvor mange der bliver født om året altså for et antal år tilbage ikke
A: hmm
C: altså bruger den der minus to hundrede tusinde til at lave nulpunkt ikke
A: hmm
C: så kan vi jo opstille en funktion for hvor mange der blev født om året
A: ja
C: så kan vi tage arealet under den og så vil det være antallet af mennesker der har levet på jorden
A: hm det kan du
C: så lige nu tager vi world population over årene hvilket vi ikke kan bruge til så meget fordi så kan vi ikke bestemme hvor mange der har levet. Så skal vi begynde og finde alle mulige (?) og lave arealet under dem alle sammen
A: Ja
C: så skal lave et eller andet med hvor lang tid mennesker lever og hvor de overlapper og det
B: ja ja
A: men find ud af hvor mange der bliver født pr år og lave en formel for det med de punkter vi har
C: ja
B: så kan vi finde arealet under den og finde ud af hvor mange der har levet på jorden cirka

A: det er nok et problem hvis vi får et lavere tal end vi er lige nu. Det skal helst være dobbelt så stort
B: birth per woman
C: nej det skal være birth per man
A: øh altså holy shit var det i øh det var nul ni det var lige efter krakket der fødte folk ikke lige der er sgu ikke råd til babyer
B: det er birth rate ikke eller er det procent
A: altså ja det er procent men den går ikke øh ja
B: det her er hvor mange der bliver født per tusind mennesker
C: ja det er den her også. Men det kan man jo bare det er lidt svært men vi kan jo beregne
A: problemet er at den her går kun tilbage til nitten
C: det er det
A: hvis vi fortsætter denne her ikke så i minus to tusinde bc der har de måske fået fem hundrede børn hver eller sådan noget hvis den fortsætter sådan
C: det er det den skal i princippet gå sådan der ikke. Vores funktion skulle gerne gå opad og så gå nedad igen
A: det ved jeg da ikke. Ved du hvor mange børn de fik
B: jeg tror de fik mange jeg tror bare ikke der var mange der overlevede
A: nej nej
C: men det er hvor mange der har levet på jorden
A: ja så er vi tilbage til det der med hvornår man lever
B: det er berettermodellen med den fede hval som vi bruger i dansk ikke
C: wow de er gode til det der med børn i Afrika

**G2, L4, 03:31-04:23**
A: og det er for at finde ud af hvor mange kvinder der er
C: mellem femogtyve og niogtyve år
A: ja åbenbart får kvinderne mange mange mange børn
B: vi kunne også bare gange det der tal med nej for det bliver endnu større
A: men hvor beregner du hvor mange børn de får
B: jamen det gjorde jeg jo hernede men da jeg fik otte komma to milliarder
A: Jamen hvordan
B: Hvordan jeg gør det. Jamen jeg ganger da antal birth per woman med antallet af kvinder under mellem femogtyve og niogtyve år
A: så tager du summen af alle dem eller hvad
B: ja så tager jeg summen af hele den kolonne for det var et eller andet

**G2, L4, 06:48-07:04**
B: får vi så et tal der giver mening
A: nej det var jo lige netop det jeg sagde. Det der otte komma to milliarder eller otte komma to gange ti opløftet i ni er ikke i orden

**G2, L4, 31:15-31:38**
A: ved i hvad problemet med den her er gutter
B: hvad er problemet
A: at vores data ligger sådan her
C: nå ja du tegner den der. Jeg ved ikke hvorfor du antager du har lavet det der. Nå ja den bliver svær at forlænge
B: fødsler per menneske per år. Hold da kæft ja vi har reproduceret

| Group 3 – Transcribed parts and the questions they identify | | |
|---|---|---|
| Lesson | Time stamps | Questions |
| 1 | 02:00-02:20 | Q1.1 |
| 1 | 08:10-08:40 | Q1.1.1. |
| 1 | 34:55-34:57 | Q2.1.1.a |
| 2 | 01:20-01:55 | Q2.1, Q2.2, Q2.1.1.b |

| | | |
|---|---|---|
| 2 | 13:18-14:54 | |
| 2 | 19:57-20:14 | Q4.1.1.1. |
| 2 | 21:35-21:55 | Q4.1.1. |
| 3 | 15:45-15:48 | Q3.2 |
| 3 | 23:05-23:23 | Q1.2 |
| 3 | 27:55-28:21 | Q3.2 |
| 3 | 28:45-29:10 | Q3.2 |
| 3 | 30:34-30:40 | |
| 3 | 33.10-33:42 | Q3.2.1.a, Q3.2.1.b, Q3.2.1.c, Q3.2.1, Q3.2.1.3 |
| 4 | 00:25-00:33 | Q3.1 |
| 4 | 01:43-02:01 | Q3.1.1 |
| 4 | 01:14:54-01:15:15 | Q3.1.2 |

**G3, L1, 02:00-02:20**
A: Det er bare hvornår vi starter. Er det det moderne menneske eller.
B: hvad snakker du om. Nå er det noget sapiens sapiens som vi er eller er det homo sapiens eller er det neandertaler eller er det

**G3, L1, 08:10-08:40**
A: Men et af vores underspørgsmål kan jo så være hvornår tæller man mennesker fra eller
B: hvornår er man et menneske
C: hvornår startede mennesket. Hvor langt skal vi regne tilbage
A: Hvornår opstod det første menneske
B: det er ikke helt dumt.

**G3, L1, 34:55-34:57**
A: Udryddelse er det ikke sådan. Er det ikke sådan

**G3, L2, 01:20 –01:55**
A: Ok hvor mange mennesker dør om året. Så skal vi lave noget gennemsnitligt
B: så kan vi lægge alle dem sammen. Alle dem der er døde hvert år
A: altså vil du regne ud hvor mange der er døde indtil nu og så lægge dem sammen med dem som der lever i dag
C: jo men er det ikke nemmere at tage dem der er født
A: det kan godt være det er bedre

**G3, L2, 13:18-14:54**
A: ej det er et ord jeg ikke kan lide det der
B: guesstimates
A: det er når det er at du skal estimere noget men du ikke helt ved hvad du skal estimere så guesstimerer du
B: ej det synes jeg faktisk er et rigtig nedern ord. Det er rigtigt det er faktisk et rigtig nedern ord. En estimering det er i forvejen lidt vagt ikke men så en guesstimering
A: den er bare sådan der to
B: jeg gætter på at det her er nogenlunde ved at være tæt sandheden det er sådan det giver jo ikke mening. Så skal du lyve fra starten af
A: ja det giver ikke mening
B: det giver ikke mening jo det gør det jo men det er et irriterende ord
A: ok kan vi komme på noget andet noget noget lidt mere håndgribeligt. Skal vi begynde at svare på nogle af spørgsmålene
B: er det det vi skal nu eller hvad
A: ja det tror jeg
B: ok så hvordan øh hvordan gør vi det. Vi starter hvor mange mennesker bliver født om året
A: årh man ved også bare når der står this semi-scientific approach

B: årh nedern

**G3, L2, 19:57-20:14**
A: der er kun et land i Afrika som føder som har et gennemsnit på under to komma en fødte børn per kvinde
B: de giver den bare gas mand

**G3, L2, 21:35-21:95**
A: et hundrede enogtredive komma fire millioner birth pr year det er estimeret til to tusinde og elleve. Vi skal længere tilbage
B: hvor lang tid er det taget fra starten af at det er faldet fra mere end seks børn per kvinde til under tre ikke

**G3, L3, 15:45-15:48**
A: det er irriterende at man ikke ligesom kan finde noget om hvor mange der har været igennem tiden

**G3, L3, 23:05-23:23**
A: Hvis vi siger vi starter med nul der i hvad sagde vi to hundrede tusind før
B: ja bang
A: nej vi kan ikke starte den på nul
A. Nej men så starter vi den på to
B: ja
A: okay så giver det ligesom mening. To får en mand og en pige får en milliard børn

**G3, L3, 27:55-28:21**
A: i sekstenhalvtreds var der en halv milliard. Se den steg ret hurtigt på ellevehundrede år men den steg ikke så hurtigt på
B: vi skal have noget modelværk snart
A: men det bruger vi ikke
C: vi har halvanden time til at lave en model
A: vi kan ikke bruge det til noget. Okay så hvis vi siger
B: kan vi plotte det
A: det kan vi sagtens

**G3, L3, 28:45-29:10**
A: og så siger vi bare at x-aksen det er navn. Øh det er i
B: milliarder mennesker
A. navnet det er år og enhed det er bare øh ja og y-aksen hedder så
B: milliarder mennesker

**G3, L3, 30:34-30:40**
A: nå nej nej nej åbn regnevinduet og så laver vi regressionsanalyse på den.

**G3, L3, 33:10-33:42**
A. all right og så plotter vi det her ind som et eller andet smart
B: så er det to variabler
A: I know this I know this
B: ja hold da op den er spændende så skal du bare finde noget der passer på den
C: lineært ej
B: tænk dig lidt om
C: okay logaritme
A: jeg tror det er en potens
B: øh det er ikke så godt potens er bedre øh okay ekspotentiel er bedre øh okay
A: den passer ikke super godt
B: passer ikke så godt faktisk

**G3, L4, 00:25-00:33**
A: Jeg synes deres ide er meget god (red. om gruppe et's fremlæggelse af menneskeårsmetoden.)
B: Ja, all right

**G3, L4, 01:43-02:01**
A: Ja, så vi har altså nogle datapunkter her ikke. Så vi siger i år hundrede var der nul komma tre milliarder ikke. Så tager vi bare det for så får vi nemlig enheden når vi tager arealet under grafen
B: for så får vi så hvilken enhed
A: Så får vi så menneskeår og så tager vi gennemsnitslevealderen for et menneske.

**G3, L4, 01:14:54-01:15:15**
A: Men i hvert fald arealet under grafen vil så give os et leveår. Hvor mange år er der blevet levet i alt. Derfra er det jo så bare at tage så dele med gennemsnitlig levealder på det ved jeg ikke fyrre halvtreds cirka måske lidt mindre end det.

| Group 4 – Transcribed parts and the questions they identify | | |
|---|---|---|
| Lesson | Time stamps | Questions |
| 1 | 04:45-05:07 | Q1.1, Q1.1.1 |
| 1 | 44:50-45:25 | Q4, Q4.2, Q4.1.a |
| 2 | 02:30-03:10 | Q3.2.2 |
| 2 | 12:00-12:35 | Q3.2.1, Q.3.2.1.1 |
| 2 | 13:05-13:30 | Q1.2 |
| 2 | 19:05-19:35 | Q3.2, Q4.1 |
| 2 | 25:58-26:10 | Q3.2.1, Q3.2.1.2, Q3.2.a |
| 3 | O4:30-04:55 | Q3, Q3.1.1 |
| 3 | 06:20-06:55 | Q3, Q3.1.1 |
| 3 | 09:32-10:01 | Q4.1.1.b |
| 3 | 10:30-10:45 | Q4.1.1.b.a |
| 3 | 12:35-13:13 | Q3.2.1. |
| 4 | 35:55-36:14 | |

**G4, L1, 04:45-05:07**
A: Øh hvornår blev mennesket født? Det må være noget
B: Ja, det første menneske Men spørgsmålet er hvornår er det et menneske og hvornår er det en udviklet abe altså hvornår
A: Vi tager aberne med
B: spørgsmålet er hvornår er det en abe og hvornår er det bare sådan snart en abe

**G4, L1, 44:50-45:25**
A: Hvis vi laver en graf ikke over fødselsraten. Ikke hvor mange mennesker der er men over fødselsraten ik. Så vi ikke forøger antal mennesker hele tiden. Øh. Hvad var min pointe
A:Så skal vi hele tiden kende slutpunktet for en funktion for at starte på den næste. Det kan vi godt.
B: Altså hvor mange stykker skal vi lave det i?

**G4, L1 46:20-47:10**
A: Det vil sige hvis vi beskriver fødselsraten. Hvis kigger på hvor mange mennesker det er der bliver født så kigger vi altså i stedet for på at øh det er en slags konstant kan du godt sige men nu har du pludselig lavet en anden vinkel ikke fordi det er en forøgelse. Du forøger hele tiden med fødselsraten hvis du vil have hvor mange mennesker der har levet Giver det mening.
B: det tror jeg

A: Eller så skal vi nemlig integrere den her. Hvis vi integrerer den så kan vi finde ud af hvor mange mennesker der er nedenunder. Så spørgsmålet er om vi beskriver fødselsraten eller om vi beskriver fødselsraten i forhold til hvor mange mennesker der er. Så integrerer vi fødselsraten så får vi antal mennesker.

**G4, L2, 02:30-03:10**
A: År ti tusind nej minus ti tusind
B: de siger godt nok nul de siger der ikke levede nogen mennesker før
A: jamen det er det jeg siger nej jeg siger to tusind og halvtreds
B: de siger heller ikke der lever nul de siger meget tæt på nul de siger der lever under hundrede mennesker
A: nej jeg siger to hundrede og halvtreds tusinde jo ja ok og så siger den ti tusinde
B det er agriculture vi kan godt tage den fra agriculture den siger bare der ikke rigtig lever nogen mennesker til at starte med. Den siger der lever stort set ingen mennesker

**G4, L2, 12.00-12:35**
A: Kan du ikke bare sige den er lineært stigende der.
B: Jeg kan sige den er lineært stigende og så fra nul og så bare over til den.
A: ja præcis det tænker jeg.
B: og til to millioner. Så der kommer så småt flere og flere mennesker. Fordi de har hårde levevilkår så det går ikke så hurtigt det går ikke eksponentilet som det gøre med resten ellers det går lidt eksponentielt til sidst ikke
A: og de har ikke opfundet agriculture endnu så det er derfor.

**G4, L2, 13:05-13:30**
A: Det er for at forsimple det ikke vi må jo starte med nul altså det giver ikke mening at starte med.
B: ja men det kommer ikke til at betyde så meget i forhold til antallet af befolkningen fordi det her er i alt hvad vi har her det bliver jo øh.
A: Men jeg synes ligesom vi skal starte med nul ikke
B: så vi starter på nul ja okay.

**G4, L2, 19:05-19:35**
A. Hmm hver enkelt der er født. Det er også som funktion af. Du laver jo bare vi laver jo fødselsraten ikke så vi tager det der er neden under den ikke så det er bare. Vil du så ikke lave i stedet for f af som funktion af tiden så. Vil du så ikke sige funktion af mennesker i forhold til tiden fordi tiden det er jo den der kommer til at ligge her langs x-aksen ikke.

**G4, L2, 25:58-26:10**
A: Nej vi skriver bare første stykvise funktion som en vektor. De sidste beskriver vi som eksponentielt tænker jeg.
B: Skal der ikke stå. Nå ja det er rigtigt nok.

**G4, L3, 04:30- 04:55**
A: Men jeg tænker hvis man har antallet af mennesker i det år så hvis man integrerer vil man vel finde ud af hvor mange der så er blevet født
B: hvis du har antallet af mennesker det år så behøver du ikke at integrere. Hvis du har datapunkterne ud fra antallet af mennesker der er. Hvis du bare laver det som en forøgelse hele tiden
A: Det vi skal finde ud af er hvor mange mennesker der nogensinde har fundtes

**G4, L3, 06:20- 06:55**
A: men hvis man har hvor mange der er i det specifikke år så kan du integrere den jo og så kan du få hvor mange der nogensinde har levet
B: ja men problemet er at det problemet er at det du gør der ik. Det kan du sådan set ikke gøre. Og grunden til at du ikke kan gøre det det er fordi at øh du skal have fødselsraten fordi ellers så ryger de ind i statistikken flere gange så er det som om at øh. Hvis du lever hundrede år så bliver du født hundrede gange kan du se det. Så bliver det helt ved siden af.

**G4, L3, 09:32-10:01**
A: Vi vil gerne vide hvor mange der bliver født ikke

B. ja
A: Hvis vi tager vi har statistikker på hvor mange der lever i året det ene år. Vi har statistikker på hvor mange der lever året efter ikke. Øh. Hvis du nu siger øh vi vil gerne sige okay fødselsraten. Hvis vi starter ud med det her så siger vi fødselsraten er hvor mange der levede før. Næh: Hvor mange der lever nu minus hvor mange der levede før ikke
B: ja
A: men det er så ikke helt rigtig for der også nogen der dør og de kommer til at indgå i statistikkerne.

**G4, L3, 10:30-10:45**
A: Så det er også vigtigt hvor mange der dør det kan jeg godt se.
Det var det der var min tankegang i hvert fald og det er derfor jeg gerne vil vide hvor mange der dør
a: ja
b: og så tænker jeg det kan vi beskrive som en procentdel
a: ja
b: altså hvor mange procent dør og så kan vi på det måde gange det op.

**G4, L3, 12:35-13:13**
A: så vi har data på var det ikke nitten hundrede og halvtreds vi har data fra. Der er gode ikke
B: det ved jeg så ikke
A: prøv at gå længere ned og tilbage
B: nå her
A: ja prøv at gå ned til nitten hundrede og halvtreds der er gode data ikke
B: ah jo
A: så det er en større opgave men hvis du kan gøre det. Så vi vil gerne have de der datapunkter dem vil vi gerne have lavet om. Og der er mange datapunkter så måske tag hvert femte år eller hvert tiende år hvis du ikke orker det for jeg forstår godt hvis du ikke orker at gøre det for hvert år
B: ja.

**G4, L4, 35:55-36:14**
A: men man kan jo vælge en anden form
B: ja så prøv
A: hvad skal vi prøve
B: øhh ikke eksponentiel. Prøv øh polynomie
A: polynomiel
B: ja befolkningen kommer ikke til at falde igen det kan jeg godt sige dig.

| Group 5 – Transcribed parts and the questions they identify | | |
|---|---|---|
| Lesson | Time stamps | Questions |
| 1 | 13:30-15:41 | Q2, Q4, Q4.1, Q4.1.1., Q4.1.1.a |
| 1 | 34:20-34:55 | Q1.1 |
| 1 | 57:20-57:39 | Q1.1, Q 1.1.1 |
| 2 | 01:42-02:20 | Q2.1 |
| 3 | 08:15-08:47 | Q4.1.1.1 |
| 3 | 10:50-11:09 | Q3.2.2.a |
| 3 | 14:59-15:36 | Q3.2.2. |
| 3 | 45:32-46:40 | Q3, Q3.2 |
| 3 | 51:00-51:53 | Q3.1, Q3.1.1. |
| 4 | 16:00-16:02 | Q3.2.b |
| 4 | 34:40-35:10 | Q3.2.1, Q3.2.1.a, Q3.2.1.d |

**G5, L1, 13:30-15:41**
A: det vi gør ikke også vi finder fødselstallet for så mange år tilbage som vi kan ikke også så laver vi kurve over det mere eller mindre og så integrerer vi den fucking kurve
B: så vi har lineær over årstal og så hvor mange mennesker der levede på det tidspunkt

A: nej hvor mange der blev født på det tidspunkt
C: Men du tager jo ikke højde for at nogen af dem er de samme mennesker der levede det samme år
A: Jo jo det er kun fødselstallet
C: Ja okay
B: ja er I klar så tager vi første spørgsmål. Hvor mange mennesker blev født i et givent årstal.
C: men hvor lang tid skal vi gå tilbage?
A: jamen
C: vi kan også prøve at kigge på hvor mange mennesker der døde. Så kan du se krige og alt mulig
B: der er bare det at det har ændret sig meget
A: ja det har ændret sig rigtig meget. Det har sprunget super meget her på det seneste.
C: det er sådan her. Vi laver en kurve så det bliver sådan her. Om det så bliver femogtyve graders polynom så er det lige meget
B: femogtyve grader. Det bliver fandme
C: jo jo det kan vi jo sagtens integrere
B: jamen det er jo en fucking hyperbel
A: okay altså så det er den her som vi skal til at lave nu
B: skal vi ikke det deroppe
A: jo jo men vi skal jo ligesom
C: vi skal have nogle data på
A: drenge lad være. Den gennemsnitlige levealder. Vi skal jo lave en funktion for pokker. Vi skal kun kigge på hvor mange der er født fordi hvis de er blevet født så har de også levet.
B: det er lige meget om de er døde
A: ja det er rigtig men det kunne også være spændende. Det kunne også være relevant at kigge på hvornår folk de dør fordi hvis der mange der dør så er det på grund af krig eller sygdom eller et eller andet
B. jamen det har jo stadig noget at gøre med altså det har noget at gøre med hvor mange der bliver født men det behøver vi jo ikke at tage højde for
A: jeg tror altså ikke at vi bare kan tage en masse punkter og så lave regression og så
B: bare for at argumentere ikke. Hvis der nu i dag holdt op med at blive født nogen mennesker så ville der jo stadig i mange år frem lev rigtig mange mennesker
A: ja ja men de ville ikke være en del af vores kurve. Så ville kurven stadig gå i nul og det er jo også det den skal. Den skal jo reflektere hvor mange mennesker der lever
C: nej vi skal jo ikke se på hvor mange der lever som sådan. Vi ser på hvor mange der bliver født og det er det eneste
A: altså vi skal finde ud af hvor mange der har levet
C: så hvor mange har levet. Hvor mange har levet. Det er jo lige meget om de er stoppet med at blive født

**G5, L1, 34:20-34:55**
B: vi har også stillet nogle spørgsmål. Her har vi så valgt at fokusere at for det første ligesom de andre hvornår beskriver vi os selv som mennesker og hvor langt skal vi kigge tilbage og så har vi valgt at kigge på hvornår stiger befolkningstallet fordi der er jo forskel på om vi er tilbage i lang tid tilbage i historien eller nu øh for eksempel som nu med Kina hvor det er steget rigtig meget og så hvilke faktorer har påvirket fødselsraten altså krige og sådan noget og så vil vi gerne på en eller anden måde beskrive det her som en funktion for at vide hvor mange der har levet og det kan man lave med en stykvis funktion. Øh og ja så har vi selvfølgelig også det sidste spørgsmål. Hvornår er den største stigning i fødselsraten.

**G5, L1, 57:25-57:39**
A: menneske øh okay. Her der siger de at forhistoriens altså hvis vi tager tidligst syv millioner år siden i Afrika. Men det er jo er jo heller ikke nødvendigvis mennesket som arten menneske. Det er jo en øh. Vi vil jo gerne finde den første homo sapiens.

**G5, L2, 01:42-02:20**
B: Vi har indtil videre kun fundet et link øh og det var ikke så relevant som vi håbede øh fordi det kun gælder Danmark. Vi vil gerne finde et dødstal i forhold til år eller fødselstal i forhold til år for hele verdensbefolkningen igennem en periode for så og lave en kurve og så prøve at integrere den mere eller mindre.

**G5, L3, 08:15-8:47**

A: What we are going to do is vi har allerede lavet en løsningsmodel og det vi mangler nu det er at finde the raw data ikke også. Vi skal ligesom finde ud af hvor mange mennesker der er blevet født.
B: ja vi skal prøve at finde informationer
A: exactly. Så hvis vi bare siger at vi gør det samme som justin gruppen gjorde det lyder sådan set meget fornuftigt at for ca tohundredetusind år siden der var de første mennesker til og så finder vi så mange dele som vi kan derfra og så derefter

**G5, L3, 10:50-11:09**
B: der bliver ved med at dø nogen og det kunne være man skulle sige nærmest nogenlunde konstant fordi at før der var der måske sygdom og alt sådan noget tilbage i jernalderen eller sådan noget og man ikke fik mad nok og sådan noget. Det er der også i dag men langt mindre men så er der krige og sådan noget
A: jeg tænker at selvom at vi har skrevet det på vores planche der så ved jeg ikke om det er så superrelevant at kigge på og selvom det har gjort en forskel så kan vi ligeså godt bare kigge på milepunkterne hvor mange er der blevet født. Selvom der er en helt masse der dør og sådan noget så kan man jo alligevel se hvor mange færre der bliver født på fødselstallene hvis vi kan finde nogle estimater på det.

**G5, L3, 14:59-15:36**
A: Når vi finder ud af hvor mange hvert menneske føder så ved vi jo ikke hvor mange der bliver født. Så ved vi bare hvor mange der er blevet født pr menneske men vi ved jo stadig ikke hvor mange mennesker der er
B: det er hvor mange mennesker der har levet
A: vi ved jo stadig ikke hvor mange der er til at føde dem. Vi ved bare hvor mange de gennemsnitlig føder hver. Hvis vi vidste kombineret med at vi ved hvor mange de føder hver og så samtidig vidste hvor mange de var så ville det jo være nemt, men det ved jeg ikke så derfor mangler vi ligesom halvdelen.

**G5, L3, 45:32-46:40**
B: så det du vil gøre det er at du vil tage nogle årstal hvor du har en population og så vil du integrerer under det øh bruge regression og så integrere
A: nej nej jeg vil gerne have årstal fødsler. Årstal x fødsler y
B: så okay
A: og så laver vi en kurve mere eller mindre en femogtyvetusind graders polynom ikke og så integrerer vi den
B: Jeg gider ikke finde fødsler mand det er så besværligt. Der er ikke nogen der ved hvor mange der er født
A: kan du så finde ud af hvor mange der lever der har levet på de forskellige tidspunkter for det kan vi sådan set også godt bruge det er bare lidt mere besværligt men hvis du kan finde det så kan vi sagtens bruge det
B: hvad skal vi bruge
A: så finder vi ud af hvor mange der har levet gennem nogle forskellige år altså
C: ja så har du bare populationen ud af y-aksen i stedet for fødslerne.
A: ja ja det er lidt nemmere

**G5, L3, 51:00-51:53**
A: nej nu skal vi til at blive matematiske og finde en fremgangsmåde fordi nu har vi jo ligesom en helt masse tal øh over hvor mange mennesker der har levet på forskellige tidspunkter. Kan vi bruge det til at finde ud af hvor mange der har levet i alt fordi hvis vi lægger dem alle sammen så får vi et alt for stort tal jo
B: er du sikker på det
A: ja det er jeg helt sikker på. Og hvis vi laver en kurve over det kan vi så diffentiere den for at finde fødselsraten. Nok ikke sådan lige. Øh giver det overhovedet mening at lave en kurve over det. Det kunne det godt ikke gøre.

**G5, L4, 16:00-16:02**
A: skriv en komma hundrede og sekshalvfjerds. Og så skriv hundrede komma hundrede seksoghalvfjerds. (Note: Eleven udvælger hvert 100. år at tage data fra).

**G5, L4, 34:40-35:10**
A: Vi vil gerne buge den der hedder polynomiel
B: Logaritme…
C: Den ser flot ud gør den ikke?
B: Altså den er heller ikke dårlig men den anden er endnu mere præcis, den niende grads-polynomielle

| Group 6 – Transcribed parts and the questions they identify | | |
|---|---|---|
| Lesson | Time stamp | Questions |
| 1 | 02:15-02:26 | Q1.2 |
| 1 | 16:37-17:01 | |
| 3 | 01:00-01:15 | Q1.1.1.1 |
| 3 | 05:28-05:40 | Q3.1, Q3.1.1 |
| 3 | 23:30-23:40 | Q3.1.2 |
| 3 | 39:26-40:25 | |
| 4 | 08:53-09:00 | Q3.2.1, Q3.2.1.1 |
| 4 | 09:50-10:10 | |
| 4 | 20:44-21:27 | Q3.2.1.b |
| 4 | 25:08-25:56 | |
| 4 | 26:18-26:33 | Q3.2.a |
| 4 | 50:50-51:15 | Q4, Q4.1.1, Q4.1.1.1 |
| 4 | 52:30-53:03 | Q4.1 |
| 4 | 54:20-55:13 | Q4.a |
| 4 | 57:15-57:36 | Q4.a |

**G6, L1, 02:15-02:26**
A: Så er der vel noget med populationen
B: Ja der er noget med noget befolkningsvækst. givetvis må vi starte med en.  sandysnligvis også to.
A: Ja det må være sandsynligvis to ikke

**G6, L1, 16:37-17:01**
A: where are your sources mate.
B: det er noget de selv har øhhm estimeret sig frem til.
A: hmm.
B: og det er også kun semi-scientifc.
A: semi scientifc.
B: ja fordi at der ikke rigtig er noget data for det.

**G6, L3, 01:00-01:15**
A: jeg synes bare år nul er for sent.
B: på den anden side kunne man jo sige at alt det der sker før år nul kunne man argumentere for ikke er ligeså vigtigt i forhold til den stigning der sker efter.

**G6, L3, 05:28-05:40**
A: jeg har ikke forstået det endnu
B: hvorfor de kigger på arealet under grafen
A: ja
B: du laver en graf over hvor hurtigt det vokser og så alt under det er jo alle dem der også har været.

**G6, L3, 23:30-23:40**
A: i jæger- fiskerfolkenes tid det må være der i stenalderen. Der var gennemsnitslevalderen femogtredive år men
B: fyrre er fint
A: men der var også nogen der kunne blive halvtreds til tres år.
B: så fyrre

**G6, L3, 39:26-40:25**
A: vil man nu ikke kunne sige at hvis man kigger på befolkningsstigningen mellem to datapunkter og dividerer den med den gennemsnitlige levealder så mp vi vide hvor mange der har levet der –ish.

B: Jo altså sådan groft sagt.
A: Jamen hvorfor skal man del med gennemsnitlig levealder. Fordi fx her går der halvtreds tusinde tohundrede år. Der er ikke nogen der lever det de lever fyrre år og så lever der et nyt menneske fyrre år måske.
B: Vi kigger på hvor mange der er hvert år.
A: Det er selvfølelig rigtigt.
B: Det gælder bare om hvor mange er der nu mange er der nu mange er der nu mange er der nu. Men er det ikke det. At hvis du bare lægger dem her sammen og laver det til en graf og så regner hvor meget der er under grafen så får du vel alle dem der har levet.
A: Fordi der er sådan også en hel masse der dør
B: Ja okay.

**G6, L4, 08:53-09:00**
A: mon ikke vi kan antage at befolkningsstigningen fra minus halvtredstusinde til år to hundrede var relativt lineær.

**G6, L4, 09:50-10:10**
A: så nu har jeg ordnet alle tallene så lad os lave en regressionslinje.
B: kan du opstille en øh
A: åh det er dejligt det går så hurtigt. Uh det var godt det der. Åh.
B: kan du finde ud af at opstille funktioner for det fordi jeg kan ikke
C: jamen jeg prøver lige at give det et skud.

**G6, L4, 20:44-21:27**
A: vi laver en graf over mennesker per år med alle de datapunkter vi kan
B: fedt mand
A: ja så vi mangler en tendenslinje
C: så med arealet under en graf, så kunne vi i teorien få menneskerne
B: det virker bare ret lavt.
C: ja men det er jo bare menneskerne per det der år.
A: det er irriterende prøv bare logaritmisk med hundrede tusinde eller en polynomie med en million.
B: så det er sådan et.

**G6, L4, 25:08-25:56**
A: hvad er et førstegradspolynomiue. Så er det bare en streg ik
B: jo
A: kan den ikke gå højere end sjettegrads
C: apparently not
B: bare giv mig et eller andet
A: så tag den der. Der er der mindst altså den har bedst øh fit on it altså du kan se der er mest hvor den bare følger
B: hvordan ser potensen ud
A: den ser fucked up ud fordi det er et sjettegradspolynomium
C: guys i skal tage et polynomium og så køre vildt mange grader op
B: ja men den vil ikke give os flere grader end seks
A: hvordan ser et seksgradspolynomium ud
B: der
C: det er den der passer bedst af dem alle sammen
D: det gør det altid. Hvis du har et polynomium med nok grader kan man fitte ethvert dataset.

**G6, L4, 26:18-26:33**
A: og den passer godt nok nøjagtigt
B: men vi skal have en stykvis
A: vi skal have en stykvis. Nu har vi en enkeltvis
B: nu laver vi en stykvis for den der går kun for tusind og frem

**G6, L4, 50:50-51:15**
A.: altså i teorien hvis vi havde fødselsraten gennem hele historien havde det her været super nemt
B: ja hvis den var konstant ikke
A: nej hvis vi bare havde hvad den var fordi så kan du sige fødselsrate gange befolkning divideret med tusind.
B: divideret med tusind
A: ja fordi det er fødselsrate per tusind mennesker per år
B: right og det er det vi har gjort med de her og så har vi lagt dem alle sammen sammen
A: ja

**G6, L4, 52:30-53:03**
A: og man kan så opstille en funktion der hedder befolkning per år og fødselsrate per pr år ikke.
B: ja det er jeg med på
A: det er antal fødte ikke skal vi så ikke bare skrive befolkningstilvækst det år

**G6, L4, 54:20-55:13**
A: skal vi så skrive at summen. Summen af befolkningstilvækst per år giver det er hvor mange der har levet
B: hvis du lægger alle årene sammen som jo reelt set er at integrere
A: ja ikke helt integration det er også at gange det
B: nå ja. Hvis du lægger alle årene sammen siden vores begyndelsestal så vil du jo reelt set få hvor mange der er blevet født
A: skal jeg så ikke bare skrive det der sumtegn og så bare år nul år et år to og så prik prik prik og så giver samlede befolknings.
A: du kan jo skrive summeret fra år nul eller hvad vi siger og frem til år to tusinde et eller andet.

**G6, L4, 57:15-57:36**
A: så hvis du nu kalder fødselsraten for f og befolkningen for b så kan du skrive summen op og så kan du skrive f i gange b i over tusind. Fordi fødselsraten og befolkningen har en bestemt værdi for hver i-værdi så f i relaterer til den specifikke.

# Appendix B

# Assignments

## B.1 Assignment group 1

# Tale til posteren

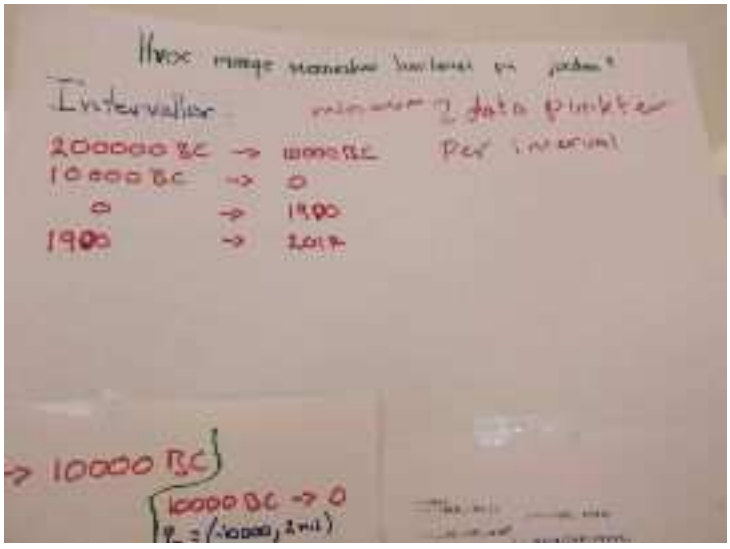
Formålet med denne aflevering er at rette vores poster til på baggrund af feedback fra fremlæggelserne og at lave en kort tekst som forklarer det som posteren skulle suppleres med at tale.

Denne gruppe vil nu skrive en salgstale, altså talen som blev udført til posteren som vi havde fremstillet. Vi startede med at opstille nogle intervaller for, hvor mange der har levet gennem tiden. Dette blev gjort i fire intervaller henholdsvis:

- 200.000BC (før kristus fødsel) => 10000BC (før kristus fødsel)
- 10000BC (før kristus fødsel) => 0
- 0 => 1900
- 1900 => 2017

Det er den kronologiske rækkefølge vi lavede intervallerne fra, først fra 200.000BC til 10000BC og så videre. Til de to første intervaller opstillede vi to lineære funktioner ud fra to datapunkter i hvert interval. Vi startede med at finde a og b værdierne i den første funktion, som symbolisere henholdsvis hældningskoefficienten og skæring med y-aksen. Herefter plottede vi funktionen. Derefter opstillede vi den samme funktion for to nye punkter i det andet interval, ved samme fremgangsmåde.

På den første poster ses de intervaller som vi har delt op i, da vi ved at det første menneske kom til verden 200.000 før kristus fødsel, så vi starter fra det år.

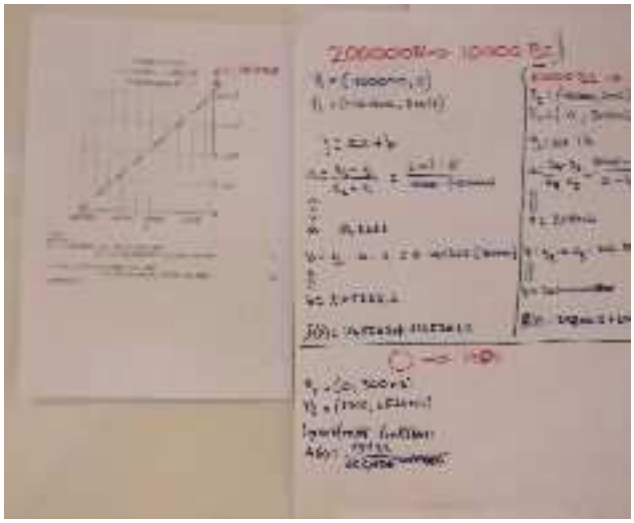



*Figur 1: På billedet ses de inddelte intervaller.*

[redacted]

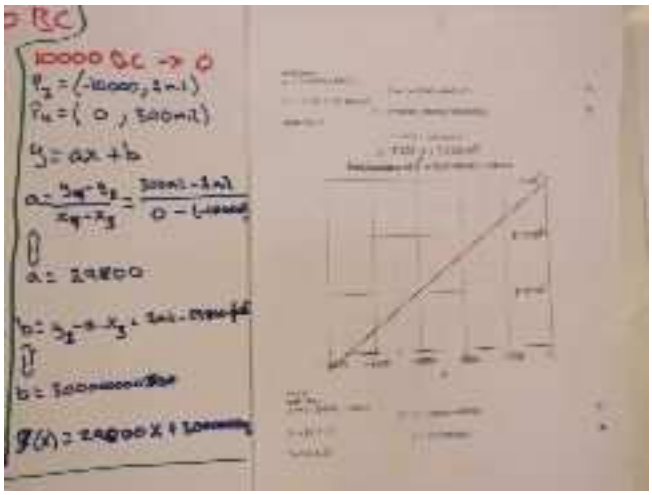
[redacted]

På den næste poster forneden ses det første interval fra 200.000BC (før kristus fødsel) til 10000BC (før kristus fødsel) som vi lavede en lineær regression over hvor vi fandt a og b, da vi ved at har med et førstegradspolynomium at gøre. Forskriften for funktionen blev $f(x) = 10,526x * x + 2105263,2$. Der ses samtidig hvordan vi har fundet frem til hældningen a og skæringen på y-aksen b, ved at at tage to kendte punkter på grafen. Antal mennesker ud af x-aksen og antal år ud af y-aksen.



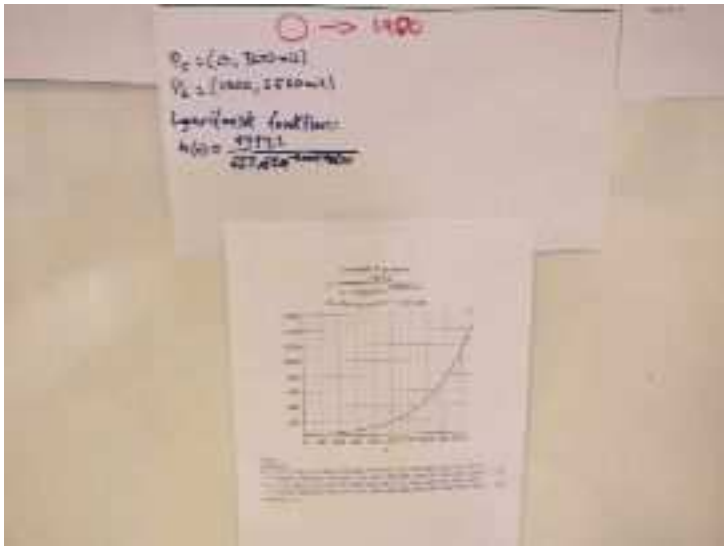
*Figur 2: På billedet ses hvordan vi kom frem til funktionen af intervallet fra* 200.000BC til 10000BC.

På den næste poster forneden ses det næste interval fra 10000BC (før kristus fødsel) til 0, som vi også lavede en lineær regression over, hvor vi på samme fremgangsmåde som det første interval, finde først hældningen af grafen ved hjælp af to kendte punkter. Derefter kan man finde skæringen af y-aksen, som kaldes b og fremstille en forskrift for et førstegradspolynomium. Forskriften hed $g(x) = 31821x + 2,9326 * 10^6$.



*Figur 3: På billedet ses hvordan vi kom frem til funktionen af intervallet fra* 10000BC til 0.

Det næste interval som ses på billedet er fra år 0 til 1900. Men denne graf er en logaritmisk funktion, som ser lidt anderledes ud, fordi at frå år 0 til 1900 skete der en markant stigning af mennesker i det interval. Forskriften for funktionen bliver følgende: $h(x) = \frac{9494,2}{622,62 * e^{-0,0024663x}}$

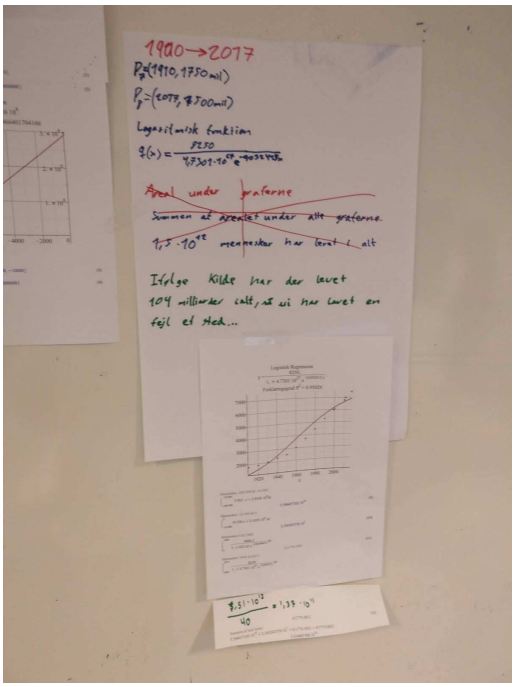



*Figur 4: På billedet ses grafen for intervallet fra år 0 til 1900.*

På billedet forneden ses det sidste interval fra 1900 til 2017, hvor der samtidig også er lavet en logaritmisk funktion, da der er en markant stigning i antal mennesker som lever på jorden. Forskriften for funktionen er følgende $h(x) = \frac{8250}{4{,}7301 * 10^{\wedge 27} * e^{\wedge -0{,}23578x}}$

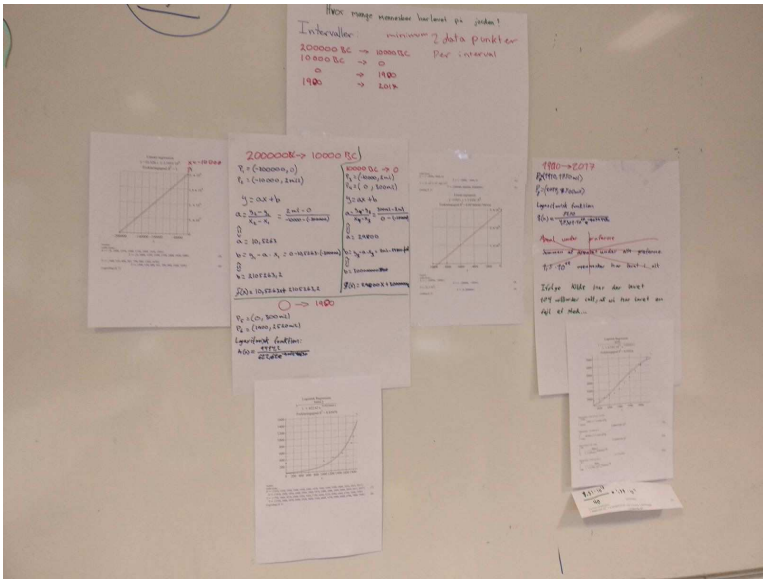
Nederste del af planchen er fokuspunktet, det er nemlig der vi tager arealet under grafen for de fire intervaller vi har valgt. Det er der vi finder ud af svaret på opgaveformuleringen, som gik ud på at finde ud af hvor mange mennesker som havde levet på jorden. Det vi gør er, at tage arealet under hvert interval og lægge dem sammen til sidst ved hjælp af integralregning. Vi fandt arealet under hvert interval og lagde dem alle sammen, dernæst skulle vi dividere det tal som vi fik frem til med gennemsnitsalderen for mennesket, som vi fandt en kilde til at være 40 år. Vi fandt frem til at der har levet $1{,}5*10^{12}$ (ved hjælp af integralregning, mere specifikt arealet under en graf, hvor udledningerne kan ses på Maple-Filen) og det skulle vi dividere med gennemsnitsalderen for mennesket, så vi ikke tæller mennesket flere gange i arealet under grafen.

$\frac{1{,}5*10^{\wedge 12}}{40} = 1,37 * 10^{11}$



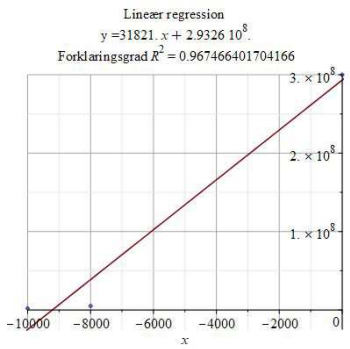

*Figur 5: Billedet viser grafen for intervallet fra 1900 til 2017.*

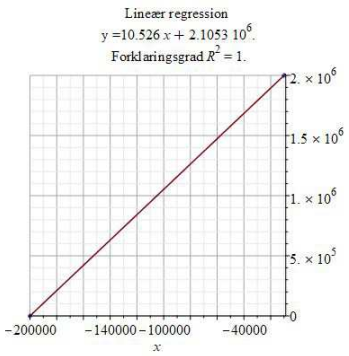
Dette billede giver et samlet overblik over alle plancherne, hvor man lægger mærke til den grønne streg i den midterste planche som deler intervallerne op i tre dele, og man lægger hurtigt mærke til en sammenhæng.

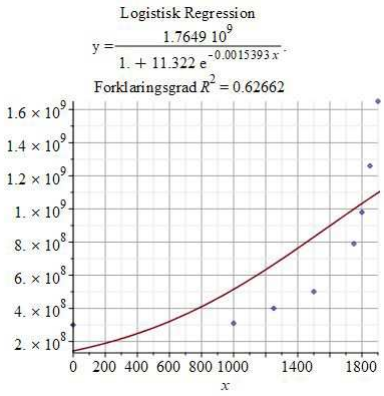


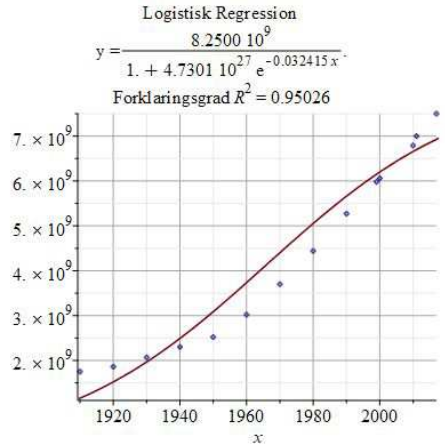

*Figur 6: Billedet viser alle vores plancher.*

## Bilag




## Diskussion og konklusion

Vi kan konkludere at $1{,}37*10^{11}$ mennesker har levet på jorden. Det har været et lærerigt projekt, som vi har begået os ud i. Man kan roligt sige at det er en åben opgave, hvor der ikke findes et konkret svar, derfor kan vi godt tillade os at, der vil forekomme afvigelse i vores svar.

## B.2 Assignment group 2

# Poster - manuskript

Vi skulle undersøge hvor mange mennesker der har levet i alt.

Vi startede med prøve at bestemme fødselsraten, altså hvor mange der bliver født i forhold til årstallet. Vi kunne derefter tage integralet af funktion og det ville være antallet af mennesker der har levet. Det var dog meget svært at finde data for dette så vi måtte finde funktionen med regression. Det betød dog at vi fik et meget skævt resultat så vi kasserede det.

Vi prøvede bagefter at finde en funktion for befolkningstallet i forhold til årstallet. Hvis vi integrerede denne ville vi få antallet af menneskeår, altså hvor mange år der er blevet levet i alt. Vi kom frem at funktionen var en eksponentiel funktion, men som man kan på figur 1 herunder passer den ikke helt perfekt:

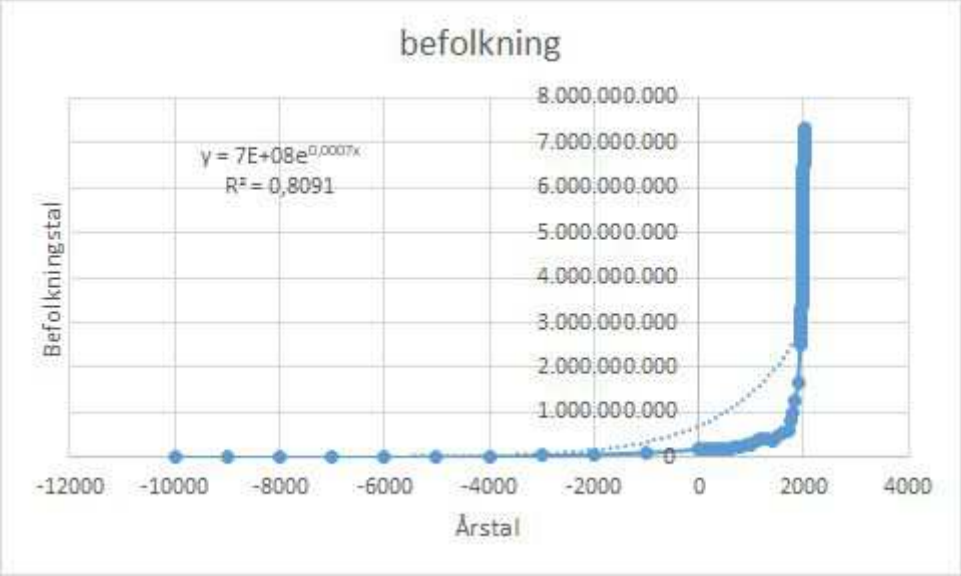
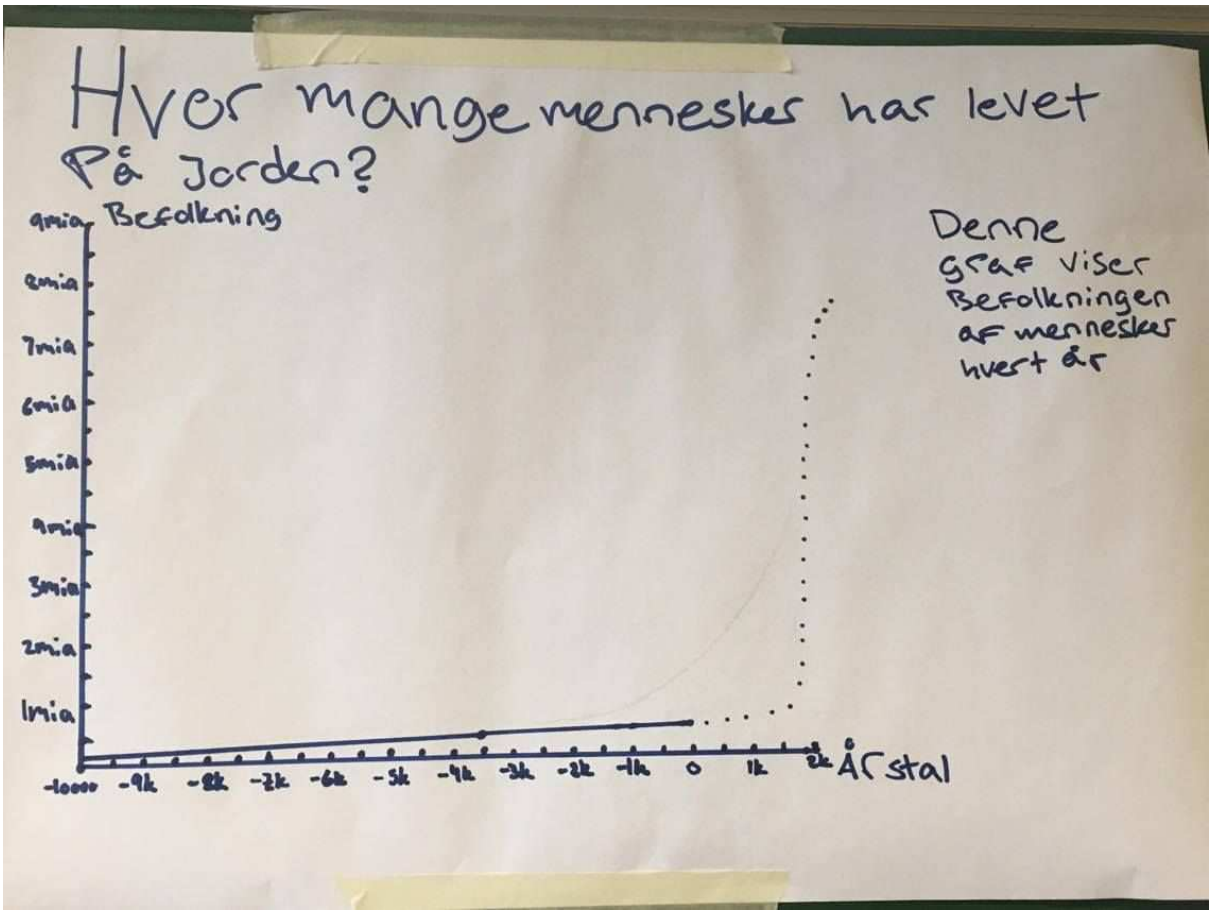


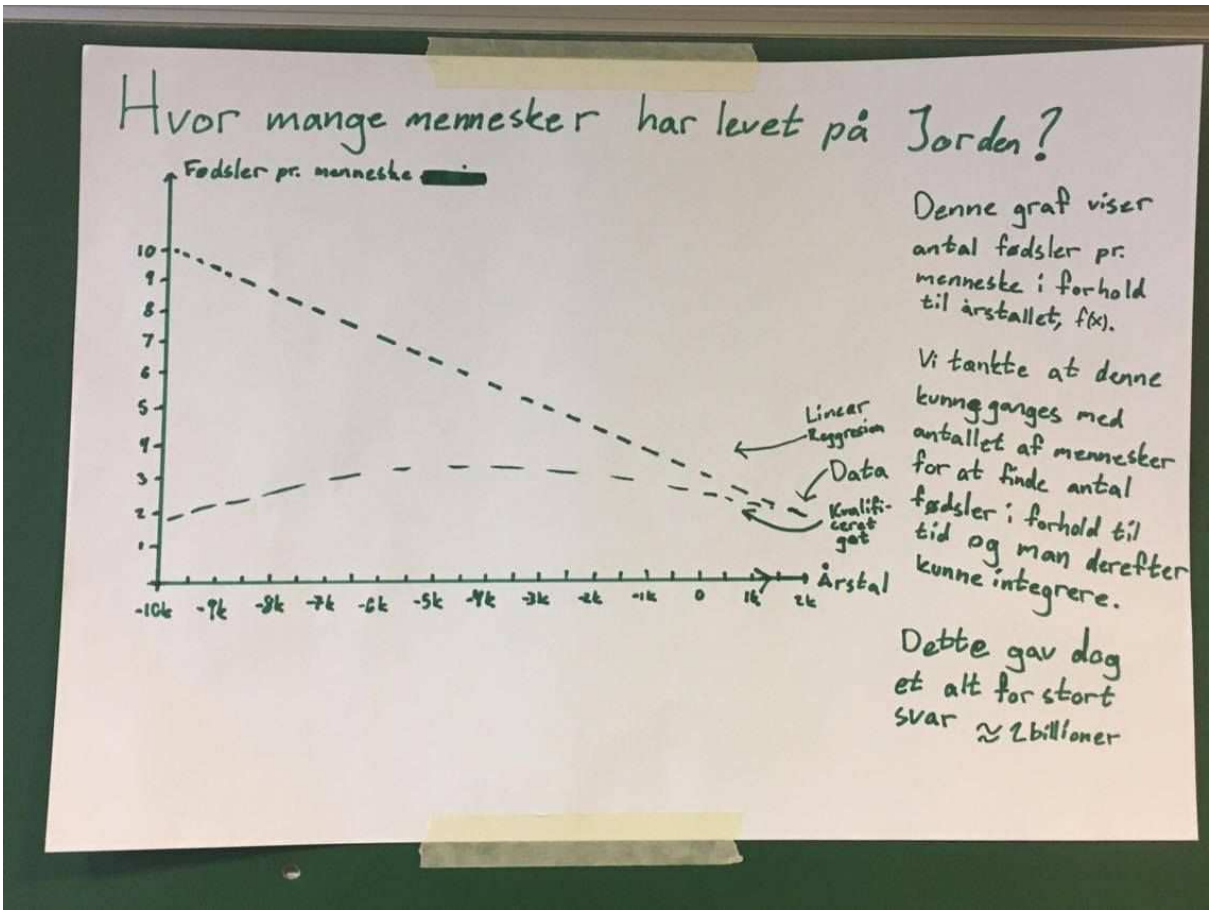

Fig. 1 Graf over befolkningstallet i løbet af tiden. Heraf ses også efter år 1950 eksisterer den bedste data.

Vi tog integralet af den fundne funktion og fik antallet af menneskeår til 4.000.000.000.000/4billioner. Vi dividerede derefter med den gennemsnitlige levealder og vi skulle derved gerne finde antallet af mennesker der har levet. Vi antog at den gennemsnitlige levealder var et sted mellem 40år og 50år og vi fik at antallet af mennesker der har levet er mellem 80.000.000.000 og 100.000.000.000.



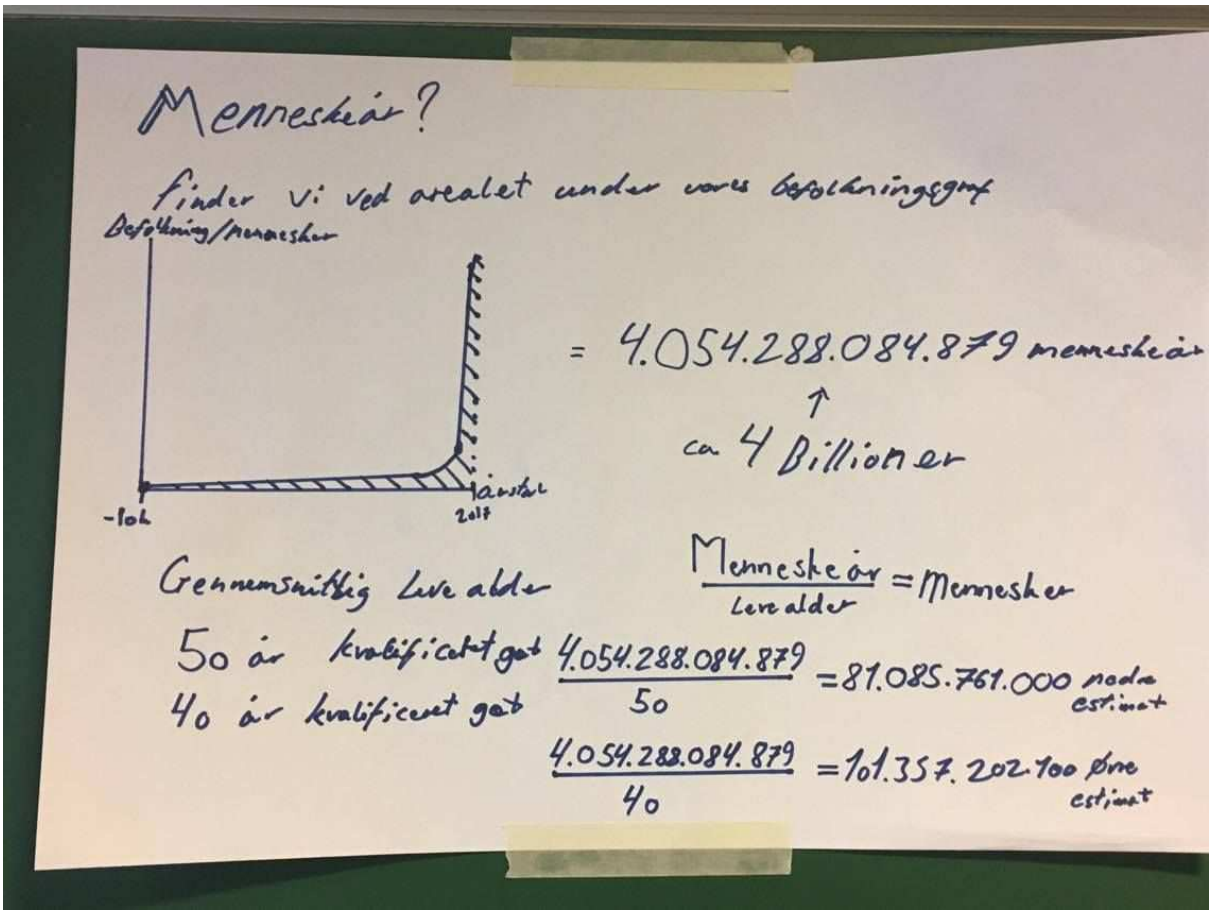

Poster del 1

På poster del 1 kan man se en graf over befolkning per år, på x aksen har vi årstal, og på y aksen har vi befolkning. Grafen starter ved 0 befolkning og år -10000.
Fra år -10000 til år 0 er grafen forholdsvis lineær, og derefter stiger grafen voldsomt.




Poster del 2

Her på del 2 kan man se fødsler pr. Menneske op ad y aksen, og årstal ud af x aksen. Den lineære funktion er fra nogle data vi har fundet, denne virker urealistisk da de i år -10000 ville skulle føde 10 børn pr menneske. Den afbøjede funktion er vores gæt på hvordan det faktisk ville se ud.

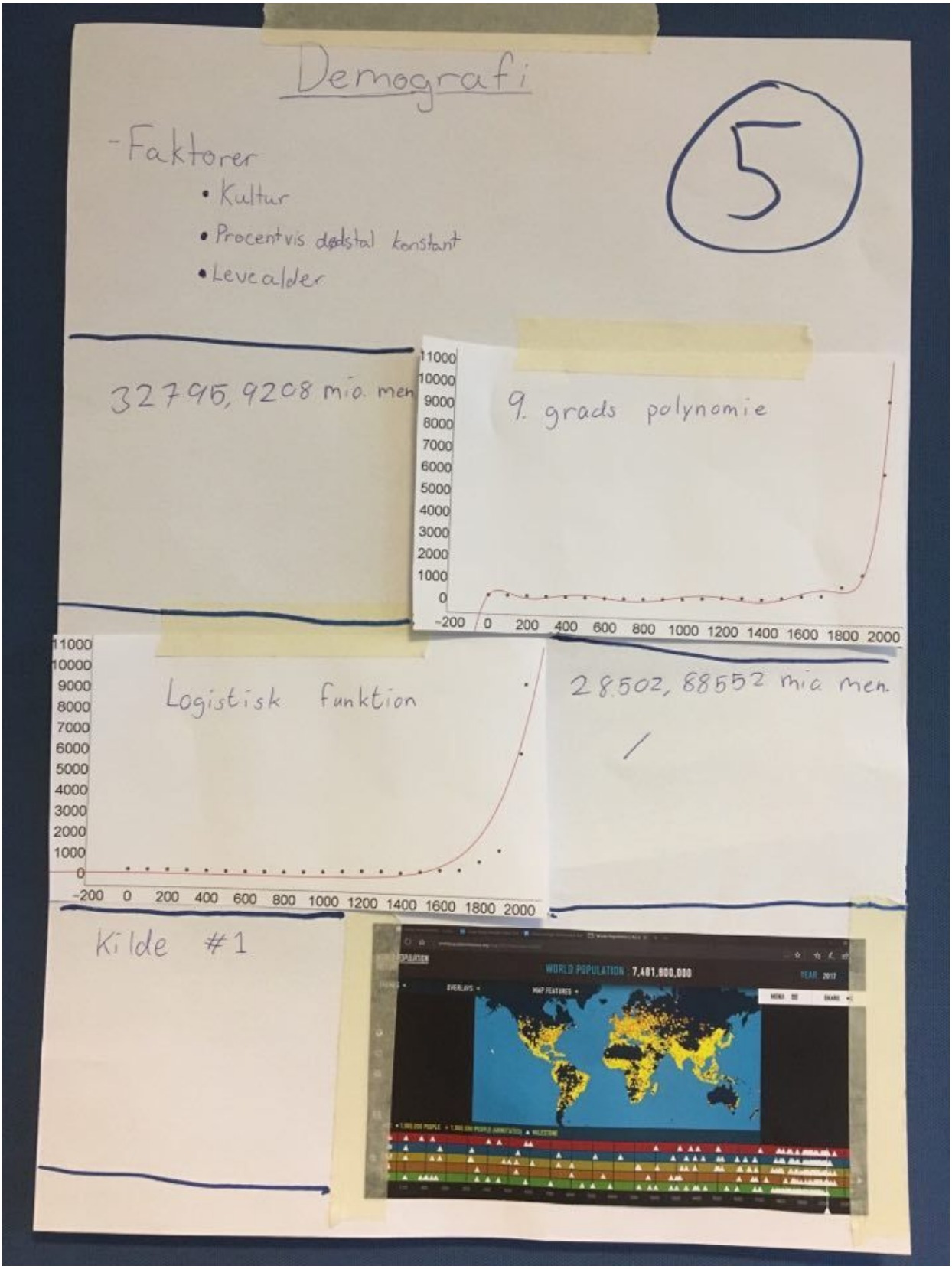



Poster del 3.

På del 3 kan man se at vi har integreret vores graf, så vi har fået vores mennesker, vores menneskeår har vi derefter delt med et kvalificeret gæt på en gennemsnitlig levetid gennem tiderne.

## B.3 Assignment group 5

**Planche**


Demografi
5
-Faktorer
• Kultur
• Procentvis dødstal konstant
• Levealder
327 95, 9208 mio. men
9. grads polynomie
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
-200 0 200 400 600 800 1000 1200 1400 1600 1800 2000
Logistisk funktion
28.502, 88552 mia men.
Kilde #1
WORLD POPULATION : 7,401,000,000
YEAR 2017
OVERLAYS
MAP FEATURES

## Fremlæggelse

**Om projektet:**

I dette projekt skulle der opstilles en funktion for antal mennesker der har levet i alt på jorden. Det skulle så diskuteres hvordan dette kunne gøres, hvilke faktorer der havde indvirkning på tallet samt hvor stort interval funktionen virker i.

**Billede 1:**

Her har vi plottet 21 punkter i et koordinatsystem med antal millioner mennesker, op ad y-aksen og antal år hen ad x-aksen. Til punkterne har Geogebra lavet et 9. gradspolynomie, som vi har integreret. Ved integrationen fandt vi de antal menneskeår, der er levet siden år nul. Ved at dividere dette tal med den gennemsnitlige levealder, fandt vi det antal af mennesker, der har levet. Vi estimerede den gennemsnitlige levealder til 40 år, da det er fra år nul til nu; år 2017. Vi konkluderede ved denne metode at der havde levet omkring 33 milliarder mennesker siden år 0.

**Billede 2:**

Her følger vi samme fremgangsmåde, som ved billede ét, bortset fra at vi her har omdannet punkterne til en logistisk funktion. Derved får vi at der har levet omkring 29 milliarder mennesker siden år nul.

Det der så blev gjort med de to integraler af funktionen, var at der blev taget gennemsnittet af dem og så delt med den gennemsnitlige levealder gennem tidsperioden fra år 0 til år 2017, hvilket er approximeret til 50 år da mange af menneskene tilbage i tiden døde i en tidlig alder da levevilkårene ikke var optimale for folk dengang. Når integralet så blev divideret med den gennemsnitlige levealder får man så tallet for hvor mange der er blevet født gennem den tidsperiode.

**Billede 3:**

På det nederste billede er et billede af hjemmesiden som vi brugte til at bestemme de punkter vi har brugt

# Appendix C

# Logbooks

## C.1 Logbooks group 1

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# D. 8/11-2017 2 lektioner

Intro til de næste 5 lektioner
Lave spørgsmål på poster til spørgsmål:
**Hvor mange Homo Sapiens har levet på Jorden?**

- Hvor mange fødsler pr. år?
    - Hvor mange er dødfødte?
- Hvad er nuværende befolkningstal?
- Hvor mange år har mennesket eksisteret?
- Hvilke parametre kan påvirke fødselstallet?
    - Hvor stor indflydelse har istiden og andre naturkatastrofer haft på menneskets fødselstal pr. år?
- Hvordan kan man opstille en model der beskriver antal mennesker der har levet på jorden?
- Hvornår defineres et individ som et menneske?

**Finde information om emnet - Kildesøgning**

**Hvor mange fødsler pr. år?**
http://politiken.dk/usynligeborn/art6137748/Hvert-%C3%A5r-d%C3%B8r-2.000-b%C3%B8rn-i-mors-mave
Der bliver født nye børn hele tiden. Ind mellem disse nyfødte børn, er der desværre også dødfødte. Som der bliver skrevet i denne artikel fra politiken, skriver de at der hvert år dør omkring 2000 fostre i mors mave. Det kan så diskuteres om de tæller som er født menneske eller de ikke gør. Vi har valgt at sige, at det ikke tæller som et født menneske. Dvs. at vi vælger at se bort fra det i vores model.

**Hvor mange år har mennesket eksisteret?**
De første mennesker som nu, *Homo sapiens*, har eksisteret i ca 200.000 år[1] (de første af slægten *Homo* eksisterede for ca to millioner år siden[2]).

**Hvornår er man et menneske?**
Det er et filosofisk spørgsmål, hvor vi påpeger at der ikke findes et konkret svar til det store spørgsmål. Det er et stort spørgsmål, som "hvad definerer et menneske", der er ikke et konkret svar, men i stedet end holdning.
https://www.etik.dk/etikpanelet/hvornår-er-et-menneske-et-menneske-og-hvem-bestemmer-det
Når man tænker selv er man noget særligt som menneske
http://www.biokemi.org/biozoom/issues/522/articles/2367
Data:
http://www.worldometers.info
https://data.worldbank.org/indicator/SP.POP.TOTL

[1] https://videnskab.dk/25-soforklaringer/menneskets-evolution
[2] https://da.wikipedia.org/wiki/Jordens_historie#Mennesket

1

**Hvor mange har levet og hvornår var de første:**
http://historienet.dk/civilisationer/hvor-mange-mennesker-har-levet
http://www.worldpopdata.org/index.php/chart

**Hvad er nuværende befolkningstal?**
Befolkningstallet stiger hele tiden. Det stiger i takt med at der samtidig er folk der dør. Vi ved at der i dagens verden er flere der bliver født en der dør. Og derfor er befolkningstallet stigende.

**Hvilke parametre kan påvirke fødselstallet?**
Der har i verden været mange parametre som har påvirket fødselstallet. Det er specielt krige og naturkatastrofer der har haft store indblik i dette.
En ting kunne også være finanskrisen. Denne krise får folk til at overvejen en ekstra gang om det er det rette tidspunkt at få børn på, grundet at folk ikke har penge nok til at kunne have børn ved deres side.
http://www.ugebreveta4.dk/krisen-faar-foedselstal-til-at-rasle-ned_14335.aspx
Istider varer ca. 10.000-15.000 år, der har været nogen siden *Homo sapiens* er opstået
http://denstoredanske.dk/Danmarkshistorien/I_begyndelsen/J%C3%A6gerstenalderen/I_begyndelsen/Den_sidste_istid

**Hvor mange har levet på jorden siden begyndelsen**
Der er blevet født 106 milliarder mennesker fra starten på vores eksistens til år 2009.
http://illvid.dk/fortiden/hvor-mange-har-levet-paa-jorden

**Hvordan kan man opstille en model der beskriver antal mennesker der har levet på Jorden?**
Først skal vi have en idé om, hvor lang tid vi skal redegøre for.
Så skal vi finde ud af, hvor mange der bliver født pr. år. Hvor mange blev født i starten og hvordan stiger antallet af fødsler gennem homo sapiens udvikling.
Vi skal også finde ud af, hvornår man definerer et født menneske, da nogen er dødfødte og nogle kan dø som foster.
Så skal vi finde ud af, hvor meget diverse naturkatastrofer f.eks. istider har indflydelse på antal fødte pr. år.

## Logbog

Vi fik til opgave i starten af timen at forsøge at besvare det overordnede spørgsmål, hvor mange mennesker har levet på jorden. Vi diskutere om man kan opstille en matematisk model, for at finde ud af hvor mange mennesker der har eksisteret på jorden. For at besvare dette spørgsmål kan vi opstille underspørgsmål, som skal frembringe mere information og viden om emnet. Vi skulle derefter lave en poster, som vi præsenterede over for både læreren og resten af eleverne for klassen. Posteren gik ud på at skrive underspørgsmål skulle lede os videre til det overordnede spørgsmål. Næste skridt var informationssøgning, der skulle vi finde

[redacted]

kilder til de underspørgsmål som vi stillede os selv, naturligvis var kildekritiske. De skulle dernæst forklares foran klassen og læreren.

**Kvajebajer**

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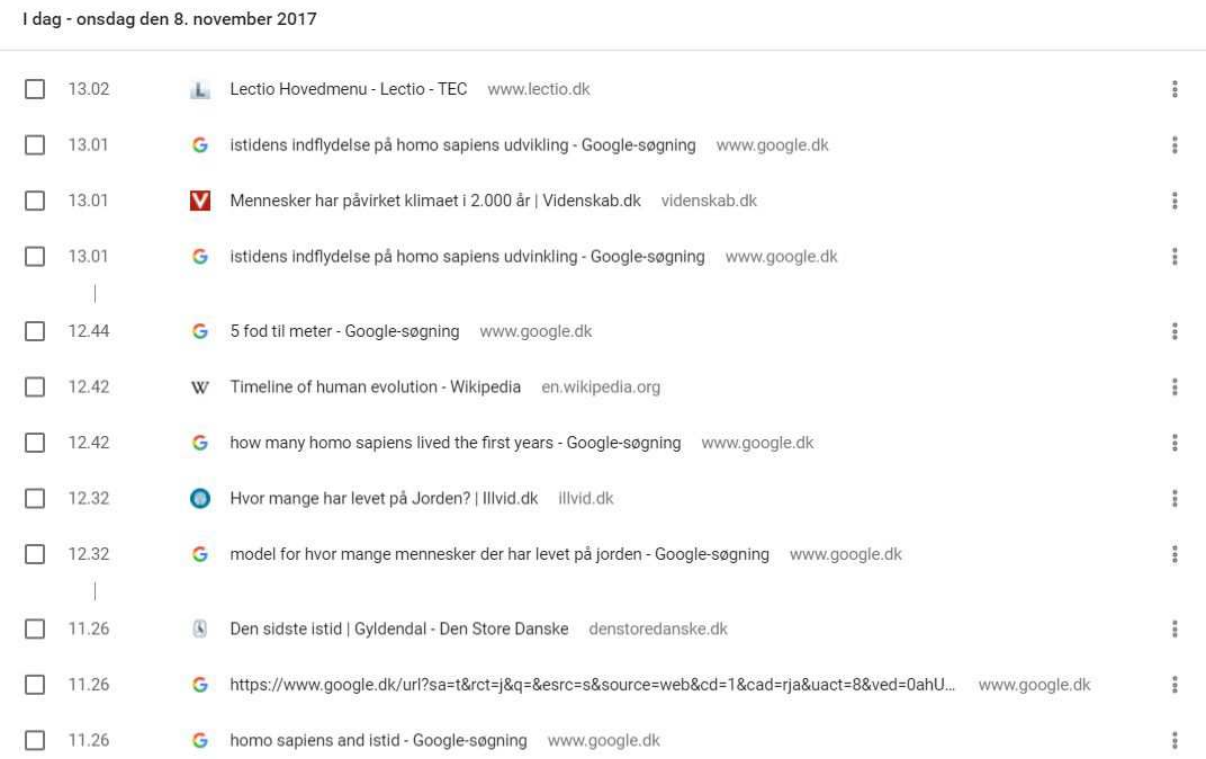
| | | 08-11-2017 | |
| | | | |



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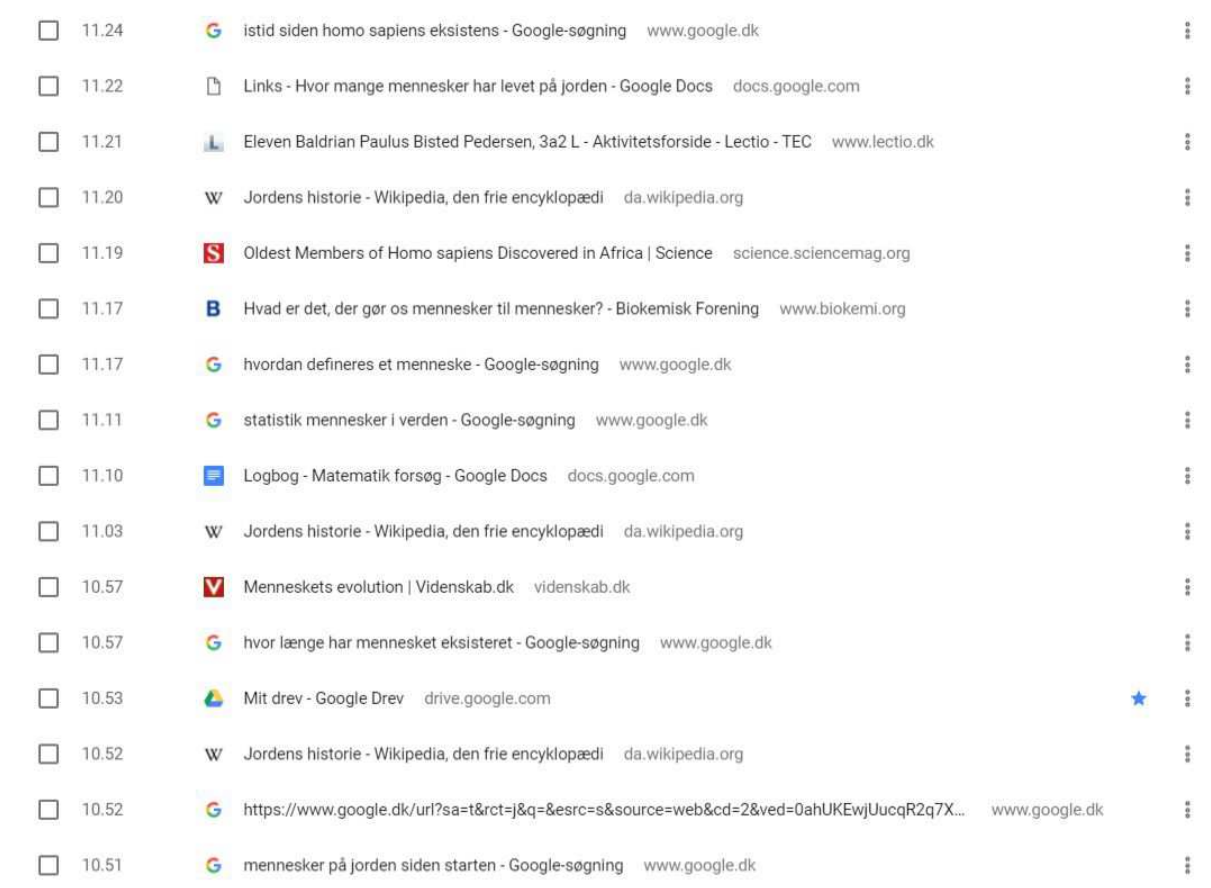

| | | | |
|---|---|---|---|
| 11.24 | istid siden homo sapiens eksistens - Google-søgning | www.google.dk | |
| 11.22 | Links - Hvor mange mennesker har levet på jorden - Google Docs | docs.google.com | |
| 11.21 | Eleven Baldrian Paulus Bisted Pedersen, 3a2 L - Aktivitetsforside - Lectio - TEC | www.lectio.dk | |
| 11.20 | Jordens historie - Wikipedia, den frie encyklopædi | da.wikipedia.org | |
| 11.19 | Oldest Members of Homo sapiens Discovered in Africa \| Science | science.sciencemag.org | |
| 11.17 | Hvad er det, der gør os mennesker til mennesker? - Biokemisk Forening | www.biokemi.org | |
| 11.17 | hvordan defineres et menneske - Google-søgning | www.google.dk | |
| 11.11 | statistik mennesker i verden - Google-søgning | www.google.dk | |
| 11.10 | Logbog - Matematik forsøg - Google Docs | docs.google.com | |
| 11.03 | Jordens historie - Wikipedia, den frie encyklopædi | da.wikipedia.org | |
| 10.57 | Menneskets evolution \| Videnskab.dk | videnskab.dk | |
| 10.57 | hvor længe har mennesket eksisteret - Google-søgning | www.google.dk | |
| 10.53 | Mit drev - Google Drev | drive.google.com | |
| 10.52 | Jordens historie - Wikipedia, den frie encyklopædi | da.wikipedia.org | |
| 10.52 | https://www.google.dk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=2&ved=0ahUKEwjUucqR2q7X... | www.google.dk | |
| 10.51 | mennesker på jorden siden starten - Google-søgning | www.google.dk | |

## Historik

Slet historie... Søg

| Websted | Adresse |
|---|---|
| naturkatastrofer - Google-søgning | https://www.google.com/search?q=naturk...HSSiBe4Q_AUICigB&biw=1435&bih=827 |
| OVERBLIK: Flere døde i kraftigt jordskælv i Mexico – Ekstra Bladet | https://ekstrabladet.dk/nyheder/krigogkat...de-i-kraftigt-jordskaelv-i-mexico/6814632 |
| De 10 mest dødelige naturkatastrofer - TV 2 | http://nyheder.tv2.dk/nytomtv/article.php/id-11554132.html/wp-includes/js/index.php |
| Top-10: De værste naturkatastrofer de seneste 50 år - politiken.dk | http://politiken.dk/viden/art5520588/Top-10-De-værste-naturkatastrofer-de-seneste-50-år |
| Naturkatastrofe - Wikipedia, den frie encyklopædi | https://da.wikipedia.org/wiki/Naturkatastrofe |
| (ingen titel) | http://www.dst.dk/Site/Dst/Udgivelser/GetPubFile.aspx?id=4576&sid=bef150 |
| Fertilitet gennem 100 år \| Kvinfo.dk | http://kvinfo.dk/2015/fertilitet-gennem-100-aar |
| perioder hvor fødselsår ikke var høj - Google-søgning | https://www.google.com/search?client=s.....0...1.1.64.psy-ab..0.0.0....0.Qzgbp-xoGyA |
| perioder hvor fødselsdato ikke var høj - Google-søgning | https://www.google.com/search?client=s...KHSruDjMQBQgjKAA&biw=1280&bih=738 |
| perioder hvor fødelsåret ikke var høj - Google-søgning | https://www.google.com/search?client=s...elsåret+ikke+var+høj&ie=UTF-8&oe=UTF-8 |
| Apple Pay - nemt, hurtigt og sikkert \| Nordea.dk | https://www.nordea.dk/privat/daglig-oekonomi/mobil/apple-pay.html?WT.MC_ID=16418 |
| At være dannet er at være menneske - politiken.dk | http://politiken.dk/debat/profiler/marstal/a...27/At-være-dannet-er-at-være-menneske |
| Hvornår er et menneske et menneske? \| ricardtriis | https://ricardtriis.wordpress.com/2013/10/05/hvornar-er-et-menneske-et-menneske/ |
| Logbog - Matematik forsøg - Google Docs | https://docs.google.com/document/d/1D3...-4V2THfzjD7LwkG1j1H073zvgoQl0M/edit |
| Hvornår er et menneske et menneske, og hvem bestemmer det? - Etik.dk | https://www.etik.dk/etikpanelet/hvornår-er...ke-et-menneske-og-hvem-bestemmer-det |
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| Delt med mig - Google Drev | https://drive.google.com/drive/shared-with-me |
| Mit drev - Google Drev | https://drive.google.com/drive/my-drive |
| Google Drev | https://drive.google.com/drive/ |
| Google Drev | https://drive.google.com/drive/?usp=docs_home |
| Google Docs | https://docs.google.com/document/u/0/ |
| Google Docs | https://docs.google.com/document/u/0/?usp=direct_url |
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| Worldometers.info | http://934725698.keywordblocks.com/To...SBBB.ymmyjJ.NmYS&kct=239194&abpl=2 |

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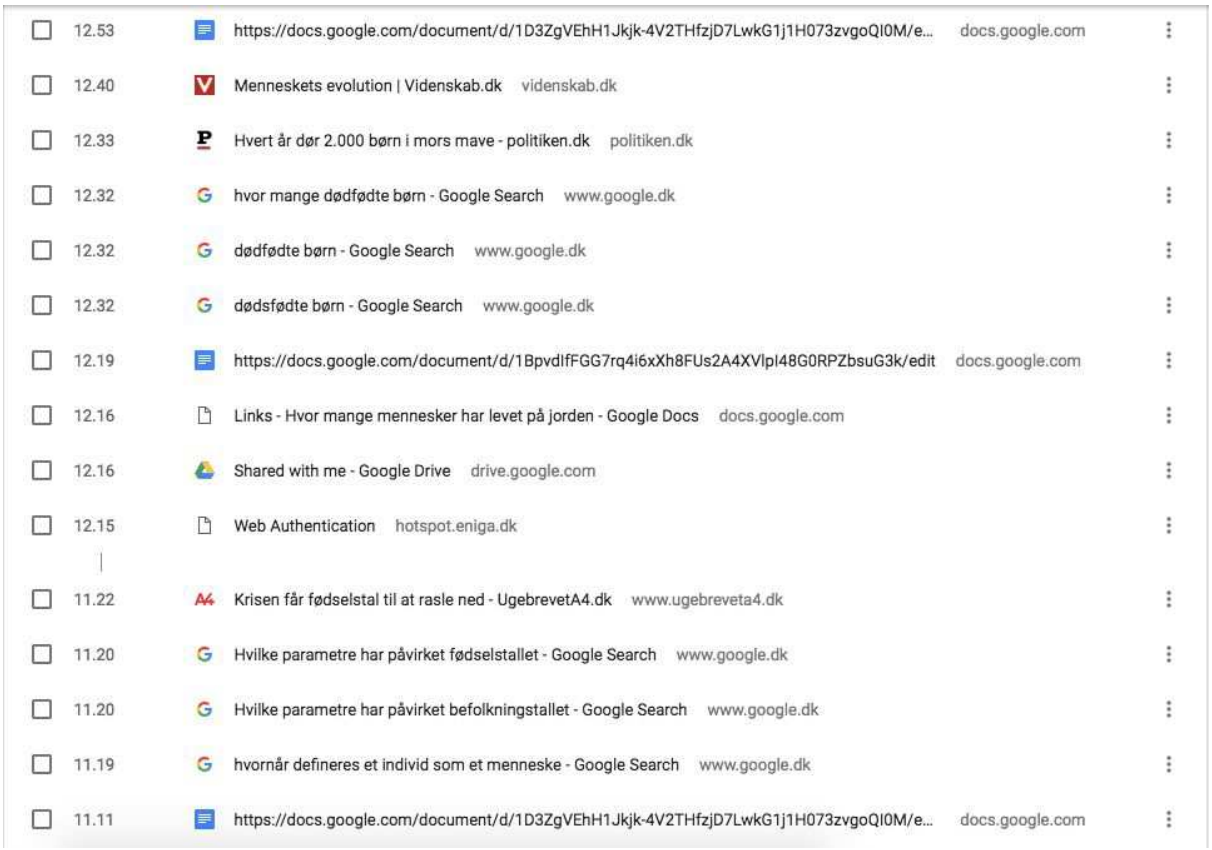

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| 11.19 | hvornår defineres et individ som et menneske - Google Search | www.google.dk | |
| 11.11 | https://docs.google.com/document/d/1D3ZgVEhH1Jkjk-4V2THfzjD7LwkG1j1H073zvgoQI0M/e... | docs.google.com | |

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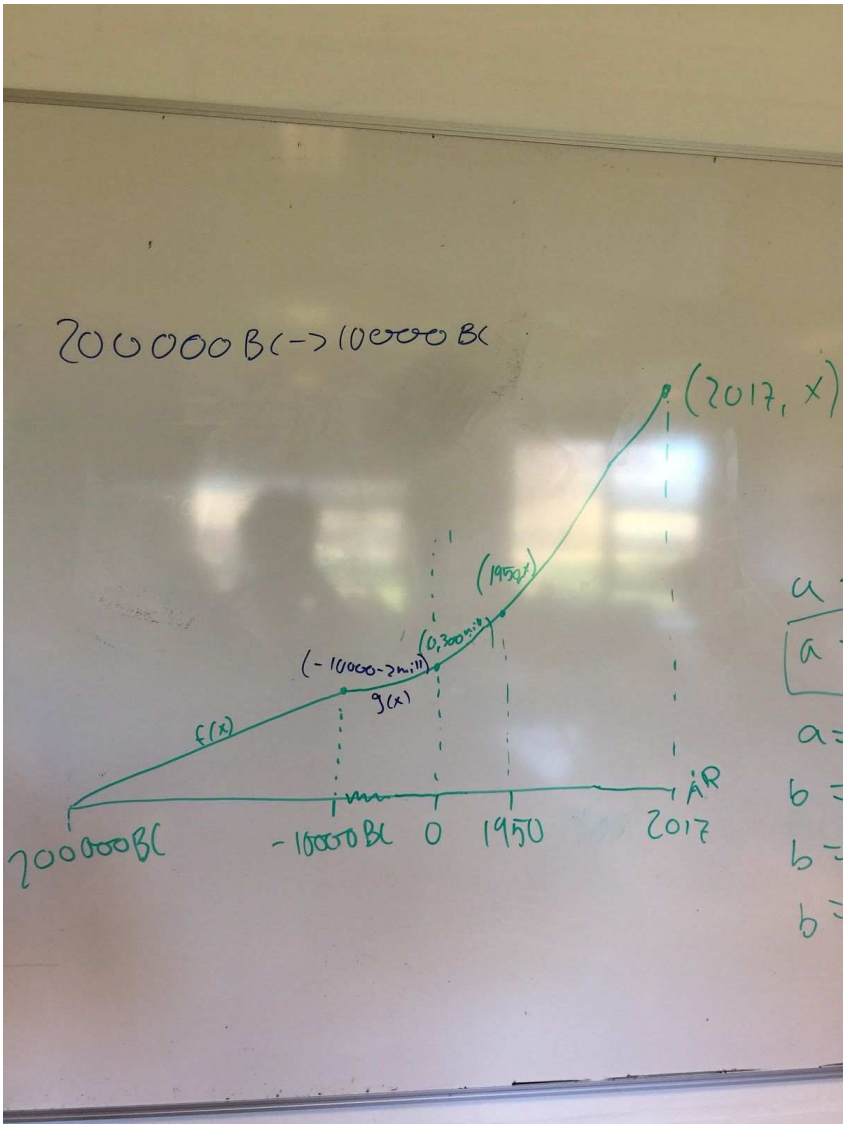
## D. 13/11-2017  1 lektion

Vi fandt en kilde der beskriver befolkningstallet i verden fra år 0 til 2011.
https://da.wikipedia.org/wiki/Verdens_befolkning

Vi har forsøgt at lave funktioner for nogle forskellige intervaller.




Her kan de forskellige intervaller ses. Den lineære funktion fra år 200000BC til 10000BC er:

[redacted]

Vi starter med at finde en lineær funktion $y = ax + b$

Hældningen $a$ findes ud fra formlen $a = \frac{y_2 - y_1}{x_2 - x_1}$

$$a := \frac{2000000 - 0}{-10000 - (-200000)}$$

$$a := \frac{200}{19}$$

Nu skal skæringen af y-aksen $b$ findes. Denne findes ud fra formlen $b = y_1 - a \cdot x_1$

$$b := 0 - a \cdot (-200000)$$

$$b := \frac{40000000}{19}$$

Vores funktion må derfor være :

$$y = \frac{200}{19}x + \frac{40000000}{19}$$

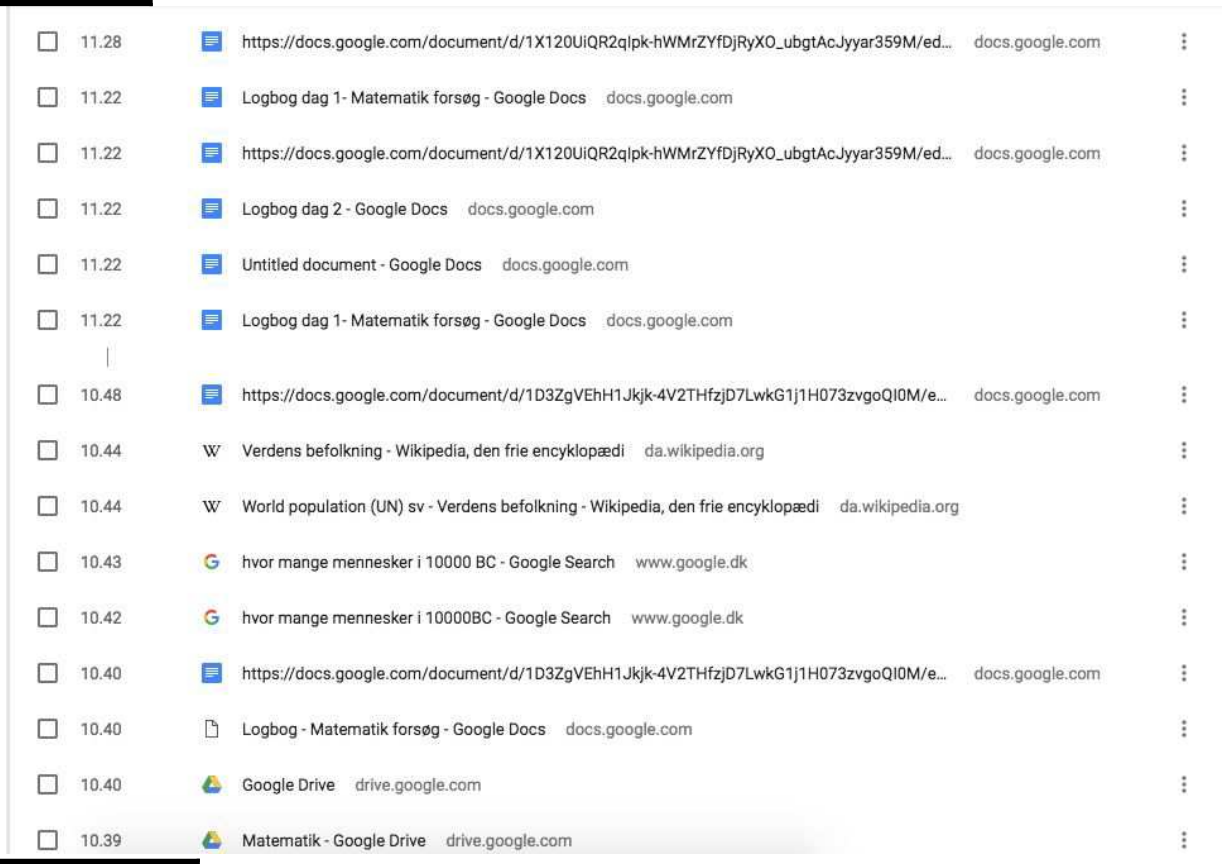
$$y = \frac{200\,x}{19} + \frac{40000000}{19}$$

Som det kan ses blev vores funktion for dette interval:

$$y = \frac{200}{19}x + \frac{40000000}{19}$$

[redacted]

[redacted]

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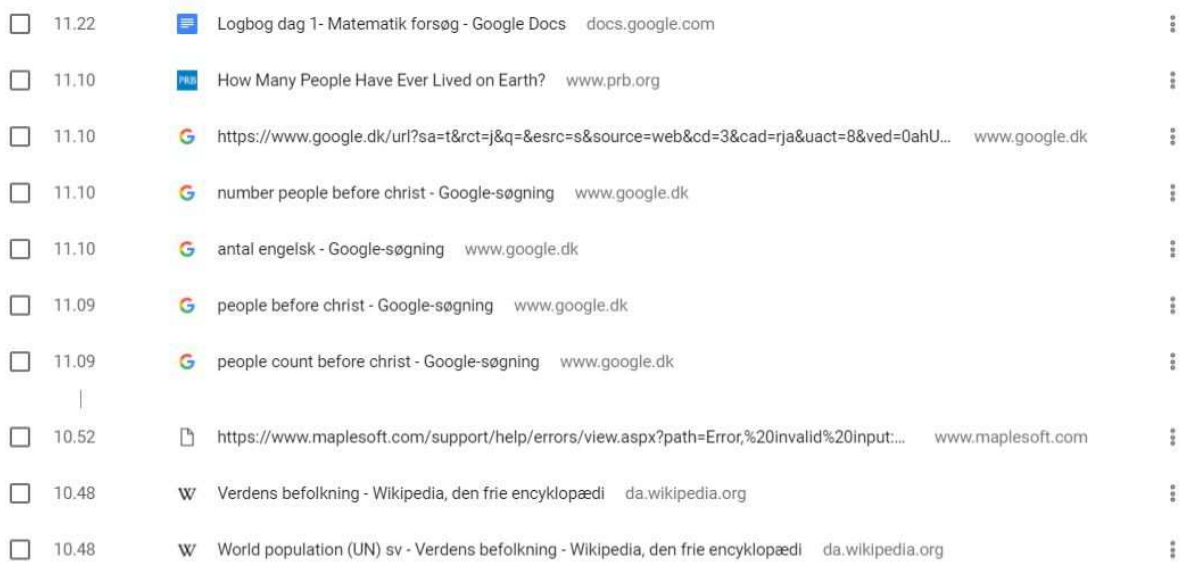
I dag - mandag den 13. november 2017



# Logbog Tirsdag 14/11-2017

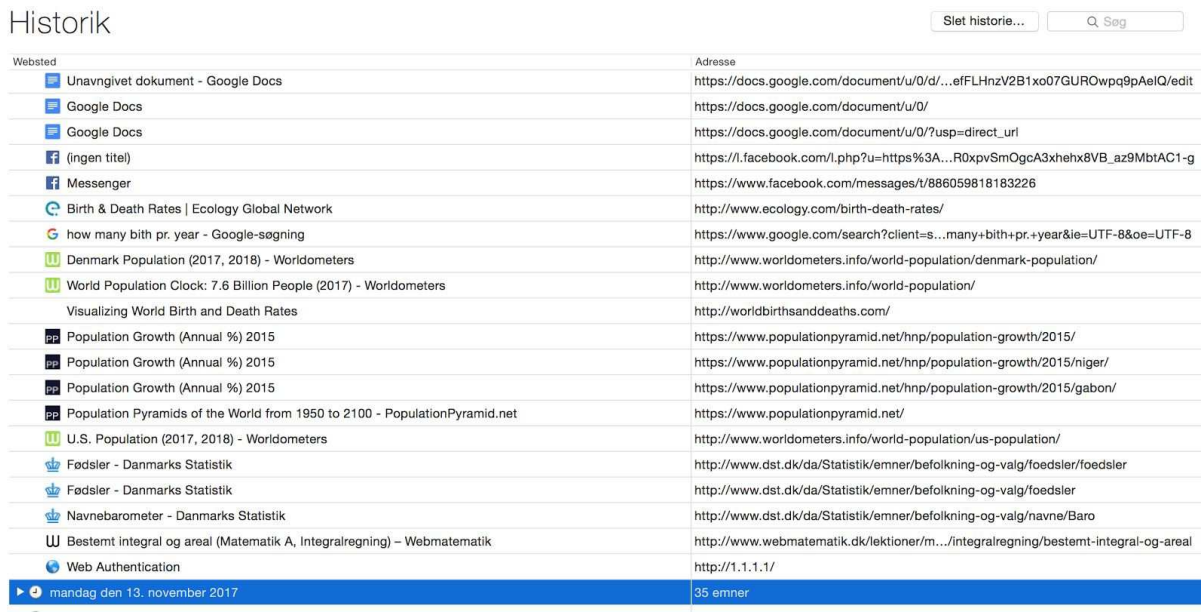
I dag var dagen hvor vi skulle fremlægge ved hjælp af en planche foran elever for klasse. Vi startede med at finde areal under grafen af de fire intervaller som vi valgte sidste gang, intervallerne går fra følgende:
200.000BC→ -10.000
10.000→ 0
0→ 1900
1950→ 2017

Vi fandt arealet under hvert interval og lagde dem alle sammen, dernæst skulle vi dividere det tal som vi fik frem til med gennemsnitsalderen for mennesket, som vi fandt en kilde til at være 40 år. Vi fandt frem til at der har levet $1{,}5*10^{12}$ (ved hjælp af integralregning, mere specifikt arealet under en graf, hvor udledningerne kan ses på Maple-Filen) og det skulle vi dividere med gennemsnitsalderen for mennesket, så vi ikke tæller mennesket flere gange i arealet under grafen.

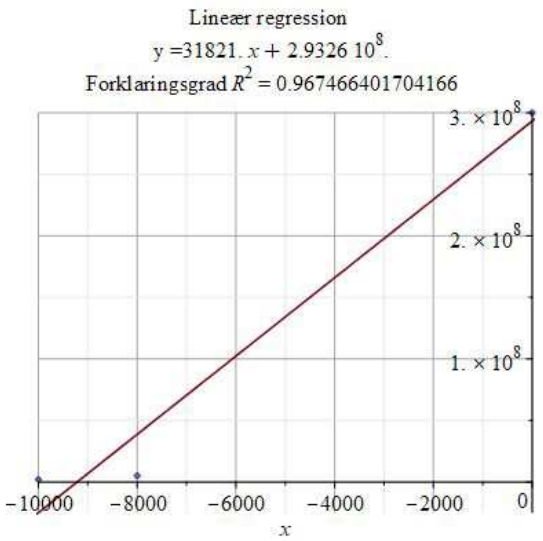
$$\frac{1{,}5*10\hat{}12}{40} = 1,37 * 10^{11}$$

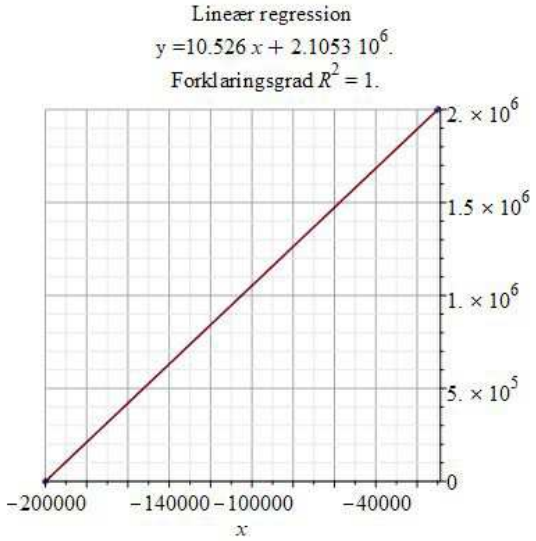
Vi kan konkludere at $1{,}37*10^{11}$ mennesker har levet på jorden. Det har været et lærerigt projekt, som vi har begået os ud i. Man kan roligt sige at det er en åben opgave, hvor der ikke findes et konkret svar, derfor kan vi godt tillade os at, der vil forekomme afvigelse i vores svar.

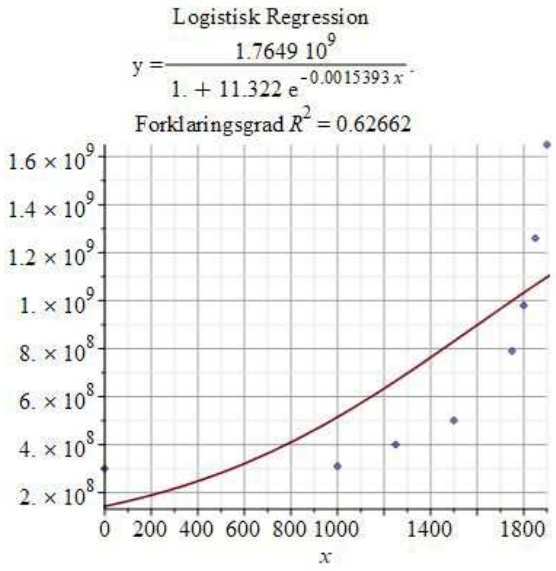
Historik:

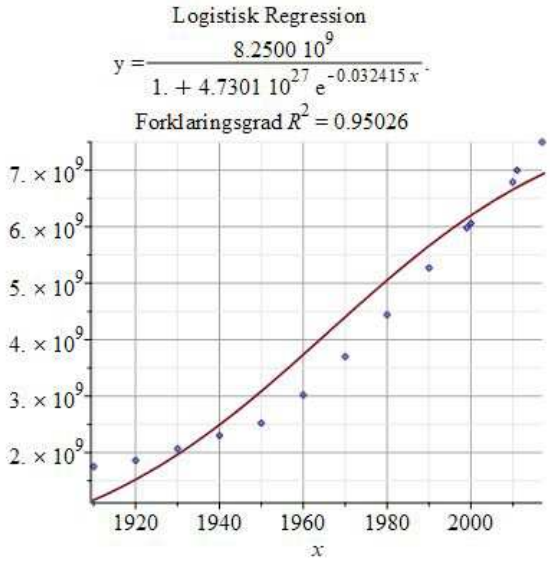


På denne side ses billederne som skal tilsammen, vise grafen for de fire intervalle, som vi valgte at dele det op i, for at finde ud af hvor mange mennesker som har levet på jorden. På x-aksen ses antal år, så man ikke bliver i tvivl og på y-aksen ses antal levende mennesker i det givne år.




2

## C.2 Logbooks group 2

I dag har vi startet forløbet op. Vi har fået en introduktion til emnet og det overordnede spørgsmål som vi skal undersøge; “Hvor mange mennesker har levet på jorden?” Vi lavede først en øvelse hvor vi skulle skrive underspørgsmål til emnet. Vi kom frem til underspørgsmålene:

- Hvor mange har levet fra 1850 og frem?
- Kan vi beskrive vækst matematisk
- Hvor lang tid lever mennesker?
- Hvor lang tid har der levet mennesker?
- Hvilke faktorer/opfindelser har haft indflydelse på væksten af verdens befolkning?

Efter vi havde fundet underspørgsmålene skulle vi søge efter kilder som vi kunne bruge til at besvare disse spørgsmål.
Både underspørgsmål og kilder blev delt i klassen ved “konferencer”.
Efter det gik vi i gang med at prøve at besvare vores underspørgsmål ved hjælp af kilderne. Vi fandt her ud af at kilderne ikke var specielt præcise og vi gik derfor på udkig efter flere og bedre kilder.

**Historikker:**

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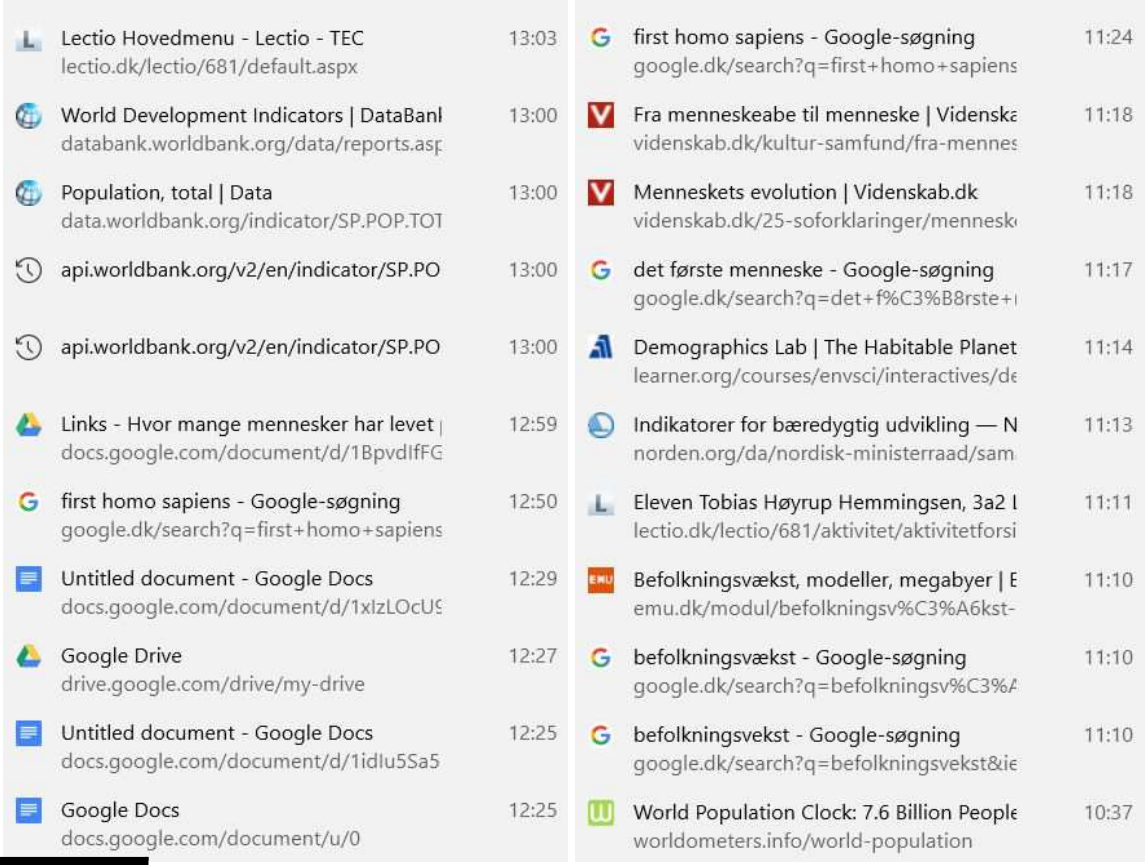


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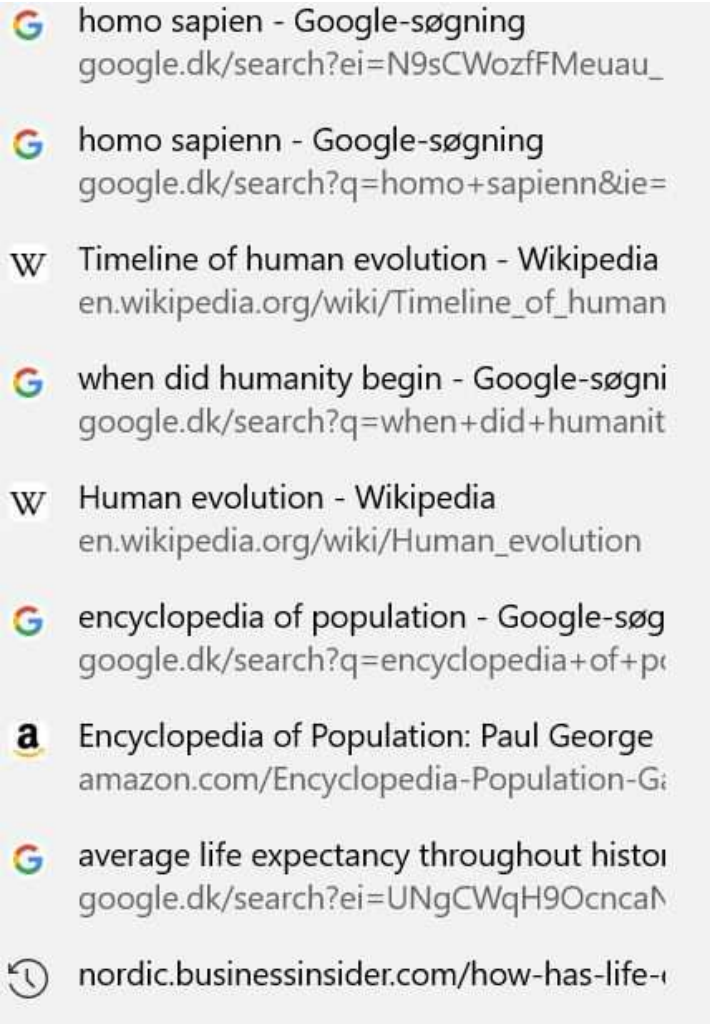
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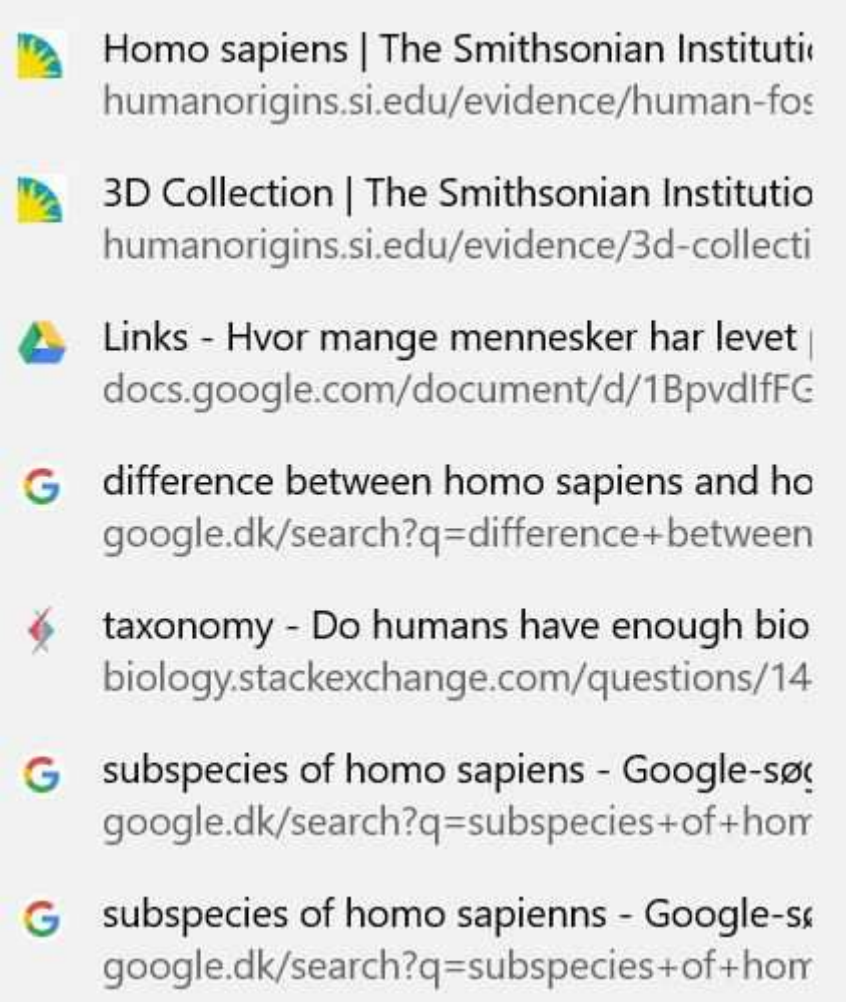


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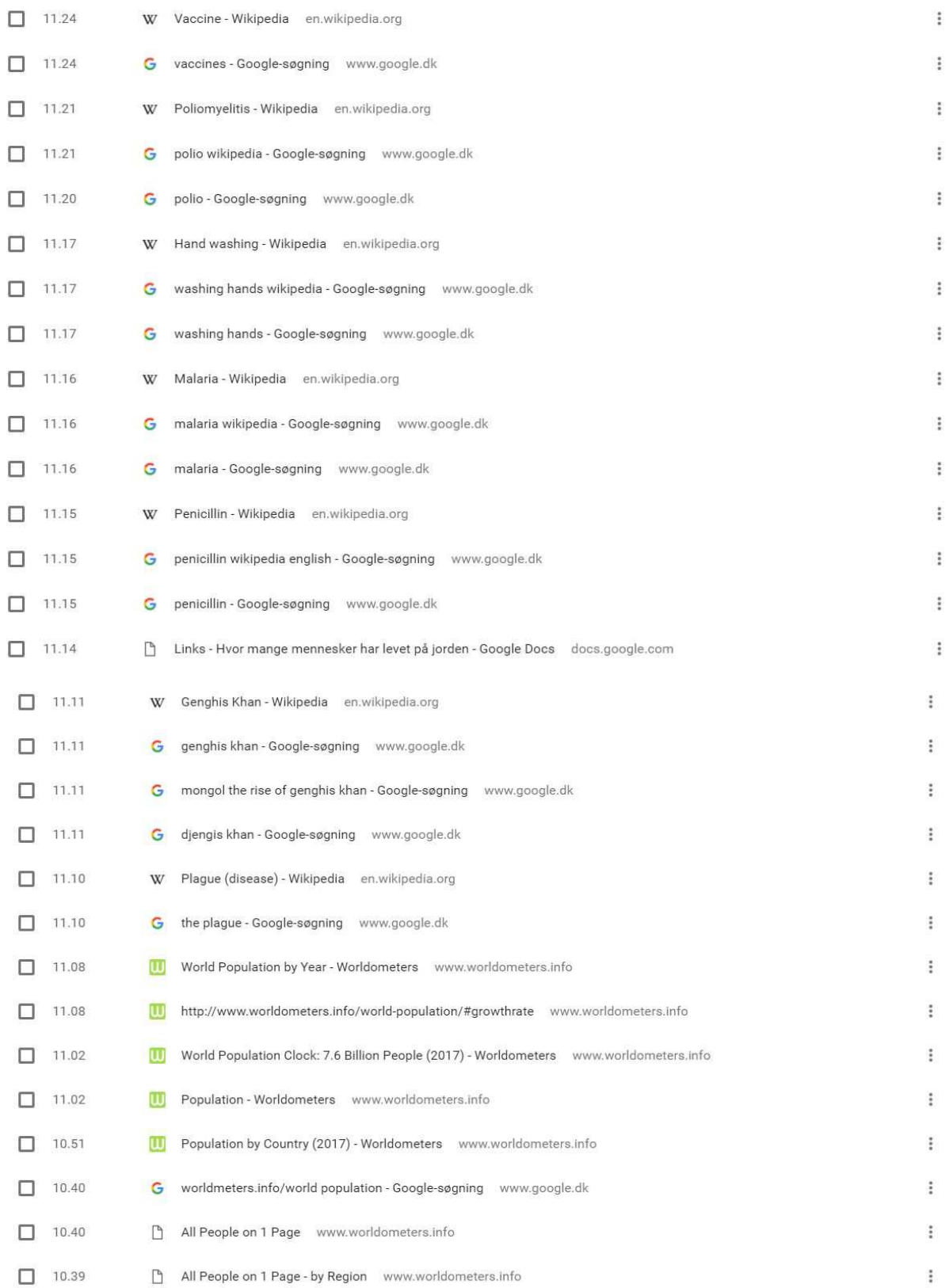
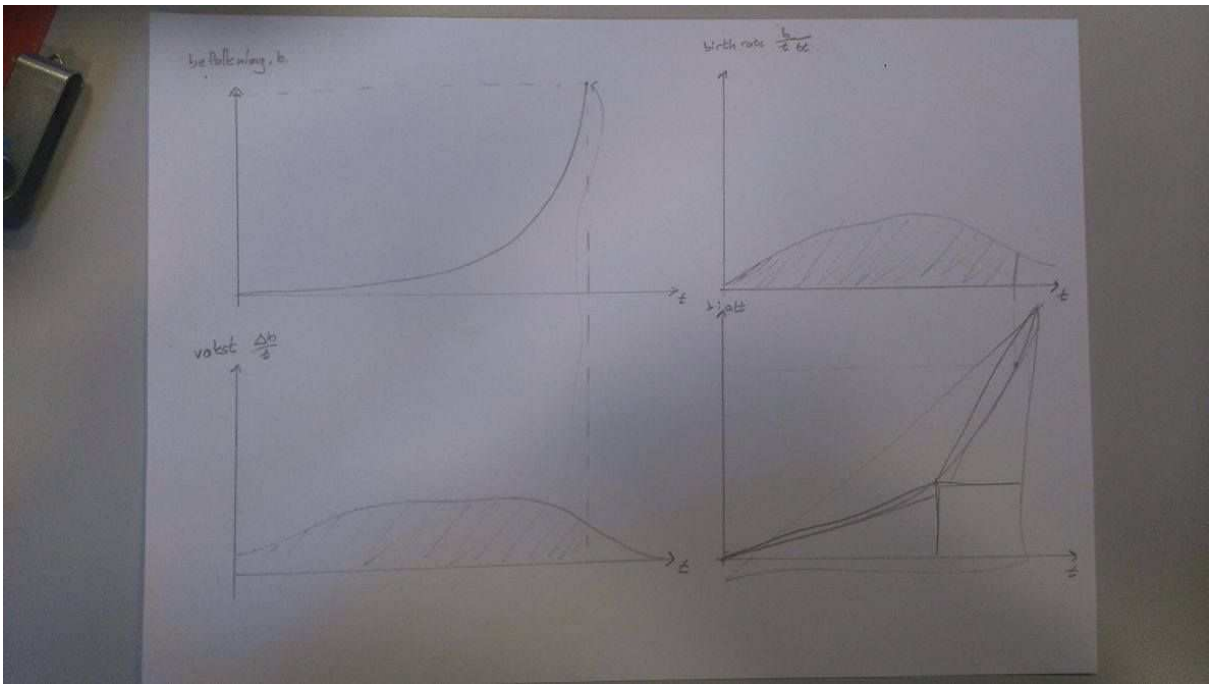
| 12.14 | https://data.worldbank.org/ | data.worldbank.org | |




Mandag d. 13/11-2017

I dag har vi gået mere konkret til værks med problemet. Vi har har fundet de data vi skulle bruge til at opstille de rigtige funktioner. Vi har tegnet skitser af de funktioner vi har for at finde ud af hvilke funktioner vi egentlig skal bruge.



Vi har med dataene opstillet en funktion for befolkningstallet som funktion af tid og antallet af fødsler som funktion af tid og befolkningstal. Vi mangler nu at finde fødselsraten i forhold til tid. Denne funktion kan vi så integrere for at finde antallet af personer der har levet.
Vi har også indset at der vil være mange fejlkilder. Det vil altså være vigtigt at påpege disse når man fremlægger resultatet.

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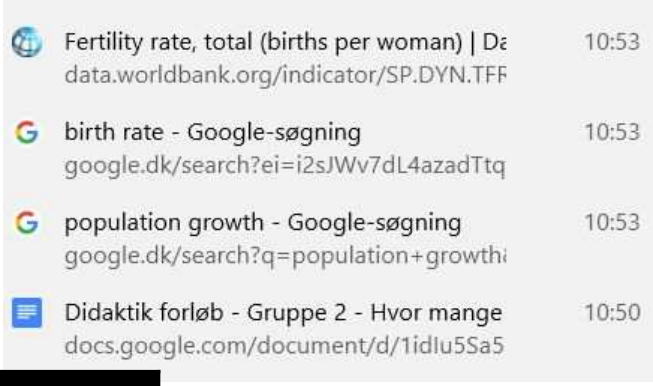


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| 10.41 | Verdens befolkning - Wikipedia, den frie encyklopædi | da.wikipedia.org | |
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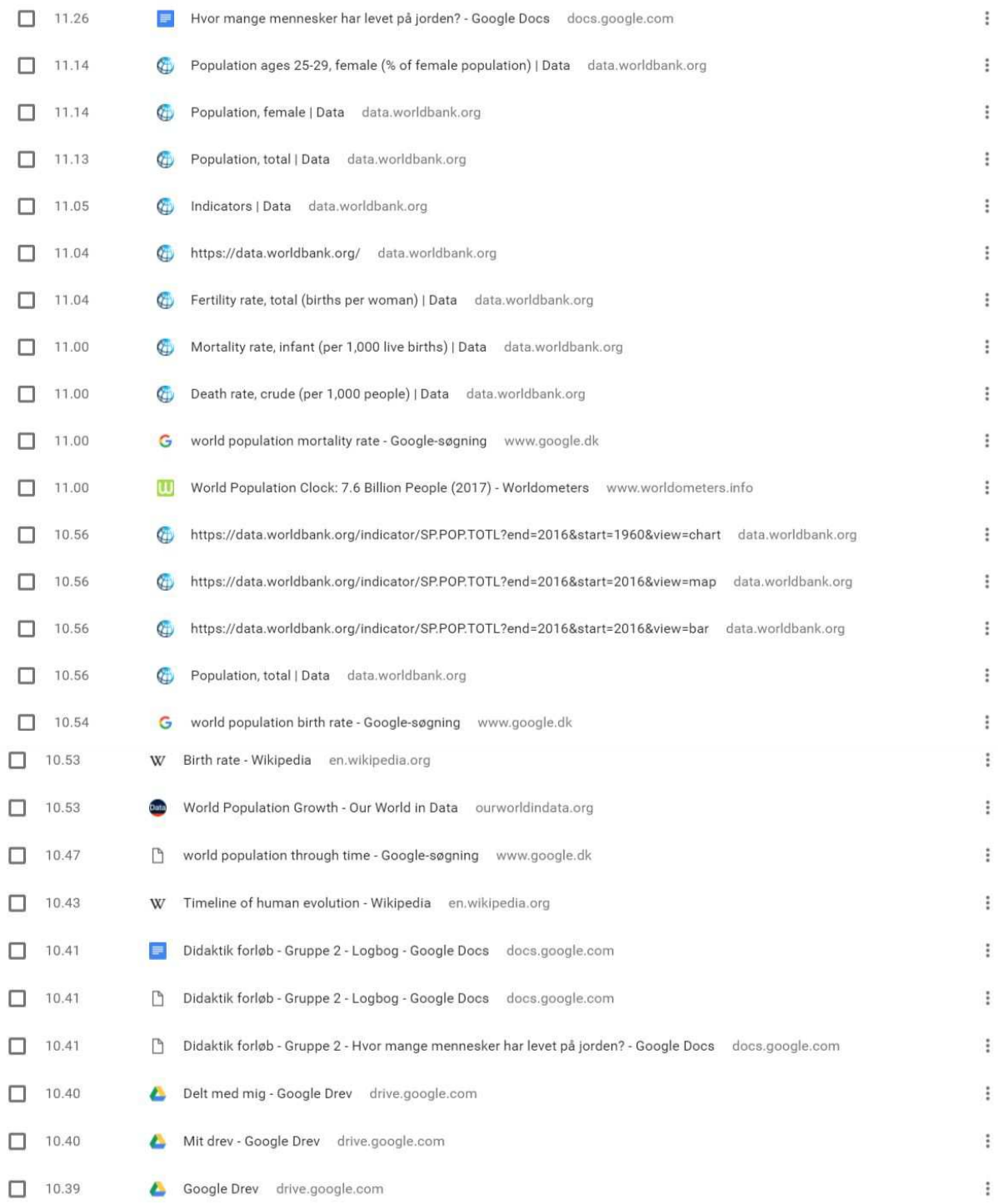
| 10.41 | verdens befolkning - Google-søgning | www.google.dk | |
| 10.40 | Didaktik forløb - Gruppe 2 - Hvor mange mennesker har levet på jorden? - Google Docs | docs.google.com | |

[redacted]




Tirsdag d. 14/11-2017

Vi opstillede funktionerne og beregnede pp disse for at kunne bruge dem på vores poster. Vi lavede vores poster som 3 A3 papirer der hver havde en graf på. Disse grafer viser udviklingen i forløbet og viser hvordan vi kom tættere på vores endelige resultater og til vores endelige konklusion. Det vi kom frem til var at den bedste måde at beregne antallet af mennesker der har levet finde nemmest (ud fra de datasæt vi brugte) var at finde en funktion for antallet af mennesker på jorden ud af en tidsakse. Denne funktion integrerede vi og fik antallet af år levet af alle

mennesker nogensinde. Dette tal for samlede antal menneskeår dividerede vi med den gennemsnitlige leveralder for mennesker siden det første menneske eksisterede. Dette tal estimerede vi til at være 40-50 år, når vi indsatte disse to tal i vores formel fandt vi at antallet af mennesker der har levet til at være 101*10^9

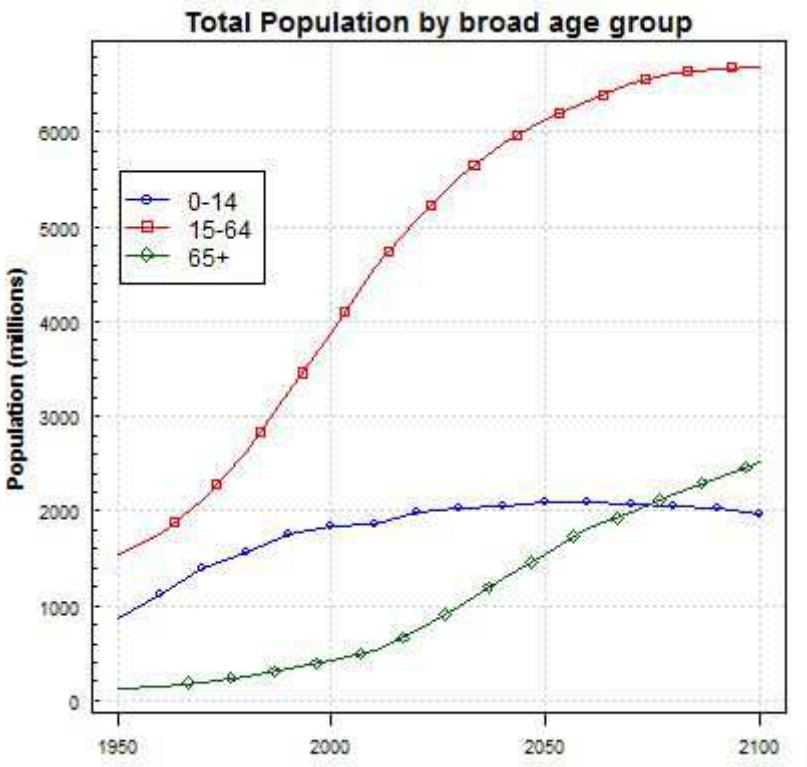
## C.3 Logbooks group 3

Spørgsmål:

**Hvor mange mennesker har levet på jorden?**

- Hvor mange mennesker bliver født om året?
  -
- Hvor mange % vokser befolkningstallet med årligt?
- Hvornår tæller vi fra?
  - De ældste kendte fossiler er dateret til at være omkring 160.000 år gamle. (250.000 år gamle)
- Hvornår er man defineret som menneske?
- Dødsbølge (krige, istider, epidemier, folke udryddelse)
- Hvilke faktorer har indflydelse på ændring i befolkningstilvæksten (Medicin, Klima, Sundhed, Samfund, Lægevidenskaben, Landbrug)
-

https://esa.un.org/unpd/wpp/Graphs/1_Demographic%20Profiles/World/Line%20Charts/Total%20Population%20by%20broad%20age%20group.png




http://www.worldometers.info/da/
http://www.globalis.dk/Statistik/Befolkningstal
https://en.wikipedia.org/wiki/Homo_sapiens#Origin
https://en.wikipedia.org/wiki/World_population#Number_of_humans_who_have_ever_lived

https://data.worldbank.org/indicator/SP.DYN.CBRT.IN?end=2014&start=1960 Fødsler pr. 1000 person

https://ourworldindata.org/fertility/

http://www.prb.org/Publications/Articles/2002/HowManyPeopleHaveEverLivedonEarth.aspx
http://illvid.dk/fortiden/hvor-mange-har-levet-paa-jorden

[redacted]

| Besøgt i dag | 69 emner |
|---|---|
| Logbog d. 8-11-2017 - Google Docs | https://docs.google.com/document/d/10oC...8ehk6L5QRlqMsorZYNhFABrrmfmlnw/edit |
| Jordens historie - Wikipedia, den frie encyklopædi | https://da.wikipedia.org/wiki/Jordens_historie |
| 1 menneske - Google-søgning | https://www.google.dk/search?client=safar...d=cr&dcr=0&ei=T-8CWouaJsGxae6cpNgE |
| (ingen titel) | http://www.dst.dk/Site/Dst/Udgivelser/GetPubFile.aspx?id=4576&sid=bef150 |
| Den Korte Avis \| Verdens befolkning vokser – og den vil vokse meget mere, især i Afrika | https://denkorteavis.dk/2015/verdens-befo...ing-vokser-og-den-vil-vokse-meget-mere/ |
| gennesnitlig tilvækst i befolkning - Google-søgning | https://www.google.dk/search?safe=off&cli...k1j33i22i29i30k1j33i21k1.101.hc4LKxArec8 |
| Hvor mange mennesker har levet? - Google Drev | https://drive.google.com/drive/folders/1s1zSo90P_HgUKQVhJUgm2lixwdguHqew |
| Delt med mig - Google Drev | https://drive.google.com/drive/shared-with-me |
| Hvor stor har Jordens befolkning været? \| Historienet.dk | http://historienet.dk/videnskab/hvor-stor-har-jordens-befolkning-vaeret |
| Hvor mange mennesker har levet? \| Historienet.dk | http://historienet.dk/civilisationer/hvor-mange-mennesker-har-levet |
| Hvor mange har levet på Jorden? \| Illvid.dk | http://illvid.dk/fortiden/hvor-mange-har-levet-paa-jorden |
| Logbog d. 8-11-2017 - Google Docs | https://docs.google.com/document/d/10oC...sorZYNhFABrrmfmlnw/edit?usp=drive_web |
| Logbog p2 - Google Docs | https://docs.google.com/document/d/14ta...lAgts3FvvF0/edit#heading=h.fl4qykxeq85h |
| Logbog p2 - Google Docs | https://docs.google.com/document/d/14ta...cVvyq_ZEL-rDt32ErfS6cwrlAgts3FvvF0/edit |
| Logbog p2 - Google Docs | https://docs.google.com/document/d/14ta...2ErfS6cwrlAgts3FvvF0/edit?usp=drive_web |
| Jobsansøgning Albert - Google Docs | https://docs.google.com/document/d/1mO...uPM6kC9aj1CdtQn9SmgqeVWIjlMHMA/edit |
| Jobsansøgning Albert - Google Docs | https://docs.google.com/document/d/1mO...Qn9SmgqeVWIjlMHMA/edit?usp=drive_web |
| Mit drev - Google Drev | https://drive.google.com/drive/my-drive |
| Google Drev | https://drive.google.com/drive/ |

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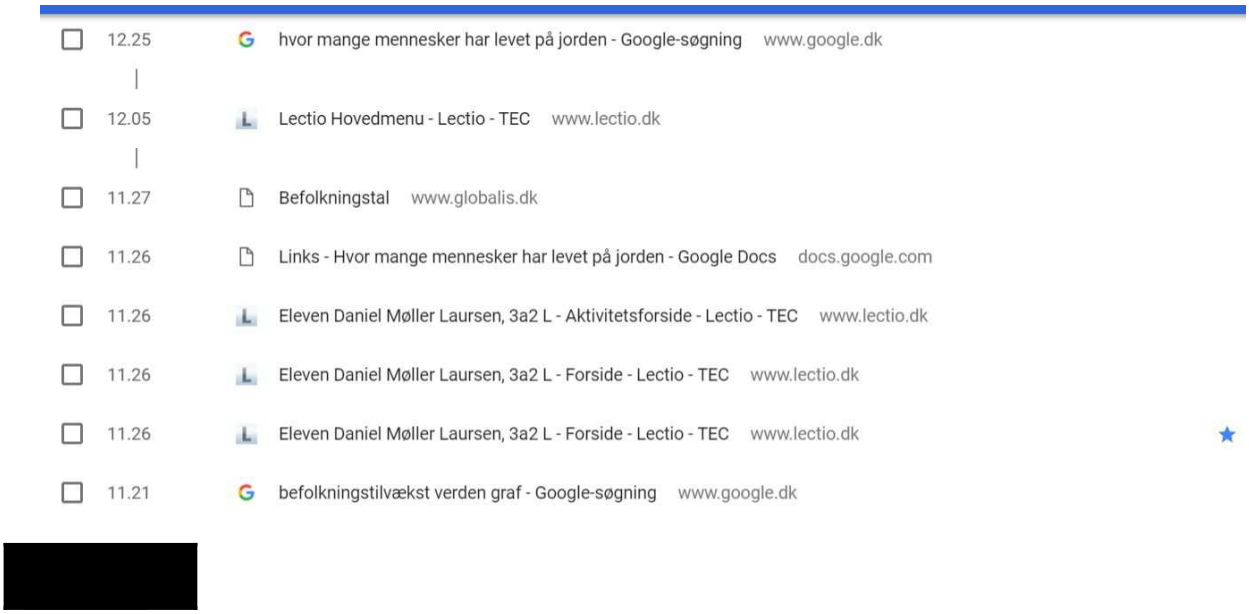
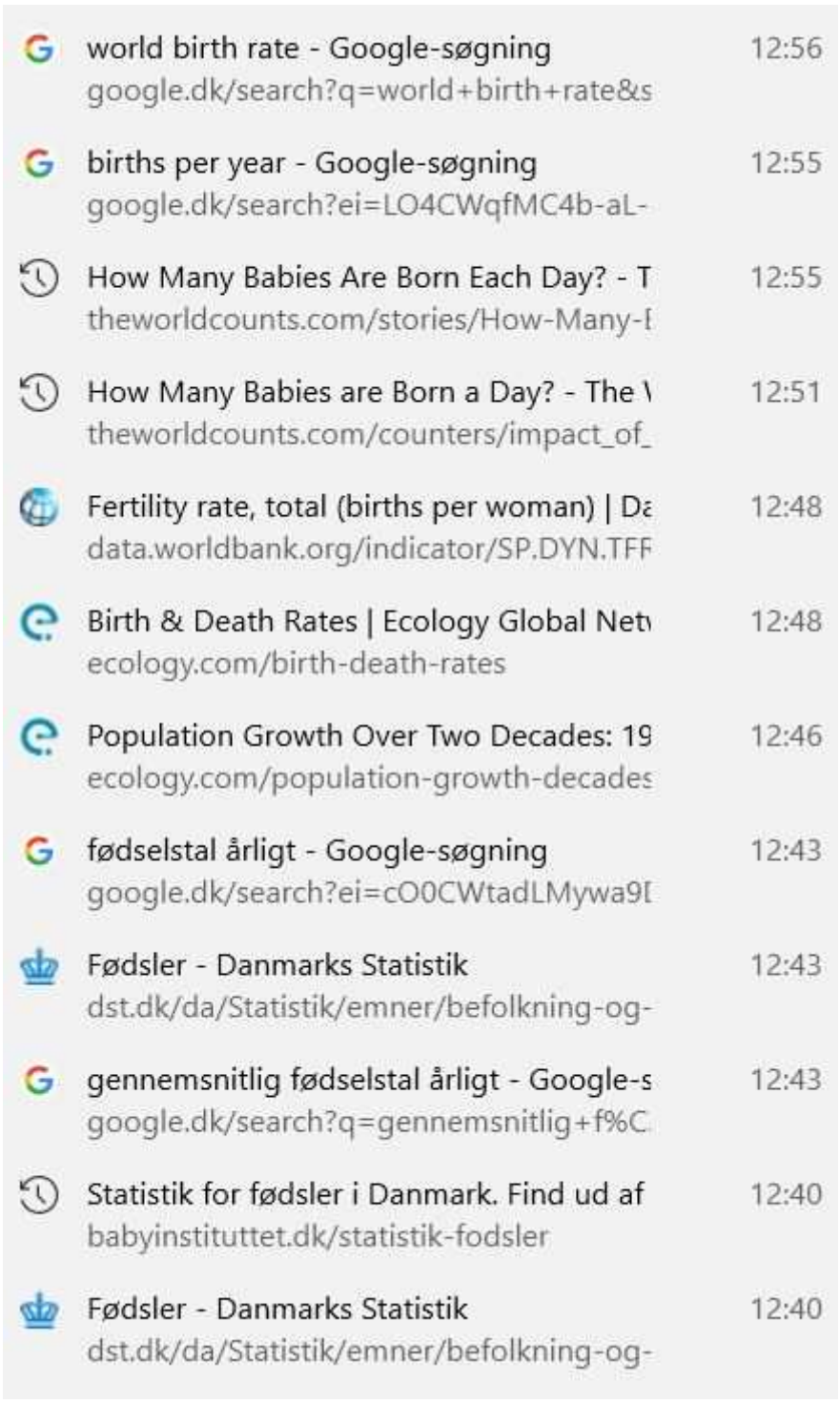
I dag - onsdag den 8. november 2017

| | | | |
|---|---|---|---|
| 13.00 | Logbog d. 8-11-2017 - Google Docs | docs.google.com |
| 12.58 | World - Birth rate - Historical Data Graphs per Year | www.indexmundi.com |
| 12.58 | https://docs.google.com/document/d/1BpvdIfFGG7rq4i6xXh8FUs2A4XVlpI48G0RPZbsuG3k/edit | docs.google.com |
| 12.45 | Fertility - Our World in Data | ourworldindata.org |
| 12.45 | https://www.google.dk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=2&ved=0ahUKEwiw_sfS867X... | www.google.dk |
| 12.45 | Births per. 1000 throughout history - Google-søgning | www.google.dk |
| 12.45 | Births pr. 1000 throughtout history - Google-søgning | www.google.dk |
| 12.33 | How Many People Have Ever Lived on Earth? | www.prb.org |
| 12.33 | https://www.google.dk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&ved=0ahUKEwjp4NL88K7X... | www.google.dk |
| 12.33 | How many people have existed - Google-søgning | www.google.dk |
| 12.33 | How many people have eksisted - Google-søgning | www.google.dk |
| 12.32 | Search | www.prb.org |
| 12.32 | Search | www.prb.org |
| 12.31 | Search | www.prb.org |
| 12.31 | INSIGHTS | www.worldpopdata.org |
| 12.31 | CHART | www.worldpopdata.org |
| 12.31 | TABLE | www.worldpopdata.org |
| 12.26 | Population Reference Bureau in Washington D.C. - Google-søgning | www.google.dk |
| 12.26 | Population Reference Bureau i Washington D.C. - Google-søgning | www.google.dk |
| 12.26 | Hvor mange har levet på Jorden? \| Illvid.dk | illvid.dk |
| 12.26 | 3 geniale idéer holdt istidens folk i live \| Illvid.dk | illvid.dk |


12.25 hvor mange mennesker har levet på jorden - Google-søgning www.google.dk
12.05 Lectio Hovedmenu - Lectio - TEC www.lectio.dk
11.27 Befolkningstal www.globalis.dk
11.26 Links - Hvor mange mennesker har levet på jorden - Google Docs docs.google.com
11.26 Eleven Daniel Møller Laursen, 3a2 L - Aktivitetsforside - Lectio - TEC www.lectio.dk
11.26 Eleven Daniel Møller Laursen, 3a2 L - Forside - Lectio - TEC www.lectio.dk
11.26 Eleven Daniel Møller Laursen, 3a2 L - Forside - Lectio - TEC www.lectio.dk
11.21 befolkningstilvækst verden graf - Google-søgning www.google.dk


world birth rate - Google-søgning
google.dk/search?q=world+birth+rate&s
12:56
births per year - Google-søgning
google.dk/search?ei=LO4CWqfMC4b-aL-
12:55
How Many Babies Are Born Each Day? - T
theworldcounts.com/stories/How-Many-
12:55
How Many Babies are Born a Day? - The
theworldcounts.com/counters/impact_of_
12:51
Fertility rate, total (births per woman) | Da
data.worldbank.org/indicator/SP.DYN.TFF
12:48
Birth & Death Rates | Ecology Global Net
ecology.com/birth-death-rates
12:48
Population Growth Over Two Decades: 19
ecology.com/population-growth-decades
12:46
fødselstal årligt - Google-søgning
google.dk/search?ei=cO0CWtadLMywa9
12:43
Fødsler - Danmarks Statistik
dst.dk/da/Statistik/emner/befolkning-og-
12:43
gennemsnitlig fødselstal årligt - Google-s
google.dk/search?q=gennemsnitlig+f%C
12:43
Statistik for fødsler i Danmark. Find ud af
babyinstituttet.dk/statistik-fodsler
12:40
Fødsler - Danmarks Statistik
dst.dk/da/Statistik/emner/befolkning-og-
12:40
Logbog d. 8-11-2017 - Google Docs
docs.google.com/document/d/10oCKdwe
13:02
Birth rate, crude (per 1,000 people) | Data
data.worldbank.org/indicator/SP.DYN.CBI
13:02
Birth rate - Wikipedia
en.wikipedia.org/wiki/Birth_rate

https://docs.google.com/document/d/10oCKdwePYmmZm8ehk6L5QRIqMsorZYNhFABrrmfmln... docs.google.com

https://docs.google.com/document/d/1BpvdIfFGG7rq4i6xXh8FUs2A4XVlpI48G0RPZbsuG3k/edit docs.google.com

Lectio Hovedmenu - Lectio - TEC www.lectio.dk

https://en.wikipedia.org/wiki/World_population#Number_of_humans_who_have_ever_lived en.wikipedia.org

World population - Wikipedia en.wikipedia.org

Verdens befolkning - Wikipedia, den frie encyklopædi da.wikipedia.org

https://www.google.dk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=3&cad=rja&uact=8&ved=0ahU... www.google.dk

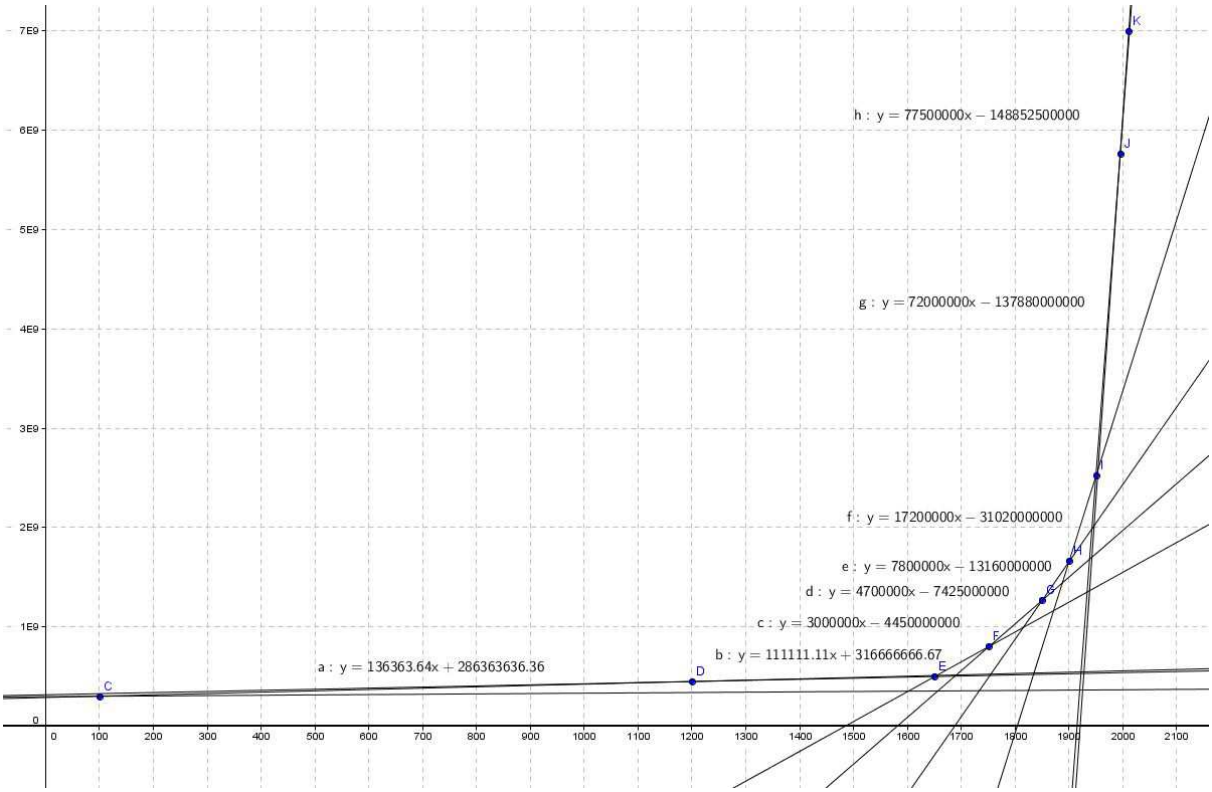
Worldometers - Statistik om verden i realtid www.worldometers.info

hvor mange dør om dagen i verden - Google-søgning www.google.dk

hvor mange dør om året - Google-søgning www.google.dk

Links - Hvor mange mennesker har levet på jorden - Google Docs docs.google.com

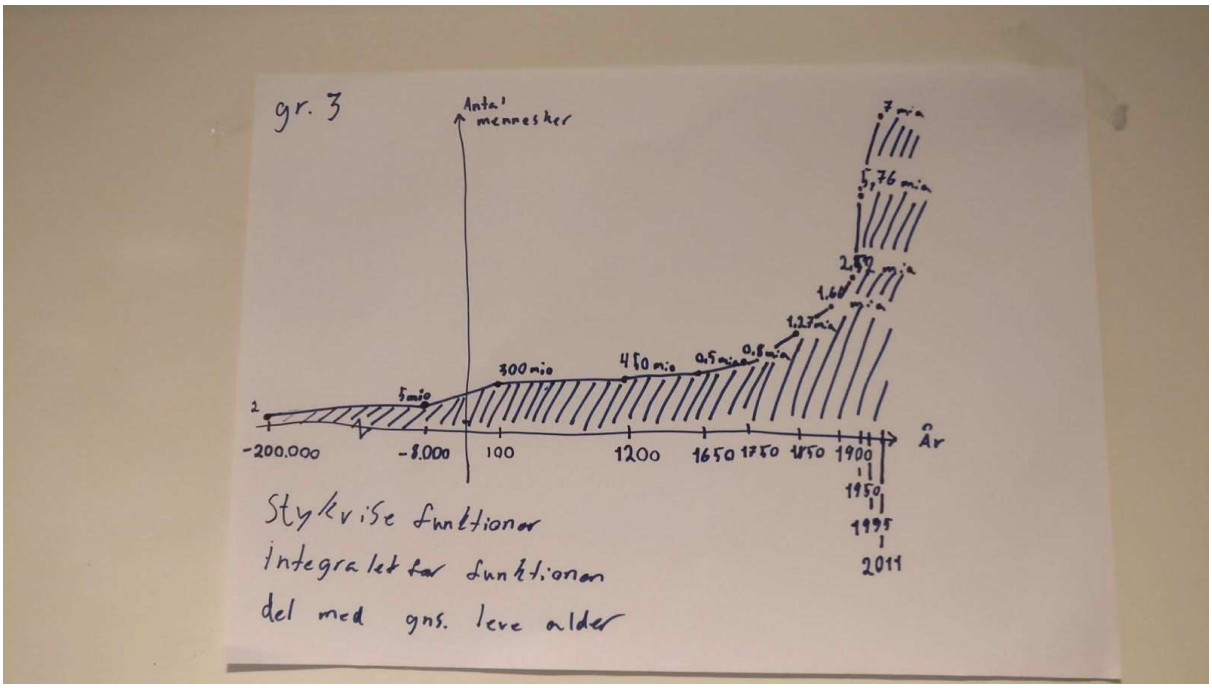
Vi opstiller en stykvis funktion ud fra de data punkter vi fandt via. historienet i går.




Vi lavede den stykvise funktion, og prøvede vha. Maple at samle det til en funktion, dette kunne maple i midlertidig ikke lide…
Planen var at tage integralet for den samlede funktion for at finde antal år der er blevet levet i alt, for derefter at dele integralet med en estimeret gennemsnitlig levealder, hvilket vil give et godt bud på et svar til hvor mange der har levet på jorden.
Da vi ikke kunne finde den samlede funktion kunne man tage integralet af hver enkelt stykvise funktion, og til sidst tage summen af disse, dette ville give samme svar som hvis vi tog integralet for den samlede funktion.

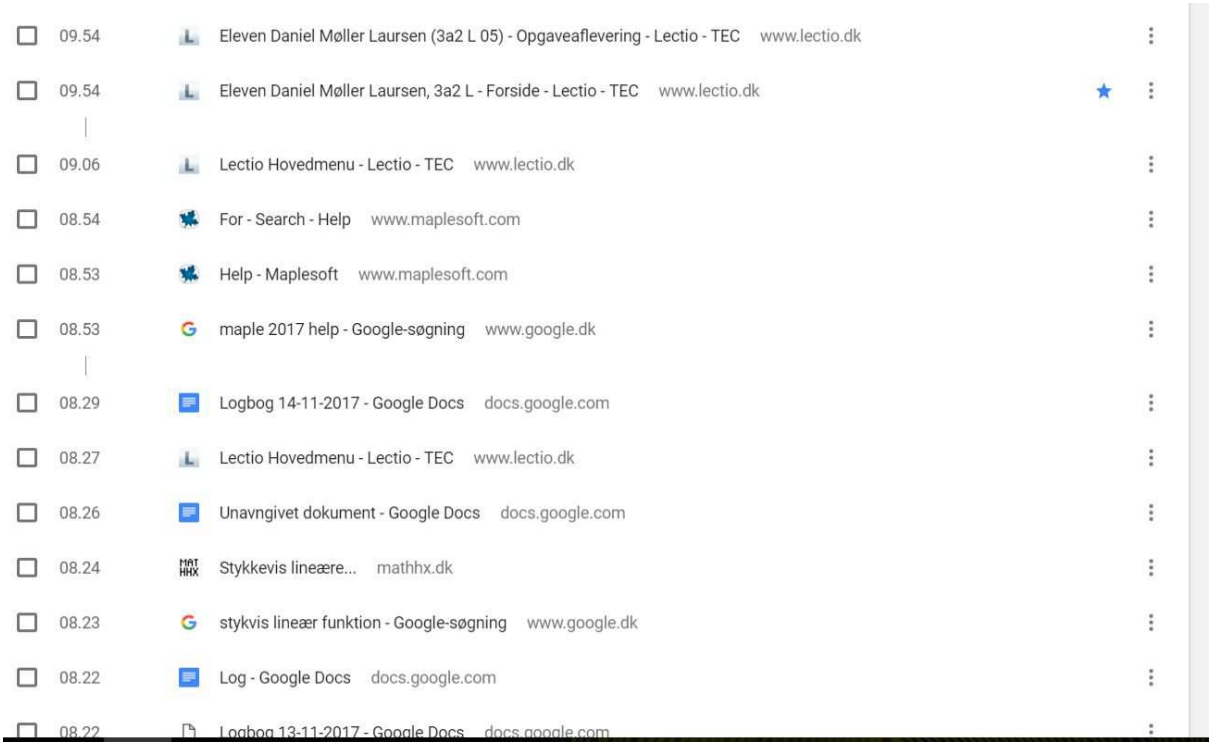
$$h(x) := \begin{bmatrix} 26.04\,x + 5208333.25\ \{-200000 < x < -8000\} \\ 36419.75\,x + 296358024.69\ \{-8000 < x < 100\} \\ 136363.64\,x + 28636363.36\{100 < x < 1200\} \\ 111111.11\,x + 316666666.67\{1200 < x < 1650\} \\ 3000000\,x - 4450000000\{1650 < x < 1750\} \\ 4700000\,x - 7425000000\{1750 < x < 1850\} \\ 7800000\,x - 13160000000\{1850 < x < 1900\} \\ 17200000\,x - 31020000000\{1900 < x < 1950\} \\ 720000000\,x - 137880000000\{1950 < x < 1995\} \\ 77500000\,x - 148852500000\{1995 < x < 2011\} \end{bmatrix}$$

$$h := x \mapsto rtable\left(1..10, \left[26.04\,x + 5.20833325\,10^{6}\ \{-200000 < x < -8000\}, 36419.75\,x + 2.9635802469\,10^{8}\ \{-8000 < x < 100\}, 136363.64\,x + 2.863636336\,10^{7}\ \{100 < x < 1200\}, 111111.11\,x + 3.1666666667\,10^{8}\ \{1200 < x < 1650\}, 3000000\,x - 4450000000\ \{1650 < x < 1750\}, 4700000\,x - 7425000000\ \{1750 < x < 1850\}, 7800000\,x - 13160000000\ \{1850 < x < 1900\}, 17200000\,x - 31020000000\ \{1900 < x < 1950\}, 720000000\,x - 137880000000\ \{1950 < x < 1995\}, 77500000\,x - 148852500000\ \{1995 < x < 2011\}\right], subtype = Vector_{column}\right)$$

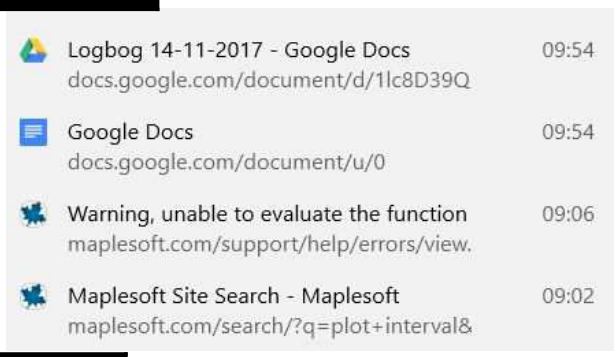

gr. 3
Antal mennesker
2
5 mio
300 mio
450 mio
0.5 mia
0.8 mia
1.27 mia
1.60 mia
2.52 mia
5,76 mia
7 mia
-200.000
-8.000
100
1200
1650
1750
1850
1900
1950
1995
2011
År
Stykvise funktioner
integralet af funktionen
del med gns. leve alder

**Historik:**

[redacted]




[redacted]




[redacted]

https://www.google.dk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=5&cad=rja&uact=8&ved=0ahU... www.google.dk

Population Calculator! www.klimadebat.dk

https://www.google.dk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=3&cad=rja&uact=8&ved=0ahU... www.google.dk

ligning for befolkningsvækst - Google-søgning www.google.dk

World Population Prospects - Population Division - United Nations esa.un.org

World Population Prospects - Population Division - United Nations esa.un.org

Logbog 13-11-2017 - Google Docs docs.google.com

Google Drev drive.google.com

World Population Prospects - Population Division - United Nations esa.un.org

World Population Prospects - Population Division - United Nations esa.un.org

FN- rapporten fra 2013:"World Population Prospects" - Google-søgning www.google.dk

Den Korte Avis | Verdens befolkning vokser voldsomt – det er en bombe under fremtiden denkorteavis.dk

Den Korte Avis | Befolkningstilvægst denkorteavis.dk

## C.4 Logbooks group 4

Gruppe nr.: 4

[redacted]

Logbog (08-11-2017):

Vi blev stillet følgende Hovedspørgsmål:
Hvor mange mennesker har der levet på jorden?

og bedt om at uddybe/udlede nogle delspørgsmål, der skal hjælpe os med at komme igennem forløbet. Spørgsmålene nedenunder er det vores gruppe vil fordybe os i.
Delspørgsmål:
Hvornår beskriver vi os selv som mennesker?
Hvornår begynder befolkningstallet alvorligt at stige?
Hvilken faktor påvirker fødselsraten?
Hvordan kan vi beskrive væksten som en funktion?
Hvornår er den største stigning i fødsler?

Efter udledelse af nogle håndterlige delspørgsmål, påbegyndte vi en informationssøgning og evaluering om de valgte spørgsmål. Efterfølgende begyndte vi at fordybe os i opstillelse af en primitiv funktion over mængden af fødte som funktion af tid, her målt i år.

Vi går ud fra at mennesket har eksisteret fra 200,000 BC og at befolkningen frem til i år 200,000 BC var 0 mio mens befolkningen i år 10,000 BC var 2 mio.
Da der er meget begrænsede informationer om denne tidsperiode har vi valgt at beskrive som et gennemsnit derved, med en befolkningsvækst der går linært fra (-200,000 , 0) til (-10,000 , 2 000,000)

Beregner vektoren (-10,000 - (-200,000) ;  2,000,000 - 0) = (  190,000 ; 2,000,000)

Datapunkter fra 10,000 BC til 1950:

1950 til 2017:

Gruppe nr : 4

[redacted]

## Browserhistorik:

| | | | |
|---|---|---|---|
| 12.25 | Jordens historie - Wikipedia, den frie encyklopædi | da.wikipedia.org | |
| 12.25 | Human evolution - Wikipedia | en.wikipedia.org | |
| 12.25 | Timeline of human evolution - Wikipedia | en.wikipedia.org | |
| 12.25 | (6) Avatar - The Eagle Has Landed - YouTube | www.youtube.com | |
| 12.25 | When Did the First Human Appear on Earth? | www.foundalis.com | |
| 12.25 | Lectio Hovedmenu - Lectio - TEC | www.lectio.dk | |
| 12.20 | YouTube | www.youtube.com | |
| 12.20 | (6) Avatar - Vultures Fly - YouTube | www.youtube.com | |
| 12.19 | Verdenskrig - Wikipedia, den frie encyklopædi | da.wikipedia.org | |
| 12.19 | https://www.google.dk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=7&cad=rja&uact=8&ved=0ahU... | www.google.dk | |
| 12.19 | Hvilket land har været mest i krig? \| Historienet.dk | historienet.dk | |
| 12.19 | https://www.google.dk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=2&cad=rja&uact=8&ved=0ahU... | www.google.dk | |
| 12.19 | Hvilke krige var de værste? \| Videnskab.dk | videnskab.dk | |
| 12.19 | https://www.google.dk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&cad=rja&uact=8&ved=0ahU... | www.google.dk | |
| 12.19 | hvilke krige har der været - Google-søgning | www.google.dk | |
| 12.15 | (6) Avatar - New Land - YouTube | www.youtube.com | |
| 11.23 | Who was the first person on earth? - Quora | www.quora.com | |
| 11.23 | https://www.google.dk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=10&cad=rja&uact=8&ved=0ah... | www.google.dk | |
| 11.23 | https://www.google.dk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=3&cad=rja&uact=8&ved=0ahU... | www.google.dk | |
| 11.23 | 'First human' discovered in Ethiopia - BBC News | www.bbc.com | |
| 11.23 | https://www.google.dk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=2&cad=rja&uact=8&ved=0ahU... | www.google.dk | |
| 11.23 | https://www.google.dk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&cad=rja&uact=8&ved=0ahU... | www.google.dk | |
| 11.23 | what was the first human - Google-søgning | www.google.dk | |
| 11.19 | lucy in the sky with diamonds - Google-søgning | www.google.dk | |
| 11.19 | lucy the first human - Google-søgning | www.google.dk | |
| 11.19 | BBC - Science & Nature - The evolution of man | www.bbc.co.uk | |
| 11.18 | Lucy (fossil) - Wikipedia, den frie encyklopædi | da.wikipedia.org | |
| 11.12 | Menneskets evolution \| Videnskab.dk | videnskab.dk | |
| 11.12 | https://www.google.dk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=4&cad=rja&uact=8&ved=0ahU... | www.google.dk | |
| 11.12 | https://www.google.dk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=3&cad=rja&uact=8&ved=0ahU... | www.google.dk | |
| 11.12 | hvem var det første menneske på jorden - Google-søgning | www.google.dk | |
| 10.44 | hvor mange børn bliver der født i sekundet - Google-søgning | www.google.dk | |

Gruppe nr.: 4

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| | | |
|---|---|---|
| 12.36 | World population estimates - Wikipedia | en.wikipedia.org |
| 12.36 | world population 10000 BC - Google-søgning | www.google.dk |
| 12.36 | world population graph - Google-søgning | www.google.dk |
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| 12.35 | https://www.google.dk/search?tbm=isch&sa=1&ei=COwCWtzWO8qua5O7oOgI&q=ancient+world+... | www.google.dk |
| 12.35 | ancient world population graph - Google-søgning | www.google.dk |
| 12.35 | https://www.google.dk/search?q=world+population+graph&tbm=isch&source=iu&pf=m&ictx=1&fir... | www.google.dk |
| 12.35 | what happened to population rate - Google-søgning | www.google.dk |
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| 12.33 | 1700–1799 (A.D.) World History | www.infoplease.com |
| 11.22 | How long did it take us to evolve from monkeys to humans? - Yttrium Zone | yttriumj12.imascientist.org.uk |
| 11.22 | from monkey to human year periods - Google-søgning | www.google.dk |
| 11.22 | from monkey to human ano - Google-søgning | www.google.dk |
| 11.22 | monkey to human ano - Google-søgning | www.google.dk |
| 11.21 | monkey to human years - Google-søgning | www.google.dk |
| 11.21 | Monkey Age in Human Years Conversion Calculator | www.easycalculation.com |

## C.5 Logbooks group 5

**Logbog**

Spørgsmål vi har stillet

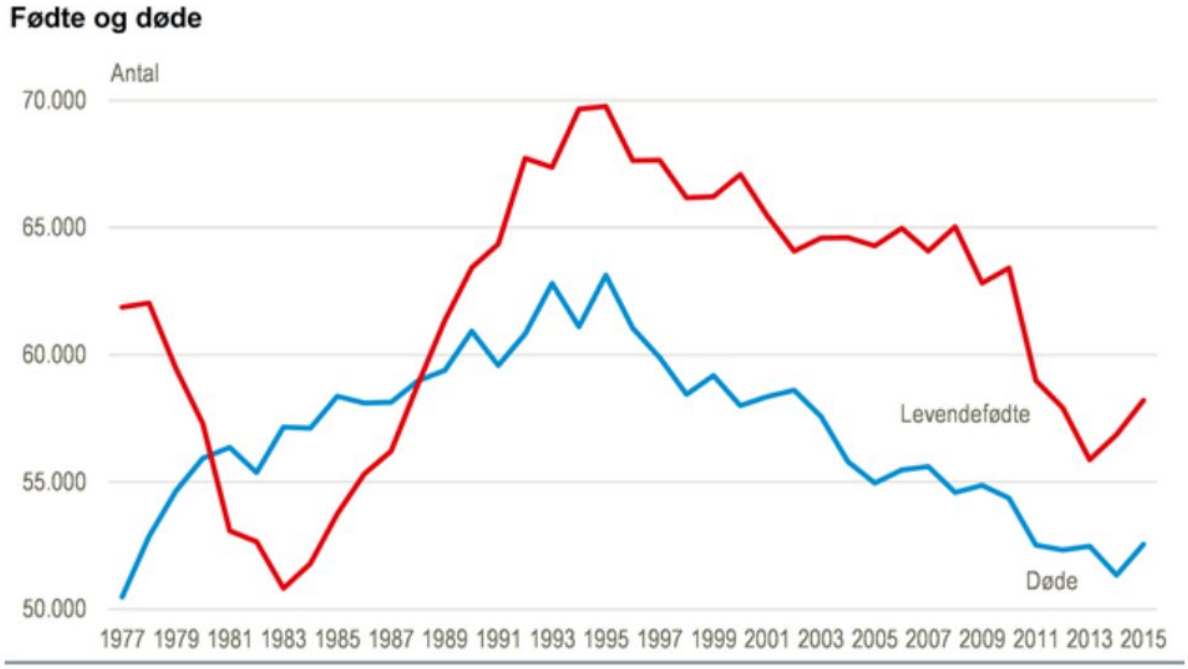
1. Hvad er er befolkningstallet i dag?
2. hvor mange mennesker er født mht. årstal?
3. Hvor mange mennesker er død mht. årstal?
4. Hvordan har krige, epidemier og naturkatastrofer påvirket befolkningstallet?
5. Hvilke parametre har betydning for fødselstallet pr. år
6. Hvorfor er befolkningstallet så højt i det 21. århundrede?

Besvarelser

1. 7,6 milliarder mennesker var i live d. 8/11/2017

http://www.worldometers.info/da/

2. I Danmark ser grafen for døde og fødte ud som:




http://www.dst.dk/da/Statistik/nyt/NytHtml?cid=20642#

3. I dag lever folk i et samfund hvor man kan forsyne sig selv og havde penge i overskud, de familier der så har overskud kan få ekstra børn så deres familie bliver større og det bliver ved, hvilket resulterer i en stigning i befolkningen
4. Mange børn er en nødvendighed for at overleve i fattige lande som Kina og Afrika. Da man skal bruge mange børn som kan hjælpe forældrene.
5. Hvis man ser groft på det vil verdensbefolkningen opføre sig som en andengradsfunktion over tid. Dette er dog uden at tage højde for parametrene.
6. Forhøjet levestandard gør det nemmere at forsyne en stor familie med mange børn.
7. pebermyndige frugter med mange børn og xxtra large mængde kontantus

Integralet af den røde kurve fra svar #2:

$$b(x) := 0\,x^5 + 0.05\,x^4 - 1.07\,x^3 + 9.6\,x^2 - 33.59\,x + 88.01$$

$$b := x \mapsto 88.01 + 0.05\,x^4 - 1.07\,x^3 + 9.6\,x^2 - 33.59\,x$$

$$\int_1^{20} b(x)\,dx$$

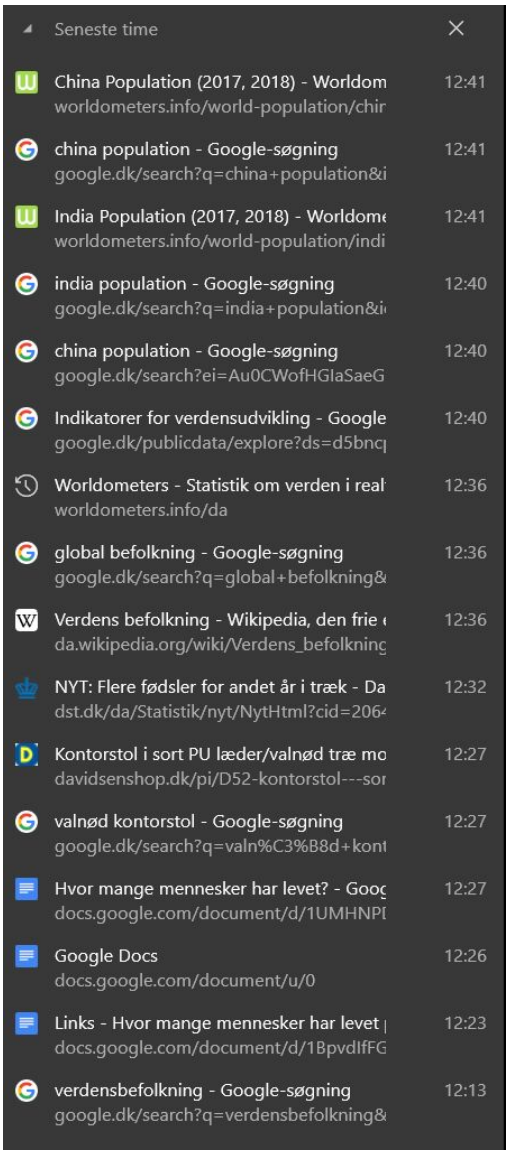
$$9768.042500$$

Altså er der i perioden 1977-2015 blevet født 9,8 millioner babies i Danmark.

Hvor mange mennesker er blevet født?

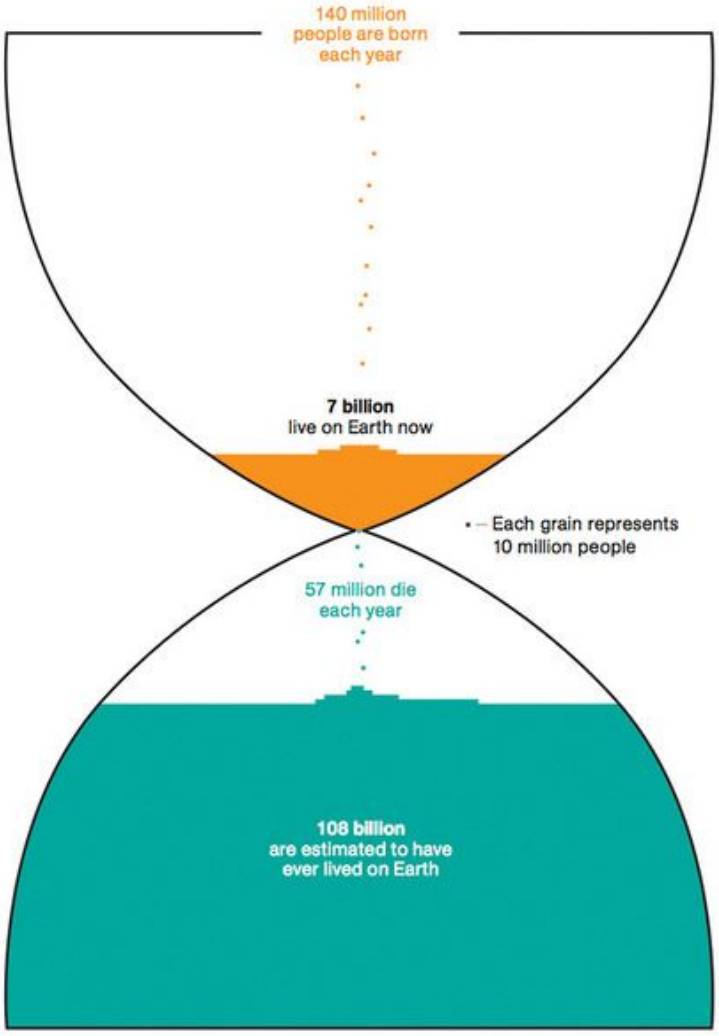
Mellem 50.000 år før kristi fødsel og 2011, blev ca. 108 mia. mennesker født
http://www.prb.org/Publications/Articles/2002/HowManyPeopleHaveEverLivedonEarth.aspx

Historik




## Dag 2:

Ud fra vores research har vi fundet frem til et realistisk estimat om hvor mange mennesker er afgået ved døden i perioden: 50.000 år fk. - 2016. Vi kan så ligge tallet sammen med verdenspopulationen i 2016 for at finde et tal der beskriver hvor mange mennesker der har levet i den periode. Vi arbejder med årstallet 2016 i stedet for 2017, da det er lidt nemmere at finde information til.




http://www.likecool.com/he_Hourglass_Of_Humans_Currently_Alive_And_That_Have_Ever--Pic--Gear.html
The Hourglass

http://worldpopulationhistory.org/map/1283/mercator/1/0/25/
History of world population

## C.6 Logbooks group 6

**Logbog d. 8/11 - 2017**

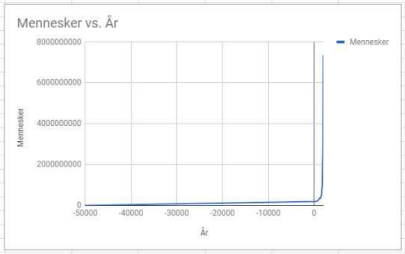
Vi er blevet præsenteret for projektet og har diskuteret overspørgsmålet og hvilke underspørgsmål vi vurderer nødvendige.

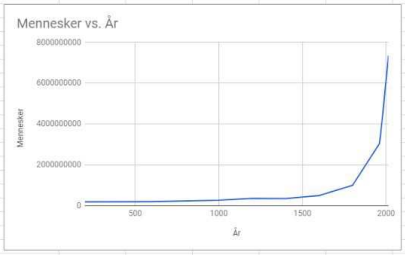
Vi er kommet frem til at disse faktorer har betydning for vores undersøgelse:

- Hvornår blev vi mennesker?
    - Ældste homosapien fund.
- Hvordan vokser populationen?
- Funktionsform
- Målepunkter/værdier
- Dødsbølger?
    - Istid
    - Genghis Khan
    - Epidemier
    - Verdenskrige
- Teknologi der forøger befolkningstilvæksten
    - Vaccinen
    - At vaske hænder…
    - Ploven?
    - Nålen
    - Kloak
- Vi ser bort fra børnedøde, således at vi først kategoriserer folk som mennesker når de kan videregive deres gener..

Vi har besluttet at vores graf starter i år 0. Vi har valgt dette år, da det giver en flot graf og befolkningstilvæksten var meget lav op til sen middelalder.

| -50000 | 200 | 600 | 1000 | 1200 | 1400 | 1600 | 1800 | 1960 | 1961 | 1962 | 1963 | 1964 | 1965 | 1966 | 1967 | 1968 | 1969 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 190000000 | 200000000 | 275000000 | 360000000 | 350000000 | 500000000 | 1000000000 | 3034193297 | 3075115342 | 3127961482 | 3192794384 | 3258201476 | 3324951621 | 3394864530 | 3464439525 | 3534821115 | 3609383725 |
| | | | | | | | | 31,81119438 | 31,09156978 | 34,94055345 | 36,1120291 | 35,00333391 | 34,38744792 | 33,35628334 | 33,08843046 | 33,23488845 | 32,72381201 |
| | | | | Den sårte død | | | | I årene op til 1960 har krig haft en betydelig indflydelse på populationen og levealderen | | | | | | | | | |
| | | | | Genghis Khan | | | | | | | | | | | | | |
| 40 | 40 | 40 | 40 | 40 | 40 | 40 | 40 | 70 | 70,196 | 70,392 | 70,588 | 70,784 | 70,98 | 71,176 | 71,372 | 71,568 | 71,764 |
| 2 | 189999998 | 10000000 | 75000000 | 85000000 | -10000000 | 150000000 | 500000000 | 2034193297 | 40922045 | 52846140 | 64832902 | 65407092 | 66750145 | 69912909 | 69574995 | 70381590 | 74562610 |
| 2 | 7600000000 | 5000000 | 6875000 | 9000000 | 8750000 | 12500000 | 25000000 | 43345618,53 | 43807558,01 | 44436320,63 | 45231404,54 | 46030197,16 | 46843499,87 | 47696759,16 | 48540597,5 | 49391084,21 | 50295185,96 |

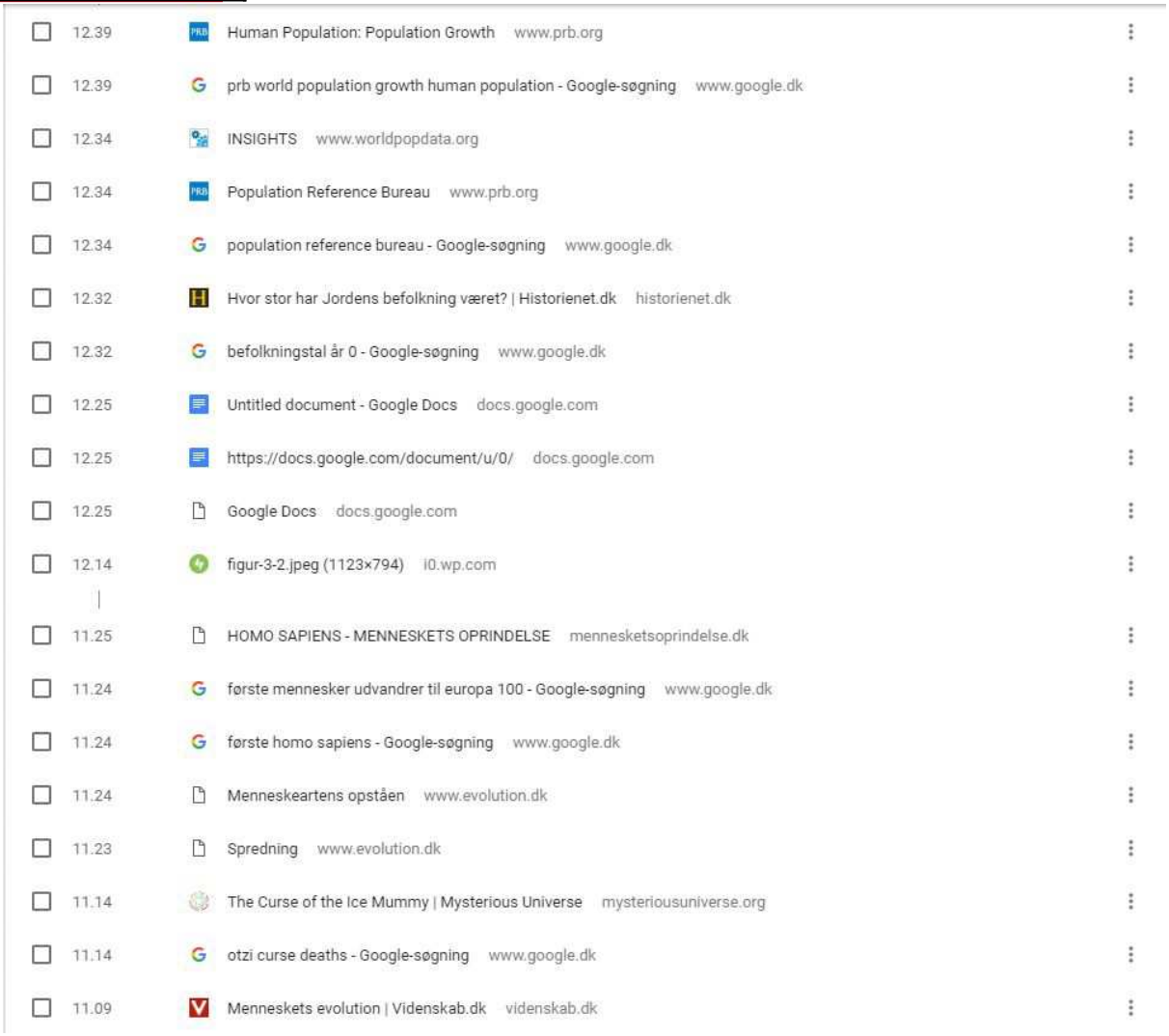



[redacted]

| | | |
|---|---|---|
| 12.59 | Lectio Hovedmenu - Lectio - TEC | www.lectio.dk |
| 12.59 | https://docs.google.com/document/d/1KYnxTiY9SJxId0KGf4hAOBM880C71xMwx7IoX_0NAbo/... | docs.google.com |
| 12.52 | https://docs.google.com/document/d/1BpvdIfFGG7rq4i6xXh8FUs2A4XVlpI48G0RPZbsuG3k/edit | docs.google.com |
| 12.27 | Mennesker på Jorden - Logbog - Google Docs | docs.google.com |
| 12.24 | Google Drev | drive.google.com |
| 12.20 | Eleven Gustav Skriver Frydensberg, 3a2 L - Aktivitetsforside - Lectio - TEC | www.lectio.dk |
| 12.16 | Timeline of human evolution - Wikipedia | en.wikipedia.org |
| 12.16 | timeline of human evolution - Google-søgning | www.google.dk |
| 11.50 | YouTube | www.youtube.com |
| 11.26 | Why Are We The Only Humans Left? - YouTube | www.youtube.com |
| 11.23 | There Was No First Human - YouTube | www.youtube.com |
| 11.23 | first humans on earth - YouTube | www.youtube.com |
| 11.23 | første mennesker - YouTube | www.youtube.com |
| 11.23 | YouTube | www.youtube.com |
| 11.20 | Fund af 300.000 år gamle knogler kan ændre historien om mennesket - TV 2 | nyheder.tv2.dk |
| 11.20 | homo sapiens kranium - Google-søgning | www.google.dk |
| 11.19 | homo sapiens - Google-søgning | www.google.dk |
| 11.16 | eva - Google-søgning | www.google.dk |
| 11.14 | eva fra adam og eva - Google-søgning | www.google.dk |
| 11.12 | Ötzi - Wikipedia, den frie encyklopædi | da.wikipedia.org |
| 11.12 | hvad hedder det ældste menneske der er fundet - Google-søgning | www.google.dk |

[redacted]



| | | | |
|---|---|---|---|
| 12.39 | Human Population: Population Growth | www.prb.org | |
| 12.39 | prb world population growth human population - Google-søgning | www.google.dk | |
| 12.34 | INSIGHTS | www.worldpopdata.org | |
| 12.34 | Population Reference Bureau | www.prb.org | |
| 12.34 | population reference bureau - Google-søgning | www.google.dk | |
| 12.32 | Hvor stor har Jordens befolkning været? \| Historienet.dk | historienet.dk | |
| 12.32 | befolkningstal år 0 - Google-søgning | www.google.dk | |
| 12.25 | Untitled document - Google Docs | docs.google.com | |
| 12.25 | https://docs.google.com/document/u/0/ | docs.google.com | |
| 12.25 | Google Docs | docs.google.com | |
| 12.14 | figur-3-2.jpeg (1123×794) | i0.wp.com | |
| 11.25 | HOMO SAPIENS - MENNESKETS OPRINDELSE | mennesketsoprindelse.dk | |
| 11.24 | første mennesker udvandrer til europa 100 - Google-søgning | www.google.dk | |
| 11.24 | første homo sapiens - Google-søgning | www.google.dk | |
| 11.24 | Menneskeartens opståen | www.evolution.dk | |
| 11.23 | Spredning | www.evolution.dk | |
| 11.14 | The Curse of the Ice Mummy \| Mysterious Universe | mysteriousuniverse.org | |
| 11.14 | otzi curse deaths - Google-søgning | www.google.dk | |
| 11.09 | Menneskets evolution \| Videnskab.dk | videnskab.dk | |

Dag 2.
Idag har vi primært arbejdet med data i et andet dokument, således at vi har noget, som vi kan bygge ud fra imorgen. Billeder der er lagt op er vores midlertidige grafer, baseret på vores data.

# Appendix D

# Four exam sets from the period 2015-2017

UNDERVISNINGS
MINISTERIET
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UNDERVISNING OG KVALITET

# Matematik A

## Højere teknisk eksamen

htx171-MAT/A-24052017

Onsdag den 24. maj 2017
kl. 9.00 - 14.00

# Opgave 5

En funktion er givet ved

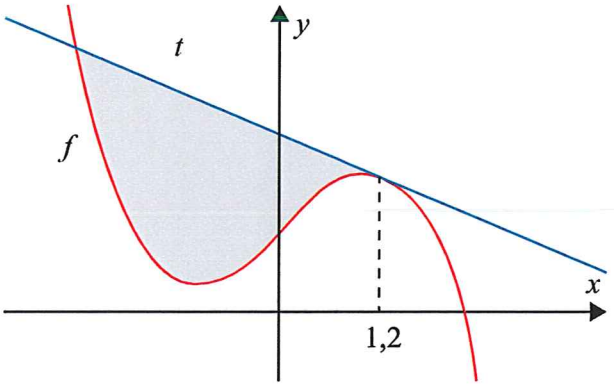
$$f(x) = -\frac{1}{3}x^3 + x + 1$$

a) Bestem monotoniforholdene for $f$.

For $x = 1{,}2$ har grafen for $f$ en tangent, $t$.

b) Vis at ligningen for $t$ er givet ved $y = -0{,}44x + 2{,}152$.

Det tonede område på figur 3 er afgrænset af $t$ og grafen for $f$.




Figur 3

c) Bestem arealet af det tonede område.

Ved hjælp af nedenstående beregninger kan man vise, at tangenterne til grafen for $f$ i $x_0$ og $-x_0$ altid vil være parallelle.

(1) $f(x) = -\frac{1}{3}x^3 + x + 1$ Forskriften for $f$ opskrives.

(2) $f'(x) = -3 \cdot \frac{1}{3}x^2 + 1 + 0$ Funktionen $f$ differentieres vha. reglerne …

(3) $f'(x) = -x^2 + 1$

(4) $f'(x_0) = -x_0{}^2 + 1$ og
$f'(-x_0) = -x_0{}^2 + 1$

d) Færdiggør forklaringen til trin (2) og redegør for trinene (3) og (4) samt argumenter for, at tangenterne i $x_0$ og $-x_0$ altid vil være parallelle.

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# Matematik A

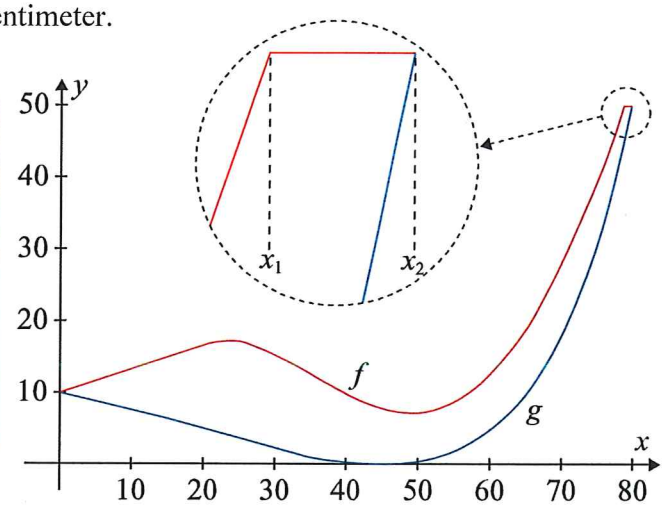
## Højere teknisk eksamen

htx162-MAT/A-26082016

Fredag den 26. august 2016
kl. 9.00 - 14.00

# Opgave 6

Billedet viser en strandstol. Fra siden kan man se stolens profil. På figur 6 er stolens profil indlagt i et koordinatsystem. Alle mål er i centimeter.




Figur 6

Strandstolens profil er afgrænset af følgende to funktioner

$$f(x)=\begin{cases}0,33x+10 & 0\le x<20\\ -0,00001x^4+0,0026x^3-0,20x^2+5,58x-34,2 & 20\le x<x_1\\ 50 & x_1\le x\le x_2\end{cases}$$

$$g(x)=\frac{-0,0088x^3+0,25x^2+31x-1100}{x-110} \qquad 0\le x\le x_2$$

Funktionen $f$ er kontinuert. Graferne for $f$ og $g$ skærer hinanden, når $x=0$ og $x=x_2$.

a) Bestem værdierne for $x_1$ og $x_2$.

b) Bestem koordinaterne til det punkt på grafen for $f$, hvor tangenten danner en vinkel på $45°$ med $x$-aksen.

c) Bestem arealet af strandstolens profil.

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# Matematik A

## Højere teknisk eksamen

Onsdag den 14. december 2016
kl. 9.00 - 14.00

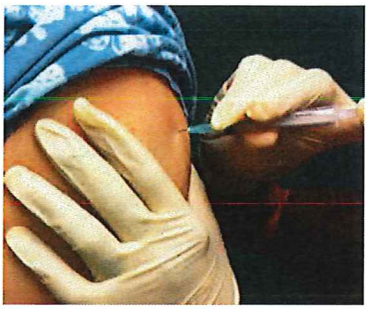
htx163-MAT/A-14122016

c) Vis, at punktet $P$ har koordinaterne $P(2,27;\ 0;\ 4,9)$.

d) Bestem afstanden fra $P$ til $\alpha$.

# Opgave 2

En patient får en indsprøjtning med et smertestillende præparat. Efterfølgende måles koncentrationen af præparatet i blodet. Koncentrationen aftager eksponentielt med tiden. I tabel 1 ses sammenhørende værdier af tiden og koncentrationen.
Tiden måles i timer og koncentrationen i mg/L.



https://www.flickr.com/photos/blakespot/4919795171

| Tiden (timer) | 0,5 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Koncentration (mg/L) | 2,5 | 2,1 | 1,5 | 1,0 | 0,74 | 0,50 |

Tabel 1

a) Indtegn data i et passende koordinatsystem.

b) Bestem en forskrift for funktionen $c$, der beskriver koncentrationen som funktion af tiden.

c) Bestem halveringstiden for $c$.

Når koncentrationen er faldet til 0,3 mg/L vil præparatet ikke længere virke smertestillende.

d) Bestem det tidspunkt efter indsprøjtningen, hvor præparatet ikke længere virker smertestillende.

# Opgave 6

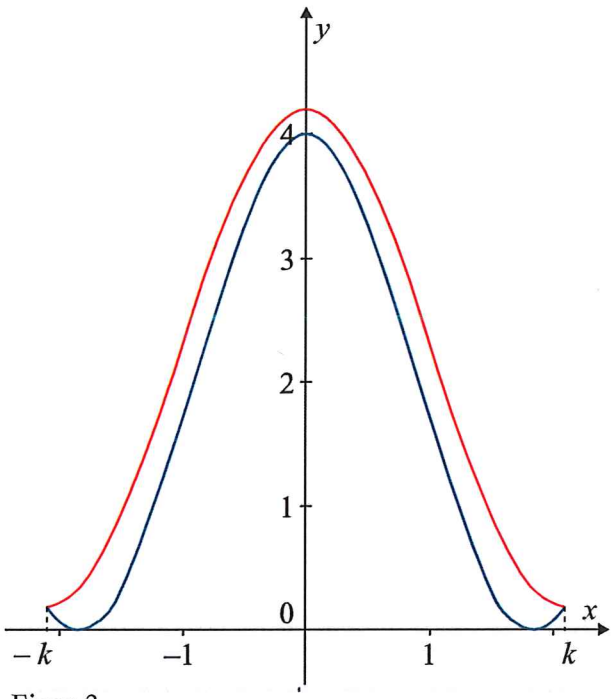
Der findes mange forskellige typer af klokker. På billederne ses nogle, som bruges til at pynte med til jul.

*http://www.jamerantik.dk/*

*http://vik-elektrooginterior.no/*


*http://www.atjemsland.no/*

*http://porcelaensbutikken.dk/*




Figur 3

På figur 3 er et tværsnit af en klokke indtegnet i et koordinatsystem, hvor målene er i centimeter. Tværsnittet af klokken er afgrænset af graferne for de to funktioner

$$f(x) = 2\cos(1,5x) + 2,2 \qquad -k \le x \le k$$

$$g(x) = 2\cos(1,7x) + 2 \qquad -k \le x \le k$$

a) Vis, at bredden af klokken er 4,23 cm.

b) Bestem den største tangenthældning for funktionen $f$.

Klokken skal fremstilles i messing. Volumen af klokken kan beskrives som et omdrejningslegeme omkring $y$-aksen.

c) Bestem volumen af den messing, der skal anvendes til fremstilling af klokken.

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KVALITETS- OG
TILSYNSSTYRELSEN

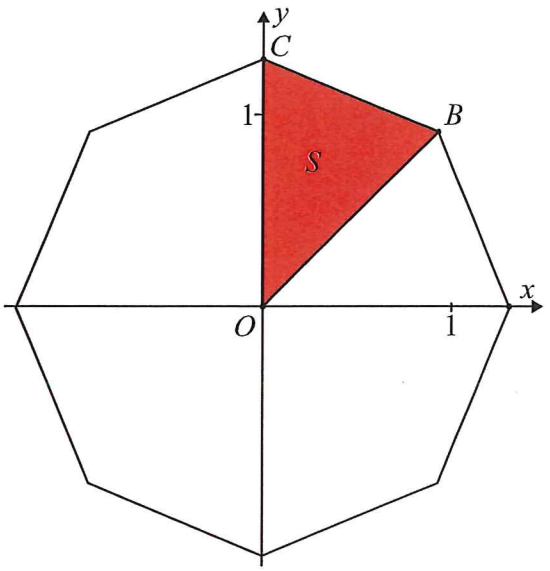
# Matematik A

## Højere teknisk eksamen

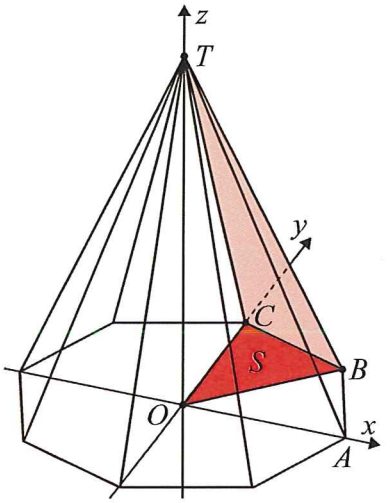
htx151-MAT/A-29052015

Fredag den 29. maj 2015
kl. 9.00 - 14.00

# Opgave 2




Figur 3




Figur 4

Figur 3 og figur 4 viser den samme pyramide som i opgave 1. Her er trekant $OBC$ markeret med rødt på figur 3 og figur 4. Desuden er sidefladen $BCT$ tonet med rødt på figur 4.

Planen $\alpha$, der indeholder punkterne $B$, $C$ og $T$, har tilnærmelsesvist ligningen

$$1{,}148x+2{,}772y+1{,}207z=3{,}621$$

a) Opskriv forskriften for funktionen $f$ af de to variable $x$ og $y$, hvis graf er planen $\alpha$.

Grafområdet $S$ udgøres af trekanten $OBC$, som er vist på både figur 3 og figur 4.

b) Gør rede for, at grafområdet $S$ er afgrænset af de to linjer $y=x$ og $y=-0{,}41x+1{,}31$.

c) Opstil planintegralet, der kan beskrive det volumen, som ligger under planen $\alpha$ og over grafområdet $S$.

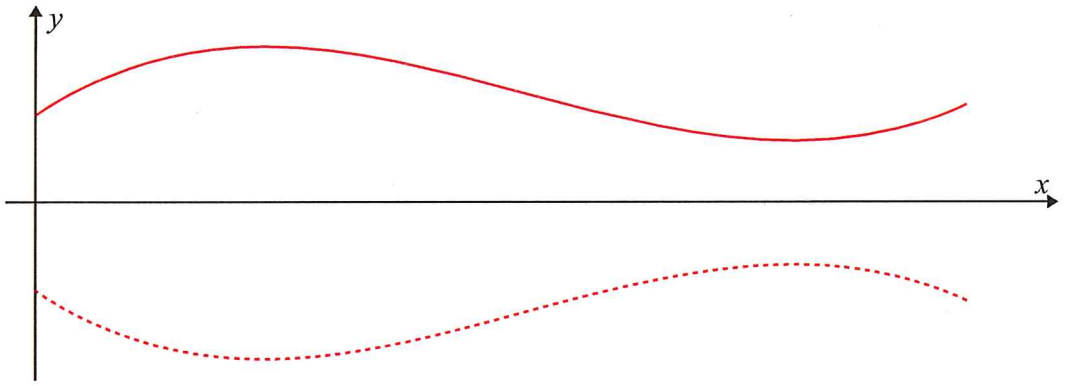
d) Bestem volumen af pyramiden.

# Opgave 4

På billedet ses en vandkaraffel med prop. Indlægges tværsnittet af vandkaraflen i et koordinatsystem, kan den indvendige radius tilnærmelsesvist beskrives ved følgende funktion

$$r(x) = 0,0014x^3 - 0,062x^2 + 0,65x + 2,64 \qquad x \in [0;\ 28]$$

Grafen for funktionen er afbildet på figur 6. Her angiver $x$ højden over karaflens bund. Alle mål er i centimeter.


Figur 6

Karaflen fyldes med vand til højden 15 cm.

a) Bestem volumen af vandet i karaflen.

b) Bestem karaflens største indvendige diameter.

# Opgave 5

I Politiken den 10. oktober 2011 kunne man læse, at antallet af personer i Danmark, som misligholder SU-gæld, stiger. At misligholde sin gæld betyder, at man ikke betaler sin gæld tilbage som aftalt.


I tabellen ses udviklingen af antal personer med misligholdt gæld i årene 2003 – 2010.

| År efter 2000 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|
| Antal personer | 39100 | 40800 | 43000 | 45900 | 48700 | 51600 | 54700 | 57800 |

a) Indtegn data i et koordinatsystem.

b) Opstil en lineær model af formen $N(t) = \alpha \cdot t + \beta$, der beskriver antallet af personer med misligholdt SU-gæld, hvor $t$ er antal år efter år 2000.

c) Opstil en eksponentiel model af formen $M(t) = b \cdot a^t$, der beskriver antallet af personer med misligholdt SU-gæld, hvor $t$ er antal år efter år 2000.

d) Vurder, hvilken model, der bedst beskriver antallet af personer med misligholdt SU-gæld i den givne periode.

# Appendix E

# Lesson plans

[redacted] rolle generelt: Skal ikke agere en som kender et facit, men skal blot sørge for at eleverne har dynamik mellem at stille spørgsmål og finde svar, og mellem at studere ressourcer og ræsonnnere med den viden de har/får.

**1.+2. time**

| Tid | Aktivitet |
|---|---|
| 5 min | Introduktion til forløb (slide 1)<br>Fortælle om forløb og om dataindsamling (Lasse):<br>Lydoptagelse i grupper<br>Føre logbog og nedskrive spørgsmål undervejs<br>Sende screenshot af webhistorik sidst i timen |
| 7 min | Appetizer til demografi (slide 2)<br><br>Introducerende snak:<br>- Kort om demografi: studie af menneskelige populationer. Befolkningsbeskrivelse samt beskrivelse af ændringer.<br>- Hvor mange mennesker er der på jorden i dag?<br>- Tror i det tal er voksende eller aftagende?<br>Åbn link fra slide.<br>- Hvorfor? Hvad afhænger det af?<br>- Hvordan kan man finde tal på det?<br>- Hvordan tror i det fortsætter? |
| 10 min | Introduktion af forløbets rammer/regler/principper<br>Spørgsmålet som de 5 lektioner skal handle om introduceres (slide 3):<br>Q: Hvor mange mennesker har levet på jorden?<br>- Svaret skal komme med en uddybet begrundelse. Derfor bør man også komme ind på hvilken indflydelse forskellige antagelser man gør har på resultatet.<br>Sikre at elever forstår spørgsmålet: Alle der nogensinde har levet, fra de første til dem som lever i dag. Ikke som "world-clock".<br><br>Arbejdsmetode (slide 4): Som forskere! Forskelle ligheder:<br>- Udforsk spørgsmålet ved at stille egne spørgsmål som kan hjælpe med at svare på det. Udforsk disse afledte spørgsmål. Hele tiden generere spørgsmål som ved besvarelse kan bidrage med viden der skal bruges til at svare på Hovedspørgsmålet ($Q_0$).<br>- Søg litteratur på området som kan anvendes til at skabe svar til det oprindelige og de opståede spørgsmål.<br>Forskere: Særligt videnskabelige artikler…<br>3a2: Videnskabelige artikler, tidsskrifter, lærebøger, matematikbøger…<br>- Konferencer: dele og formidle resultaterne med hinanden mundtligt.<br>- 4 konferencer undervejs, forskellig varighed, den sidste er en afsluttende hvor i præsenterer jeres endelige resultater på overskuelig vis. |

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| | - Aflevering: Formidle resultaterne skriftligt**.**<br>(slide 5) Aflevering<br>Giv eleverne et **handout** m.:<br>- Hovedspørgsmålet<br>- Formulering af afleveringsopgaven (poster + forklaring).<br>- Reminder om at føre logbog og nedskrive spørgsmål.<br>- Guide til at gemme og sende webhistorik |
| 3 min | Gruppeinddeling: 6 grupper af 4 elever = 24 elever.<br>Inddeles som de sidder ved bordene.<br>Husk at de skal kende deres gruppenummer!<br>Klargøring af lydoptagelse (Lasse). |
| 5 min<br>Eller til alle grupper er klar med 2 spgm. | <u>Indledende udforskning af spørgsmålet (slide 6)</u><br>Uden brug af media (internet, bøger…), skal eleverne i deres grupper undersøge spørgsmålet.<br><br>De skal finde ud af hvad de skal vide for at kunne svare på det, og skrive **de 2 vigtigste skal skrives** på kæmpe-post-it som gives hver gruppe.<br><br>Tavle inddeles i 6 felter som post-it kan sættes på efterfølgende.<br><br>Andre spørgsmål som opstår i udforskning af Q skal **eksplicit skrives ned, så de kan inkluderes i logbogen.**<br><br>Hedinns rolle: Skal ikke agere en som kender et facit, men skal blot sørge for at eleverne har dynamik med at stille spørgsmål.<br><br>Fortæl: Dette følges op med en konference, hvor de præsenterer deres tanker omkring deres 2 vigtigste spørgsmål. |
| Op til 10 min | <u>Konference</u><br>1 post-it fra hver gruppe sættes på gruppens felt på tavlen.<br><br>1 person fra hver gruppe præsenterer deres arbejde: Deler deres afledte spørgsmål og evt. svar.<br><br>Der skabes enighed om hvad der er ”gode” spørgsmål at gå videre med og hvad der ikke er ved efterfølgende at iagttage hvilke spørgsmål som er ”ens”/minder om hinanden.<br><br>Ting som er gode at arbejde videre med:<br>*Definition af menneske og hvor længe de så har været på jorden*<br>*Fødselstal/rater på forskellige tidspunkter*<br>*Befolkningstal på forskellige tidspunkter*<br>*Overvejelser om interpolation af de tal*<br>*Overvejelser om at gå fra befolkningstal for tidspunkter til antal født over perioden.* |
| Ca. 20 min (indtil pause | <u>Reel udforskning af Q m. alle media.</u><br>Opsøge litteratur og sætte sig ind i det.<br>Arbejde mod et svar på Q.<br>Fokus: nye spørgsmål… stille opfølgende spørgsmål. |

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| n) | Fortæl: De skal (i sidste halve time af næste lektion) præsentere (vise og fortælle om) **de 2 bedste websider de har fundet, + 2-3 afledte spørgsmål.** |
| 40 min | Spisepause |
| 40 min | Fortsat: Reel udforskning af Q m. alle media.<br>Opsøge litteratur og sætte sig ind i det. Forsøge at anvende det de finder til at svare på spørgsmål de har stillet.<br>Evt.: Hvis de ikke finder noget eller blot søger danske resultater, kan vi generere $Q_0$ på engelsk: "How many people have ever lived".<br>Hvis de mener at have et færdigt velbegrundet svar:<br>Arbejde for at forbedre deres model, præcisere.<br>Vurdere følsomhed og forskellige parametres indflydelse.<br>Uddybe argumenter: teori som ligger bag. |
| 20 min | Konference<br><br>6 felter på tavlen. Elever skriver deres 2 bedste websider og 2-3 afledte spørgsmål.<br>Websider noteres blot med hoveddel af URL og overskrifter<br><br>Hver gruppe præsenterer kort deres websider (Hvad indeholder de, og hvad kan de bruges til/give svar på) og spørgsmål (Hvad kan de bidrage med) og de svar de har fundet.<br><br>Links med kort beskrivelse lægges ind i fælles google document: https://docs.google.com/document/d/1BpvdIfFGG7rq4i6xXh8FUs2A4XVlpI48G0RPZbsuG3k/edit?usp=sharing<br><br>I nødstilfælde: Hvis de ikke har fundet noget brugbart så sig: Hedinn og Lasse har i vores undersøgelse fundet … https://www.quora.com/How-many-human-beings-have-ever-lived |
| 1 min | Opfordring: Sende deres spørgsmål og deres webhistorik |
| | Lektie til næste gang:<br>Studere den fundne litteratur. |

**3. time**

| Tid | Aktivitet |
|---|---|
| 3 min | Opfriskning af hvad vi nåede sidst. |
| 32 min | Fortsat arbejde: Studie af det relevante media i grupperne mhp. at besvare spørgsmål.<br>Elever skal forsøge forstå media, og forstå hvordan det kan bruges til at besvare deres spørgsmål.<br>Fortæl: til næste konference skal de opstille et regnestykke baseret på data, der begrunder et svar til hovedspørgsmålet.<br><br>Husk noter spørgsmål som opstår undervejs – før logbog.<br>Til sidst skal i sende webhistorik. |

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| 25 min | Konference: præsentation af hvad de har fundet ud af.<br><br>Hver gruppe præsenterer kort et regnestykke baseret på data, som begrunder et svar.<br>Præsenter ud fra hvad de har i logbogen.<br>Det forventes ikke et finpudset færdigt arbejde, men blot som de måtte have det i deres logbog. |
| 1 min | Opfordring: Sende deres spørgsmål og deres webhistorik |

4.+5. lektion

| **Tid** | **Aktivitet** |
|---|---|
| **75 min** | Arbejde færdig med argumenterne for en formel for svaret.<br>Forberede en præsentation af deres svar.<br>Lave poster til præsentationen<br>Der forventes en grundig velforberedt præsentation af deres arbejde. |
| **45 min** | Præsentationer af deres resultater.<br><br>Poster-sessions (4 stk):<br>Hver gruppe har en poster og der er ved hver session 1 fra gruppen som præsenterer den, mens resten af gruppen ser præsentationer af andre posters. |
| **1 min** | Opfordring: Sende deres spørgsmål og deres webhistorik. |